Effects of Demand Share Patterns and Allocative Disturbances on Okun's Law and the Beveridge Curves

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Abstract

We extend the simple stochastic multi-sector model in Aoki (2002, Sec. 8.6) by incorporating a scheme for hiring and firing employees, and calculate the coefficient of the Okun's law, and show by simulations that the loci of the Beveridge curves move with GDP.

When large fraction of demand is allocated to some set of less productive sectors, the sizes of these sectors increase to meet demands. If more demand of GDP are shifted to more productive sectors from less productive sectors under such circumstances, more workers will be laid-off by less productive sectors than the number of workers being hired by more productive sectors under certain conditions. Under the conditions derived in the paper GDP ends up less than before with this type of allocative disturbances in spite of the fact that demands have been shifted to favor more productive sectors.

The model behavior is simulated to estimate the value of the Okun' law coefficient and the loci of the Beveridge curve in the unemployment-vacancy plane.

The Introduction

We propose a stochastic dynamic model of a multi-sector economy with non-identical productivities in order to demonstrate the dependence on total output (GDP) of both the coefficient of the Okun's law and the loci of the Beveridge curves in the two-dimentional unemployment-vacancy plane.

The basic model is that of Aoki (2002, Sec.8.6). The economy has K sectors with productivity coefficients arranged in decreasing order, $c_1 > c_2 > \cdots > c_K$. We normalize the coefficient by setting $c_1 = 1$.

Given a vector of demand shares, s_i, of GDP for sector i, where $\sum_i s_i = 1$, each sector is either in positive excess demand or negative excess demand. Positive excess demands are regarded as signals for such sectors to attempt to increase production, and negative ones are signals for those sectors to plan to contract production. Due to the presence of externalities among the K sectors, only one sector succeeds in implementing its plan.

We formalize this behavior by modeling the dynamics as a continuous-time Markov chain. In such chains, only one sector with the shortest holding or sojourn time will be able to carry out its plan for changing production level, and changing the components of the state vector, which are the number of employees, sizes of pools of laid-off workers,and the vacancy signs posted by some sectors, to be described more fully later. Each sector is either in positive or negative excess demand, denoted by $f_i > 0$ or $f_i < 0$.

We call a sector active, when it is the one that can carry out its desired adjustment, i.e., sector i is active when it has the shortest holding time. We use the subscript a to refer to active sector state variable, $f_a < 0$ means that the active sector has a negative excess demand for its product, for example.

States of Sectors

Each sector is characterized by three state variables: (1) the number of employed, n_i, (2) the number of laid-off workers by this sector, u_i, and (3) v_i, which is either 1 or 0 to be described next. This additional state variable v_i, $i = 1, \ldots, K$ where $v_i = 1$ indicates that sector i has posted a vacancy (help-wanted) sign, and $v_i = 0$ means no vacancy sign is posted by sector i.

There are four possible combinations of f_a and v_a. With $f_a > 0$, and if $v_a = 0$, then v_a is changed to $v_a = 1$, and sector posts a vacancy sign, and it goes into overtime status. This is interpreted as increasing output by changing the hours of work. The existing employees, n_a in number, do the work equivalent to $n_a + 1$ number of employees, i.e., $Y_a = c_a(n_a + 1)$. Since $Y_i/n_i = c_i + c_i/n_i > c_i$, the labor productivity is increased due to possibble underutilization of labor, Okun (1973, p. 212).

When $v_a = 1$ already when $f_a > 0$, the active sector hires one additional employee. thus, n_a increases by one, and v_a is reset to zero, that is, the vacancy sign is removed. In this case, one unemployed is hired from one pool out of K pools of umenployed, say sector i, and u_i is reduced by one, the probability of which is determined by the ultrametric selection rule described below.

With $f_a < 0$, when v_a is zero, one employee is laid-off, i.e., u_a increases by 1, and n_a is reduced by 1. If v_a is 1, and $f_a < 0$, v_a is reset to zero and the vacancy sign is removed.

Probability calculation of hiring by ultrametrics

To convey the basic idea simply, consider a three sector economy in which sector 1 and 2 are more similar with each other than sector 3. Denote the ultrametric distance between sector i and j by $d(i, j)$. Ultrametric distance is symmetric, i.e, $d(i, j) = d(j, i)$, and satisfies not the usual triangle inequality but an inequality

$$d(i, j) \leq max_k[(d(i, k), d(k, j)].$$

In this example $d(1, 2) = 1$, and $d(1, 3) = d(2, 3) = 2$, say. See Aoki (1996, Sec. 2.5) where it is shown that the ordinary correlation coefficients are not satisfactory because they are not transitive.

