Technical Report

Themes on Okun's Law and Beyond

by

Reiner Franke

S C E P A
May 2006

Contents

1 Introduction

The present report starts out from the widely accepted view that employment is directly affected by the growth of economic activity. This is a central connection for both a characterization of empirical data and the identification of structural change, and for the conception of theoretical macroeconomic models dealing with policy issues. The basis for a discussion on the effects here involved is the statistical relationship between the variations of (un)employment and GDP which is well-known as Okun's law (the seminal reference is Okun, 1962). This law states that though employment rises with output, the changes are not one-to-one. Okun found that the GDP growth rate must be equal to its potential growth just to keep the unemployment rate constant, and more specifically that a 1 percent increase in output above its trend line would only lead to a 0.3 percentage decrease in the rate of unemployment.

While the relationship has for along time been considered to be a fairly stable regularity in the industrialized countries, this notion is now no longer taken for granted. Regarding the US economy, over the past few years the general impression has gained ground that the stronger increase in output was not accompanied by a commensurate increase in employment, such that the job creation in relation to growth has been falling in the US whereas it has been rising in major European countries like France and Germany. Therefore, the present report also seeks to inquire into the time variability in the relationship. However, since in this discussion a variety of topics will be addressed, our investigation focusses on US data only.

Because of the central importance of the time-varying coefficients in the empirical relationships, the report begins with an overview of the econometric concepts that are underlying their estimation. An elaboration on these points in an extra section seems necessary since in many non-technical presentations the details of the method employed do not become exactly clear. This is especially true for the widely used Kalman filter, which often is apparently used in conjunction with additional judgement by the researchers without them making this explicit. One might furthermore conjecture that similar time paths of a regression coefficient, for example, would be obtained by less fashionable procedures that rest on more intelligible presumptions.

Section 2 is a methodological discussion of these often neglected niceties in the presentation of practical work. It cannot do without some mathematical notation, but it tries to keep it to a minimum and to discuss the basic properties of different procedures by means of illustrative examples. In particular, in contrast to the (stochastic, random walk) procedures using the Kalman filter to estimate time-varying coefficients, we will also propose a (deterministic) approach that is based on so-called spline functions. At the end of the section we will make clear it why we prefer the latter method over the Kalman filter variants, on what (inevitable) parameterization we settle down as a default, and what general properties of the time paths it tends to generate.

This comparison of the deterministic and stochastic method is done in subsection 2.3, which we hope can be largely understood on its own. The formal presentation of the two approaches themselves in the previous tow subsections might thus be skipped.

Section 3 takes up an idea from the literature that exploits Okun's law and the purported relatively stable link between output and the employment rate to obtain an alternative estimation of a natural rate of unemployment. By avoiding any theorizing about wage and price inflation, this approach is more parsimonious than the usual natural rate concept of the NAIRU. Because of the central role of the output gap here and also in other parts of the report, the section also discusses the topic of detrending, with the conclusion that the familiar recipes in this field should be seriously reconsidered. We will, specifically, conclude that Hodrick-Prescott filtering already does a good job, where, however, it seems appropriate to increase the familiar smoothing parameter $\lambda = 1,600$ for quarterly data considerably.

Section 4 turns to an investigation of Okun's law itself, studying both a level and a first-difference version regarding output and employment. Time variations of the Okun coefficient are examined by means of regressions with a rolling sample period and, allowing more flexibility, by the spline function method introduced and discussed in Section 2. It will, in particular, be found that the main tendency of the evolution of the Okun coefficient over time is somewhat different from the time path obtained by Semmler and Zhang (2005) in a most recent contribution to Okun's law.

These investigations are carried out in Sections 4.1 and 4.2. Subsequently, Section 4.3 goes beyond the regularities between output and the employment rate and examines the most important macroeconomic variables that contribute to this connection. To this end, the employment rate is decomposed into its constituent parts of demand and supply, i.e. employment and the labour force. It is furthermore useful to consider working hours and decompose total output into labour productivity (output per hour), hours per job, the employment rate, and the labour force. Concentrating on a business cycle perspective, we compute the trend deviations of these variables and study their comovements. It can in this way be revealed that for some variables the patterns of the business cycle fluctuations or their amplitudes have changed over the last 15 years. This, in particular, holds true for employment and hours and so, as a consequence, for labour productivity, too; but also the cyclical behaviour of the labour force has changed substantially.