Define

$$\bar{u}_1 = \frac{u_2}{1 + d(1, 2)} + \frac{u_3}{1 + d(1, 3)} = \frac{u_2}{2} + \frac{u_3}{3}.$$

This is the effective number of unemployed outside the sector 1, and where

$$U_1 = u_1 + \bar{u}_1$$

is the effective total number of unemmployed available to sector 1.

Similarly sector 2 has own pool of laid-off workers, u_2, plus the effective unemployed of the other two sectors

$$\bar{u}_2 = \frac{u_1}{U_1},$$

and, similarly for sector 3

$$\bar{u}_3 = \frac{u_1}{1 + d(1, 3)} + \frac{u_2}{1 + d(2, 3)}.$$

For example, suppose sector 1, with $v_1 = 1$ becomes active, and hires a worker. In this case the conditional probability that a vacancy in sector 1 is filled with a worker from its own pool of laid-off workers is

$$\Pr(u_1 \to u_1 - 1|\text{sector 1 is active}) = \frac{u_1}{U_1}.$$

The conditional probability that a new hire comes from the pool of laid-off worker of sector 2 is

$$\Pr(u_2 \to u_2 - 1|\text{sector 2 is active}) = \frac{u_2/[(1 + d(1, 2)]}{U_1},$$

and so on.

Continuum of Equilibria

Denote the equilibrium values with superscript e. In equilibrium states all excess demands are zero, hence the size (the number of employed) of sector i is given by

$$n_i^e = \frac{s_i}{c_i} Y^e, i = 1, 2, \ldots, K.$$

Adding these over the sectors the total number of employed in equilibria is given by

$$L^e = \sum_i n_i^e = \kappa Y^e,$$

where the parameter defined by

$$\kappa = \sum_i s_i / c_i \tag{1}$$

characterizes the relation between the macroeconomic variables L^e and Y^e. The inverse $1/\kappa$ is the aggregate productivity coefficient.

Allocative Disturbances, and Effects of Changes of Demand Allocation Patterns

As the patterns of demand shares are changed, the value of κ, and the equilibrium levels of GDP change. As the patterns of demand shares are changed before the equilibrium level is reached, GDP will exhibit some transitional patterns before it settles at some level, which may or may not be the equilibrium level of new patterns. In this sense, changes or switches of demand share patterns are analogous to what is called allocative shocks in Davis, Haltiwanger and Schuh (1996, p.104). This type of disturbances is different from what is generally understood by aggregate shocks.

We report first of the effects of two patterns, called D1 and D4. We take $K = 10$. One allocates more demand shares on more productive sectors, and the other is the opposite. Pattern D1 has share vector

$$\mathbf{s_1} = \frac{1}{17}(5, 3, 2, 1, 1, 1, 1, 1, 1, 1).$$

In this pattern, the most productive sector receives about 29 perc ent of GDP. The top five productive sectors receive about 71 per cent of the total GDP.

The other pattern is the reverse of D1, called D4. The share vector of this pattern of demand shares is

$$\mathbf{s_4} = \frac{1}{17}(1, 1, 1, 1, 1, 1, 1, 2, 3, 5).$$

With D4 the most productive sector receives only 6 per cent of GDP, and the top five most productive sectors receive about 29 per cent of GDP

Simulation results

We set the initial values of n_i at 100 for all ten sectors. Productivity coefficients are given by

$$c_i = \frac{11-i}{10},$$

for $i = 1, \cdots 10$, that is,

$$c_1 = 1,\ c_2 = 0.9\,, \ldots, c_{10} = 0.1.$$

With this choice the initial value of GDP is $\sum_i \frac{i}{10} \times 100 = 550$.

The value of κ deined in (1) with D1 is 2.16, and that for D4 has value 4.86.

All simulations are run for 7000 periods, and repeated 100 times.

The mean of $L_1^e = 971$, that of $Y^e = 449$ with demand pattern D1, and $L_2^e = 1117$ and $Y^e = 230$ with D4 demand pattern. These numbers give L/Y values that are consistent with the theoretical values.

With D1 the standard deviation of Y^e is 9.3. With D4, the standard deviation is about 9.2.

Fig. 1 show two bar graphs of the sizes of the ten sectors.. Note that n_{10}^e is very large with D4 to meet the demand with very inefficient production. With D4 the mean of the employees of the top five sectors is 87.4, and that of the bottom five sectors is 1030.

If the transition from Pattern D4 to D1 is accomplished in quasi-stationary way, one would expect changes in GDP to be

$$\Delta Y_{4\to 1} = \sum_i c_i \Delta n_i^e,$$

and $Y_4^e + \Delta Y_{4\to 1} \approx Y_1^e$. Numerically this is verified with the simulation runs.