The next two sections are devoted to modelling the output-employment nexus and estimating the reaction coefficients that characterize the structural relationships. The two models that we put forward assume both delayed adjustments of employment in response to gaps between desired and actual magnitudes. Regarding hours and the utilization of the workforce, the first model, which is dealt with in Section 5, works with the simplification of a—within the short period—fixed relationship between hours and output (a production function for hours, so to speak). In this way the relationships between the variables can be reduced to a parsimonious building block of a recruitment policy of firms, which only incorporates the employment rate and its rate of change as endogenous variables, and the output gap as an exogenous variable. This module is characterized by just two structural parameters, which come out very satisfactorily in the estimation.

The second model in Section 6 is theoretically more ambitious and treats the determination of hours and the number of jobs at the same level, assuming the same kind of decision making of firms. Formally, the employment rate as well as the utilization of the workforce become dynamic variables then, and we also point out in which way this model augments the atheoretical short-cut formulation of Okun's law. On the whole there are now seven structural parameters to estimate, six of which turn out to be highly significant and make perfect economic sense. On the basis of this evidence we afterwards investigate possible variations of these coefficients over time and relate the results to the changes of the Okun coefficient that have been found in Section 4. Section 7 concludes the report.

The empirical analysis uses data for the nonfinancial corporate business sector (the firm sector, for short), which we have extracted from the database provided by Ray Fair on his homepage (http://fairmodel.econ.yale.edu). The regressions and, in particular, the estimations of the time-varying coefficients have been carried out by the AELSA–FP software package that was especially developed for this research project and accompanies the report (Franke, 2006; the acronym stands for "advanced econometric least squares analysis integrating the Fair–Parke database"). Thus, the estimations of the report can be readily reproduced and possibly extended by modifying the sample periods or some of the technical or theoretical specifications.[1] The user of the software will also see that most of the diagrams in the paper are imported from this program, which allows her or him additional checks of the results here presented.

[1]The GMM estimations in Section 5 are the only exception; they were performed by using Eviews.

2 Foundations for regressions with time-varying coefficients

In applied work on time-varying coefficients in the relationships of economic variables—such as, typically, estimations of a time-varying NAIRU—the focus is usually on the results and possible alternative specifications in the selection of variables or lags. The underlying approach and the estimation method is supposed to be understood. However, properly understanding the method and the assumptions associated with it is, at least to the nontechnical outsider, no straightforward issue. This section, therefore, contains a short discussion of some basic points and how the present report takes account of them.

To begin with, let generally y_t be the series that is to be 'explained' by k independent, explanatory variables.[2] Suppose time-varying coefficients are associated with the first q variables, while the coefficients on the other variables are fixed over the sample period. Denoting the vectors of these variables by the letters x and z, respectively, the regression equation reads

$$y_t \;=\; \gamma_t' x_t \;+ \beta' z_t \;+\; u_t \;, \qquad \text{var}(u_t) = \sigma^2 \qquad (x_t, \gamma_t \in \mathrm{I\!R}^q,\; z_t, \beta \in \mathrm{I\!R}^{k-q}) \tag{1}$$

(all vectors are column vectors, γ_t' and β' are the corresponding row vectors). The disturbances u_t are assumed to be independently and identically distributed around zero.

Regarding the law governing the changes in γ_t, two different approaches can be found in the literature. One is that of a stochastic random walk, and the other conceives the γ_t as deterministic functions of time.

2.1 The random walk approach

The first approach is dominant in the literature. It is indeed hard to find any exceptions, and the stochastic framework as well as the random walk specification are hardly ever justified over possible alternatives. It is already much of a discussion if it is mentioned that estimation can be done by making use of the Kalman filter, a key advantage of which is that it can generate standard errors for nonlinear functions of the parameters (such as the ratios occurring in estimations of the NAIRU).

The random walk approach views the coefficients $\gamma_{j,t}$ in (1) as changing slowly and *unsystematically* over time ($j = 1, \ldots, q$). The latter means that in each period t an increase or decrease of a single coefficient is equally likely to occur, so that the expectation of $\gamma_{j,t}$ is equal to the value $\gamma_{j,t-1}$ that this coefficient has attained in the period before.

[2] To avoid clumsy notation, we write y_t for the value which variable y attains in period t as well as for the entire time series from $t=1$ until T (which would correctly be denoted as $\{y_t\}$ or even $\{y_t\}_{t=1}^{T}$). It will be clear from the context which of the two notions applies.

Formally, the statement amounts to the hypothesis of a random walk,

$$\gamma_{j,t} \;=\; \gamma_{j,t-1} \;+\; v_{j,t} \;, \qquad \mathrm{var}(v_{j,t}) = \sigma_j^2 \qquad (j = 1, \ldots, q) \tag{1.a}$$

where likewise the errors $v_{j,t}$ are i.i.d. with mean zero; more specifically a normal distribution is usually assumed (especially if estimation is done by maximum likelihood).

It is important to note that the $\gamma_{j,t}$ obtained from estimations of eqs (1), (1.a) do not estimate the "true" values of the coefficients. This would in fact be a meaningless statement since the latter are random variables. What is estimated are the *expected means* of the *probability distributions* of the $\gamma_{j,t}$ (with respect to, in particular, the variances $\sigma_1^2, \ldots, \sigma_q^2$).

Estimation of (1), (1.a) is not standardized. A first distinction one has to pay attention to concerns the role of the random walk variances $\sigma_1^2, \ldots, \sigma_q^2$; whether they are exogenously prespecified or endogenously determined within an estimation procedure.

2.1.1 Exogenous random walk variances

In the early applications of eqs (1), (1.a) to the problem of time-varying coefficients, the variances $\sigma_1^2, \ldots, \sigma_q^2$ were treated as exogenous parameters. It is clear that any specification of them prejudices the qualitative features of the time paths of the $\gamma_{j,t}$. The polar case of assuming $\sigma_j^2 = 0$ implies a completely constant coefficient, while a positive variance allows the coefficient to vary by a limited amount each time period. If no limit were placed on the ability of the coefficient to vary each period, then it would jump up and down and soak up all the residual variation in eq. (1).

Hence, what assumption on such a variance should be made? With respect to a coefficient, or a nonlinear combination of coefficients like the NAIRU—which according to intuition or economic theory should shift slowly—Gordon (1997, p. 22) makes the explicit proposal of what he calls a 'smoothness' prior, which avoids overly strong volatility. As he puts it, the coefficient "can move around as much as it likes, subject to the qualification that sharp quarter-to-quarter zig-zags are ruled out".

To be exact, the degree of smoothness of a time-varying $\gamma_{j,t}$ does not depend on the variance σ_j^2 in (1.a), but on its size relative to the (estimated) variance in (1), the so-called signal-to-noise ratio σ_j^2/σ^2. Nevertheless, setting σ_j^2 requires judgement; judgement of a kind that Gordon (p. 22, fn 14) describes as analogous to the choice of the smoothness parameter λ for the Hodrick-Prescott filter in the detrending of growth variables.[3]

[3] It might be argued that there is one value of λ for the Hodrick-Prescott (HP) filter that is most commonly employed for quarterly data, i.e., $\lambda = 1600$; and that although there is no such convention as yet for the choice of the signal-to-noise ratio for the random walk variances, the profession might tend to agree on a suitable order of magnitude. However, in other work Gordon makes it clear that the recommended

Figure 1 gives an example of the implications of different variances for the time paths of the parameter of interest.[4] The diagram is taken from Gordon (1997, p. 21), where on the basis of three alternative random walk variances he estimates a Phillips curve with time-varying coefficients which, then, deliver a time-varying NAIRU. After presenting the plot, Gordon notes that it is the (bold) solid line that meets his smoothness criterion, a "series that exhibits substantial movements but just avoids sharp quarter-to-quarter zig-zags" (p. 22).[5]