However, when the transition of demand patterns is sudden, that is introduced at the 2000th time step of simulation runs, the simulation results show that GDP does not approach the value of $Y_1^e = 449$ but rather it becomes **less** than $Y_2^e = 230$. The value is about 220. See Fig. 2. This surprising fact indicates that more of less productive sector employees are fired and a fewer unemployed become employed by more productive sectors, that is there is a net increase in unemployment.

We have also investigated the effects of aggregate output shock in which an exogenous 10 percent increase in Y lasting 10 time points is introduced at 3000 time instant. There is a temporary increase in GDP for these 10 time interval, but return to the level statistically insignificantly large Y as soon as the shock is removed.

Examples

To understand the effects of net increase in unemployment associated with allocative disturbances we conduct the following thought experiment using a two-sector model.

Let productivity coefficients be $c_1 = 1$ and $c_2 = c < 1$. In phase 1 of this thought experiment, the initial demand pattern is given by $s_1 = \sigma$, and

$s_2 = 1-\sigma$, with $\sigma < 1/2$. Sector 2 (low productivity sector) thus has higher demand share. We use N_1^e and N_2^e to denote the numbers of employees to avoid possible confusion with the model above. Let

$$\frac{N_1^e}{N_2^e} = \frac{(s_1/c_1)Y}{(s_2/c_2)Y} = \frac{\sigma}{1-\sigma}c < 1.$$

Now, suppose that the demand pattern is reversed, $s_1' = 1-\sigma$, and $s_2' = \sigma$, where a prime is used to denote variables of this second phase of the thought experiment. We obtain

$$\frac{N_1'}{N_2'} = \frac{1-\sigma}{\sigma}c.$$

From these two equations, we obtain

$$Y = c_1N_1 + c_2N_2 = N_2(\frac{c}{1-\sigma}),$$

and

$$Y' = c_1N_1' + c_2N_2' = N_2\frac{c}{\sigma}.$$

Hence the condition

$$\frac{Y'}{Y} = \frac{1-\sigma}{\sigma}\frac{N_2'}{N_2} < 1,$$

holds if

$$\frac{N_2'}{N_2} < \frac{\sigma}{1-\sigma} < 1, \tag{2}$$

and conversely.

In other words, if the reduction of the size of sector 2 satisfies this inequality, then GDP is less in phase 2 than in phase 1, despite the fact that the demand share for high productivity sector increased.

For example, with the 10 sector model with the demand pattern D4 corresponds to the two sector model with demand pattern (1/3, 2/3), that is $\sigma = 1/3$. Then, (1) becomes the conditon $n_2'/n_2 < 1/2$. In words, when size of sector 2 shrinks by more than 1/2 of the original size in equilibrium, then GDP becomes less in phase 2 (more demand on productive sector) than in phase 1.

Sector size changes by a binary tree stochasticpProcesses

In models with two sectors we can associate a binary tree stochastic processes with sector size changes in response to some allocative switches or disturbances, just as in elementary binary tree explanations of stock price movements.

Take the example above with switch from D4 to D1. Initially the less efficient sector has size 1030.4 and the more efficient sector size is merely 87.4. Initially, the size of the less efficient sector shrinks with probability 1030/1117 =.92, and the more efficient sector gain one unit with probability of 0.08 approximately. This is a birth-and-death process on the binary tree. Initially the probability is much higher for the less efficient sector to fire employees more often than that of the more efficient sector to hire new employees. For example, in the first 10 steps after the switch, the less efficient sector firs 10 employees in a row with probability 0.43, 9 employees are fired in the 10 steps with probability 0.3, 8 employees are fired with probability 0.15, and so on.

Okun's law

Okun's law is an empirical relation discovered by Okun between the changes in unemployment rate and changes in GDP. We think of it as deviation from some equilibrium level of GDP, ΔY and changes in the unemployment rate u from some reference level $u*$

$$\frac{\Delta Y}{Y} = \text{constant} - \alpha(u - u^*),$$

where we approximate u by U/N, $N = L + U$, where U is the number of unemployed, L is the total number of employed.

We have derived above that the equilibrium level of employment L^e and the equilibrium level of output Y^e are related by

$$L^e = \kappa^e(Y^e)Y^e, \tag{3}$$

where superscript e refers to the equilibrium level values.

We assume that demand share for sector i is positive fraction s_i of GDP, $Y(t)$, $\sum_i s_i = 1$. We assume that demand shares are endogenized as

$$s_i(t) = s_i(t_0) + d_i(Y(t) - Y(t_0)),$$

where GDP is $Y(t) = \sum_i Y_i(t)$, where $t > t_0$, and where $\sum_i s_i(t) = 1$ at all times, i.e., $\sum_i d_i = 0$.