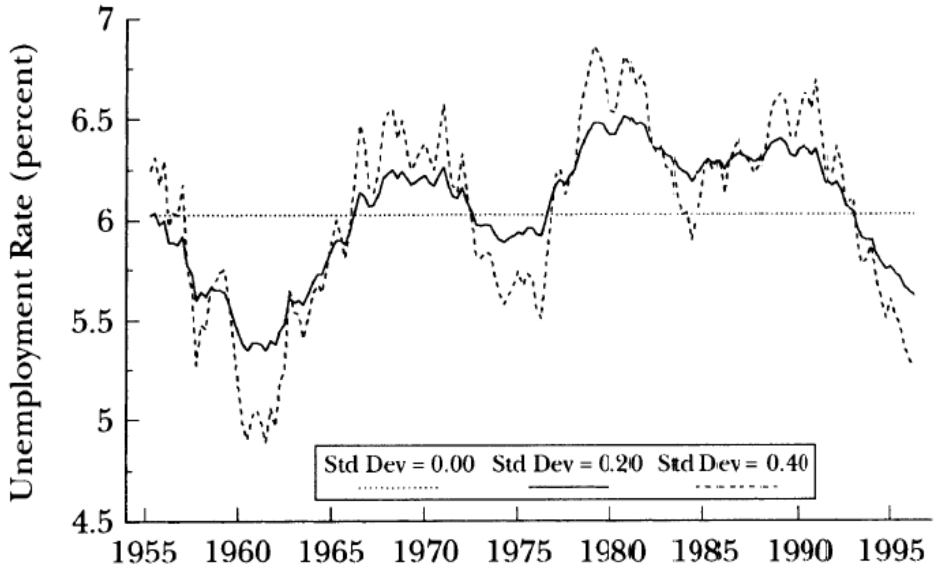

Figure 1: NAIRU time paths resulting from different exogenous random walk variances.

Note: Reproduced from Gordon (1997, p. 21, Figure 1).

The example also demonstrates that Gordon's view of what he still accepts as "smooth" may not be generally shared. In particular, there may well be other proponents in the

λ should not be adopted so automatically and that the appropriate degree of smoothness needs to be considered for each time series in its own context. We will have to return to the parameterization of the HP filter later in the report.

[4]From the references given and other empirical work by Gordon, one can assume that these estimations employ the Kalman filter. This is a one-sided (backward-looking) filter that additionally requires choosing an initial condition for the time-varying parameter (or the NAIRU itself in the present case). However, there does not seem to exist a foolproof procedure for this choice, as one can infer, for example, from the short discussion in Laubach (2001, p. 222) .

[5]From the standard errors of the regressions that Gordon reports in Table 1 (p. 25), one can infer that the solid line is generated by a signal-to-noise ratio of $0.20/0.88 \approx 0.23$.

NAIRU discussion who would find it more appropriate to have the temporary decline between 1972 and 1975 smoothed out.

2.1.2 Estimated random walk variances

While the intention of endogenizing the random walk variances in the estimation procedure is to get rid of the judgement, or arbitrariness, just discussed, this goes at the expense of transparency since matters become even more technical (beyond the problem indicated in fn 4). In addition, there are several distinct methods to be found in applied work.

Assuming that the random walk disturbances are normally distributed, the coefficients γ_t can be jointly estimated with the signal-to-noise ratios by maximum likelihood using the Kalman filter (standard references are Harvey, 1989, and Hamilton, 1994, Chapter 13.8). A grave problem here arising is the so-called "pile-up problem" (surveyed by Stock, 1994, Section 4). From analytical results and the existing experience with this method, one would expect to find estimates of the signal-to-noise ratios to be zero with high probability, even if their true values are strictly larger (cf. Laubach, 2001, p. 221). Some researchers even consider the problem so serious that they prefer to have recourse to the exogenously fixed variances, choosing signal-to-noise ratios that are in line with other empirical studies (Llaudes, 2005, p. 15) or are checked "interactively" with estimated variances (Laubach, 2001, pp. 222f).[6]

Schlicht (1989, 2003) has proposed a procedure that he simply calls the VC method (VC for "varying coefficients"), or more specifically the VC moment estimator, to distinguish it from its close cousin, the VC likelihood estimator. The general claim is that, for linear models, the VC moment estimator is mathematically and descriptively more transparent, and also statistically superior to the Kalman filter (see Schlicht and Ludsteck, 2005, pp. 3f, for brief statements of these points, before they go into the details).[7] In many cases, nevertheless, the VC moment estimator, the VC likelihood estimator, and the Kalman filter produce almost identical results (Schlicht and Ludsteck, 2005, p. 4; from a computational point of view, the moment estimator is sometimes, in poorly conditioned cases, superior to the other two methods).

To get an idea of the main conceptions and also to avoid as many technical details as possible, we cast the approach in terms of a consistency property that the—endogenous—

[6] Another method often referenced is Stock and Watson (1998), which promises to obtain median-unbiased estimates of the signal-to-noise ratios. Unfortunately, the article itself can only be understood by trained econometricians. Llaudes (2005, p. 16) reports to have tried the method in his NAIRU estimations but did not find the results very satisfactory (because of too little precision). So he discards this alternative approach, too (and goes on by fixing the variances exogenously).

[7] Despite these claims, the VC method is not to be considered a substitute but a complement to the Kalman filter, which is applicable to a much larger set of problems (Schlicht and Ludsteck, 2005, p. 28).

variances have to satisfy. It actually amounts to a fixed-point argument. To this end, consider the following three steps. First, given variances $\sigma^2, \sigma_1^2, \ldots, \sigma_q^2$ and the data y_t, x_t, z_t in (1) and (1.a), one can define suitably formatted matrices X, y, P, S and then, stacking the $\gamma_{j,t}$ in one $(q \cdot n)$-vector γ, compute the expected values of the $\gamma_{j,t}$ as

$$\hat{\gamma} \;=\; (X'X + \sigma^2\, P'S^{-1}P)^{-1}X'y \tag{2}$$

(for a proof of (2) and also (5) below, see Schlicht, 1989, pp. 11f). A least squares interpretation of what is behind these "expected values" is given in a moment.

In the second step, the n-vectors of the corresponding residuals $\hat{u}, \hat{v}_1, \ldots, \hat{v}_q$ are obtained from (1) and (1.a). In the third step, their variances $\hat{\sigma}^2, \hat{\sigma}_1^2, \ldots, \hat{\sigma}_q^2$ are computed. Taken together, we have a mapping $F\colon \mathbb{R}_+^{q+1} \to \mathbb{R}_+^{q+1}$, which can be briefly sketched as

$$F : (\sigma^2, \sigma_1^2, \ldots, \sigma_q^2) \;\longrightarrow\; \hat{\gamma} \;\longrightarrow\; (\hat{u}, \hat{v}_1, \ldots, \hat{v}_q) \;\longrightarrow\; (\hat{\sigma}^2, \hat{\sigma}_1^2, \ldots, \hat{\sigma}_q^2) \tag{3}$$

Consistency prevails if an array $\hat{\sigma}^2, \hat{\sigma}_1^2, \ldots, \hat{\sigma}_q^2$ constitutes a fixed-point of (3), such that

$$F(\hat{\sigma}^2, \hat{\sigma}_1^2, \ldots, \hat{\sigma}_q^2) \;=\; (\hat{\sigma}^2, \hat{\sigma}_1^2, \ldots, \hat{\sigma}_q^2) \tag{4}$$

The formulation is still somewhat loose, since a precise argument has to refer to expected values (see eq. (50) in Schlicht, 1989, p. 17), but (4) can nevertheless be considered to describe the core of the estimation problem and the principle that determines the signal-to-noise ratios.

Though the determination of $\hat{\gamma}$ by (2) is derived from a different calculus, it can be shown that (at least with respect to the fixed-point variances) this expression minimizes the weighted squares of sums of the residuals,

$$\sum_{t=1}^{n} u_t^2 \;+\; \sum_{t=2}^{n} \sum_{j=1}^{q} (\sigma^2/\sigma_j^2)\, v_{t,j}^2 \tag{5}$$

It is interesting to notice that the squared random walk residuals $v_{t,j}^2$ are weighted by the reciprocal values of the signal-to-noise ratios.

For the practical purpose of solving the fixed-point problem (4) one has to go deeper into the details. It here suffices to mention that it can be equivalently expressed as the unconstrained minimization of a suitably defined loss function in the variances. Schlicht (1989, especially pp. 27-31) notes that the solution is not quite equal to a more common maximum likelihood estimation, but the two are asymptotically equivalent (that is, as the number of data points becomes large enough). With respect to the wide-spread use of the Kalman filter (Kalman–Bucy filtering, to be more accurate), Schlicht (2003, p. 8) also contains a short section where he argues that his procedure is statistically and conceptually superior.

Another great advantage of Schlicht's VC method is that the algorithm that minimizes the loss function is freely available in the net (Schlicht, 2005) and fairly convenient to use. It is therefore integrated into the AELSA software that accompanies this report, and all of the original estimations of eqs (1), (1.a) documented in the following are based on it.