This means that when GDP becomes larger then demand shares for more productive sectors increase, and as GDP declines, demands for high productive sectors decline. This assumption reflects shifts of productive resources towards sectors of higer than average productivities in good times, as mentioned by Okun (1973, p.214) to the effect that the difference between a high-pressure and a low-pressure economy is not simply a proortionate addition of output and employment across all sectors. Empirically a distinct patterns of resource shifts are observed.

In our earlier simple theoretical models we assumed that κ is exogenously given.

Now we assume that $s_i(Y)$'s has an exgoeneously fixed component and a component depending on Y. More specifically for simpler analysis we assume

$$s_i(Y) = s_i^0 + \gamma k_i(Y - Y^0),$$

for some constant γ, and some constants k_i with $\sum_i k_i) = 0$, such as

$$k_i = \frac{(4.5 - i)}{8},$$

for $K = 8$, and

$$k_i = \frac{(5.5 - i)}{10},$$

for $K = 10$. Here s_i^0 is some fixed demand share, corresponding to GDP of Y^0 at time step t_0, and k_i is a coefficient by which deviation of GDP from the reference level changes s_i^0. In our simulation t^0 is taken to be step 1500 which runs 5000 steps.

In our next set of simulations we set $K = 8$, and to include the effects that as Y becomes larger, shares for more productive sectors increase. Note that $\gamma = 1/1000$.

We use this relation to estimate the coefficient of the Okun's law by

$$\alpha = -\frac{\Delta Y}{\Delta U}\frac{N}{Y^e},$$

where the ratio $\Delta Y/\Delta U$ is calculated from (1).

We rewrite the definitional relation

$$\frac{\Delta Y}{Y^e} = -\alpha\frac{\Delta U}{N},$$

to

$$\alpha = \frac{-\Delta Y}{\Delta U}\frac{N}{Y^e},$$

where we measure the slope of the first factor from the scatter diagram of simulation runs as

$$-\frac{\Delta Y}{\Delta U} = x,$$

and noting that the ratio L^e/Y^e is κ, and that $N/Y^e = (L^e/Y^e)(N/Y^e)$, we obtain

$$\alpha = \kappa x(1 + \frac{U^e}{L^e}).$$

In our simulation runs with two different demand patters, P3 and P4 have been run with demand pattern vectors

$$\mathbf{s_3} = \frac{1}{26}[6, 5, 4, 3, 2, 2, 2, 2],$$

and Pattern 2 has the reference demand share vector

$$\mathbf{s_4} = \frac{1}{26}[2, 2, 2, 2, 3, 4, 5, 6].$$

The simulation has been run 400 times with N=800. With P3 $\kappa = 2.2$, $Y^e = 509$, and the slope of OLS fit of $U - Y$ scatter diagram is 2.1 in magnitude, hence we have $\alpha = 3.1$. With D4, $\kappa = 3.5$ and the OLS fit to the $U - Y$ scatter diagram has slope 0.4, hence $\alpha = 1.6$.

Error Analysis

Simulations are run for 100 times with time steps of 5000 steps. We thus expect about 10 per cent errors or statistical variations in the values we use as L^e and Y^e.

From (x),

$$\frac{\Delta\alpha}{\alpha} = -\frac{\Delta Y}{Y} + \frac{\Delta x}{x}.$$

From the results of 100 runs, standard deviation of Y is about 10 with Y^e of the order 200. Assuming that the slope of $Y - U$ scatter diagram is measured also about 10 per cent relative error, the error in α is 20 per cent or less.

Beveridge Curves

The Beveridge curve depicts the relation between the number of unemployed U and the number of job-offers, V.

It is usually taken that the position of the Beveridge curve on the $U - V$ plane is determined soley by "structural characteristics" of the labor market and independetn of aggregate demand.

Our simulation results show tha the Beveridge cure depends on aggregate demand. As the average level of aggregate output goes up, the curve shifts down towards the origin, and vice versa. The Beveridge curve depends on the aggregate output because the productivities are distributed across sectors.

See Fig.1 and 2 for illustration.

Concluding Remarks

To see if temporary demand shocks on GDP have any lasting effects a ten percent exogeneous increase has been applied to the model at the 3000-th step lasting for 10 time steps. The simulations show that $Y(3000) = 457.9$, $Y_s(3010) = 508.2$, $Y(3011) = 458.0$, $Y_s(3011) = 462.5$ are the averaged values of 100 runs, where subscript s indicates the value with the shock. In other words, the effects of a temporary GDP shock of 10 per cent exogenous increase in Y produce a barely 1 per cent increase after the shock is removed. Temporary increase is nearly the same order of magnitude as the Monte Carlo statistical deviation.

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