The algorithm itself is an iteration procedure (a gradient method). Its convergence cannot be generally proved, but Schlicht reports that under reasonable circumstances, if the estimation is not excessively ill-conditioned, convergence is no problem.[8] Thus, Schlicht's estimation procedure appears to be a reliable alternative to the approaches mentioned above. An general evaluation of the general features of the resulting time paths of the coefficients, whether there is too much or too little variation in them, is postponed until Section 2.3.

2.2 Deterministic spline functions of the coefficients

The alternative to stochastic variations of the coefficients in (1) are deterministic time paths. In this framework a coefficient γ_j may be generally conceived as being determined by a vector $\tilde{z}$ of several, or perhaps many, variables, which can be generally described as $\gamma_j = \tilde{f}_j(\tilde{z})$. From all of the vectors $\tilde{z}$ in an economically meaningfully domain, only $\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_T$ have been actually realized, giving rise to $\gamma_{j,t} = \tilde{f}_j(\tilde{z}_t)$. If one does not aim at information about the general law $\tilde{f}_j(\cdot)$, which might be a futile idea anyway, but is content with referring to the realized vectors only, $\gamma_{j,t}$ can be directly expressed as a function of time, $\gamma_{j,t} = f_j(t)$.

The functions $f_j(\cdot)$ should not be too arbitrary, of course, and not too irregular, either. The analysis is thus restricted to functions that can be parameterized by S coefficients $c_{j,s}$, where all $f_j(\cdot)$ belong to the same class of functional specifications. This leads us to specify the time-varying coefficients in (1) as

$$\gamma_{j,t} \;=\; f(t; c_{j,1}, \ldots, c_{j,S}) \qquad (j = 1, \ldots, q) \tag{1.b}$$

Given the functional form of the mapping $(t, c_1, \ldots, c_S) \mapsto f(t, c_1, \ldots, c_S)$, the estimation of (1), (1.b) amounts to estimating the parameters $c_{j,s}$ ($1 \le j \le q$, $1 \le s \le S$). This can be done by more ordinary methods than the ones discussed above, that is, by least squares minimization. However, since the regression equations are no longer linear in these coefficients, (1) and (1.b) represent a nonlinear least squares estimation.

A most straightforward specification of a time path, which pronounces possible 'structural breaks', is one composed of linear segments. Here the sample period is subdivided

[8] Private communication. The gradient method here employed is comparable to the usual maximum likelihood estimation procedures. Under the quoted 'reasonable circumstances', the results from this algorithm are identical to those from a global and more sophisticated algorithm. In a Mathematica package, the latter can be downloaded from `http://library.wolfram.com/infocenter/MathSource/5195`.

into $S-1$ equally spaced intervals $[t_s, t_{s+1}]$, and coefficient $\gamma_{j,t}$ assumes value $c_{j,s}$ if $t = t_s$, while the connection between γ_{j,t_s} and $\gamma_{j,t_{s+1}}$ is a straight line $(1 \le s \le S-1)$. Since the outcome often does not look too 'nice' and tends to overemphasize regime changes in the coefficients, where also the dating of the 'breaks' may be inappropriate, this specification will not be employed for presentation purposes. It can, however, be quite informative in exploratory work.

Smoother time paths are obtained by polynomials over the sample period. An even more attractive device are so-called 'splines', which combine smoothness and flexibility. A spline function is (not a linear function but) a polynomial between each pair of the knot points $(t_s, c_{j,s})$ $(s = 1, \ldots, S)$, whose coefficients are determined 'slightly' nonlocally. The nonlocality is designed to guarantee smoothness in the interpolated function, such that the left-hand and right-hand side derivatives coincide at the knot points. Most often cubic splines are employed, which means the local polynomials are of third degree and constrained to have equal first and second derivatives at the knot points.[9] Despite the high flexibility between two adjacent knot points, it is remarkable that the approach introduces no additional degrees of freedom over such a segment: regarding the time-varying coefficient $\gamma_{j,t}$ there are still no more than the S knot point parameters $c_{j,s}$ to be estimated.

Splines are therefore the method of choice in this report to specify the deterministic time paths (1.b) of the coefficients in a regression equation. Nevertheless, another choice still remains to be made, namely, the number of segments. It is obvious that the variability in the estimated time paths can be increased (and the fit in (1), (1.b) "improved") by increasing the number of segments. On the other hand, one is usually interested in only the great tendencies in the evolution of a coefficient; or the theoretical background suggests narrow, though informal, bounds on its volatility. Hence, the number of segments should be rather limited. For concreteness, let us say that normally a segment should not be much shorter than ten years. Certain episodes might nevertheless be usefully checked for considerable up and downs, which would reduce such a segment to perhaps five years.

Staiger et al. (1997, pp. 36) are one example in the literature on the NAIRU where this unemployment rate is estimated by using a cubic spline. The outcome can be seen in Figure 2, which reproduces their Figure 2 (p. 38). We need not discuss the slight differences from Gordon's regression, concerning the underlying price index for inflation and the additional explanatory variables representing the supply shocks. For the present purpose it suffices to notice the general features of the paths in Figures 1 and 2, where evidently the deterministic approach produces much less variability in the series.[10]

[9] In finer detail, the splines employed in this report are 'natural splines', which have zero second derivatives on both sides of their boundaries (see Press et al., 1986, pp. 78, 86ff).

[10] Besides, Figure 2 illustrates that also for the deterministically determined coefficients a confidence band can be constructed around the estimated time path, which in this case is a nonlinear function of regression coefficients; the issue is discussed in Staiger et al. (1997, p. 37).

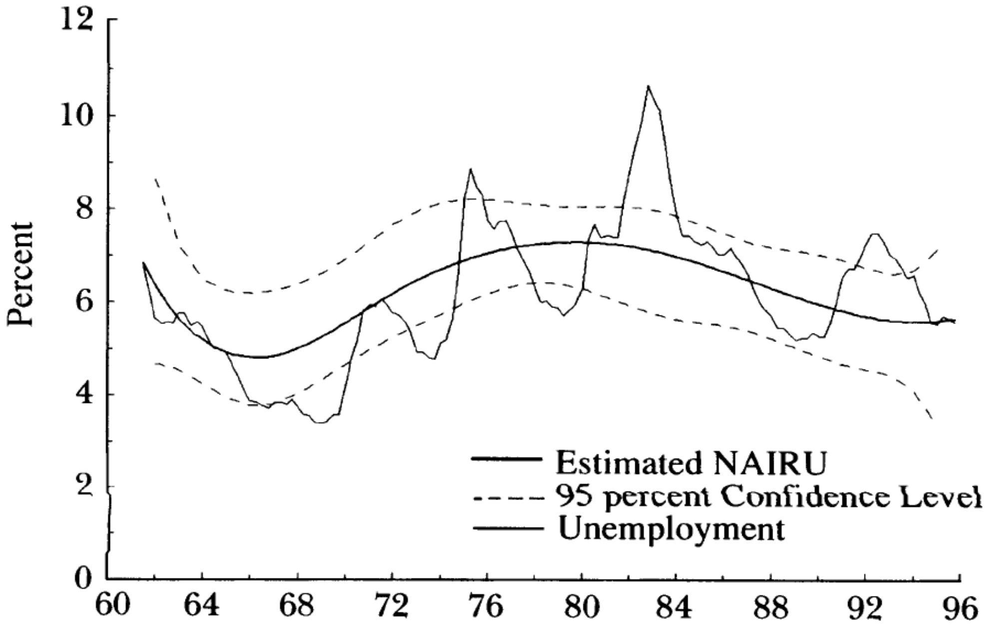

Figure 2: NAIRU time path estimated from a cubic spline function.

Note: Reproduced from Staiger et al. (1997, p. 38).

2.3 A comparison of the stochastic and deterministic approach

In modelling time-varying coefficients, the comparison of Figures 1 and 2 already gives some insights into the implications of the random walk and the deterministic spline approach. It has, however, to be noted that Gordon's time path in Figure 1 is based on his exogenous setting of the random walk variance. His 'smoothness criterion' that rejects volatile short-term reversals as implausible is seen to work, but it still leaves a considerable degree of variability. The question we have to ask is if this will still be the kind of outcome when the random walk variances are endogenously determined and the Kalman filter approach is replaced with Schlicht's (1989) estimation method. The question is nontrivial, considering the observation made in the literature and quoted in Section 2.1.2 that maximum likelihood estimation with endogenous signal-to-noise ratios using the Kalman filter suffer from the "pile-up problem", which introduces a severe downward bias for the random walk variances, and considering the asymptotic equivalence of Schlicht's approach to a maximum likelihood problem.

To compare the outcome of the random walk and spline approach, consider a relationship between two highly correlated variables. For the present illustration, let us use quarterly data on e_t^{dev}, the deviation of the employment rate from some (deterministic) trend, and the output gap y_t.[11] Figure 3 shows the comovements of the two variables, which are both measured in percent, and the lower amplitude of the fluctuations of the employment rate.

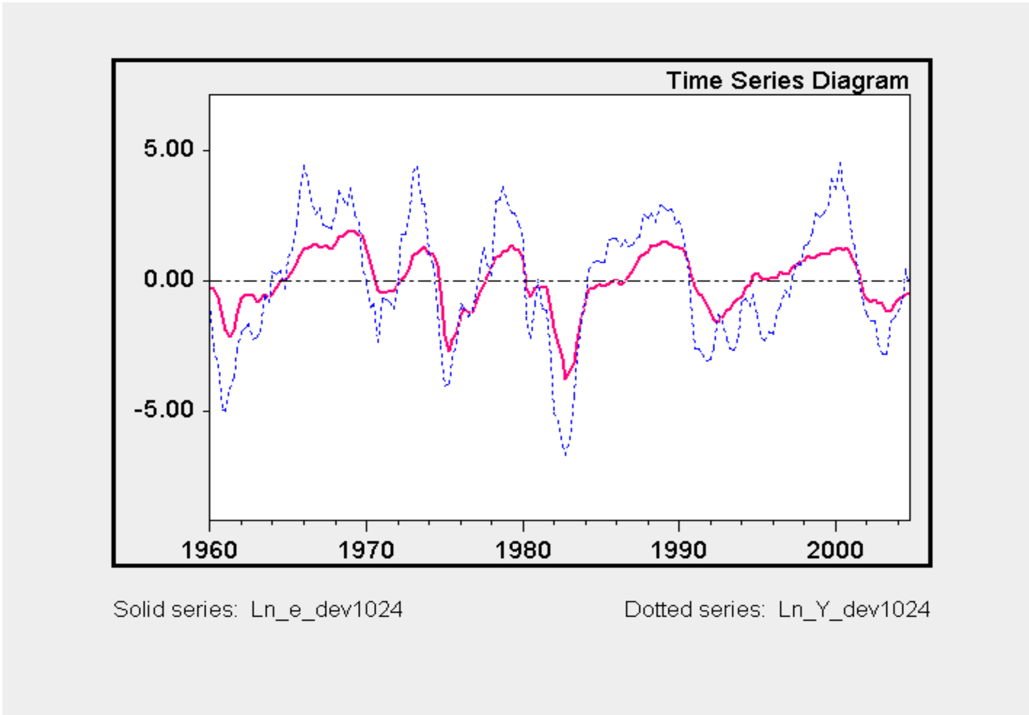

Figure 3: Trend deviations of output (dotted) and the employment rate (solid series).

The contemporaneous correlation coefficient between e_t^{dev} and y_t over the sample period 1960:1 – 2004:4 is 0.89. It is slightly higher for the lagged output gap, $\text{Corr}(e_t^{dev}, y_{t-1}) = 0.92$. A short description of the relationship between the two variables, which also accounts for possible variations in the degree of the correlation, is thus given by the regression equation

$$e_t^{dev} \quad = \quad \beta_t \, y_{t-1} \; + \; u_t \qquad\qquad (1960{:}1 - 2004{:}4) \qquad (6)$$

(u_t being the residuals). An OLS estimate of a constant coefficient yields $\beta = 0.43$, with $R^2 = 0.84$ (and, of course, considerable autocorrelation). The spline estimate (based on 5 segments) varies around this value, as shown by the bold line in Figure 4. Despite the flexibility of a spline function, the fit in (6) is not much improved by this approach (and it does not remove the autocorrelation, either); R^2 increases no higher than 0.86.

[11]The details of the specification of these variables, and their appropriateness, are discussed later. Here they only serve for illustration. The variables were constructed within the AELSA–FP software from the unemployment rate and output data in the Fair–Parke database, where partly the names of the new time series have been automatically assigned by the software (and we maintained them). The names can still be seen from the legend of Figure 3, which is directly taken over as a software output.

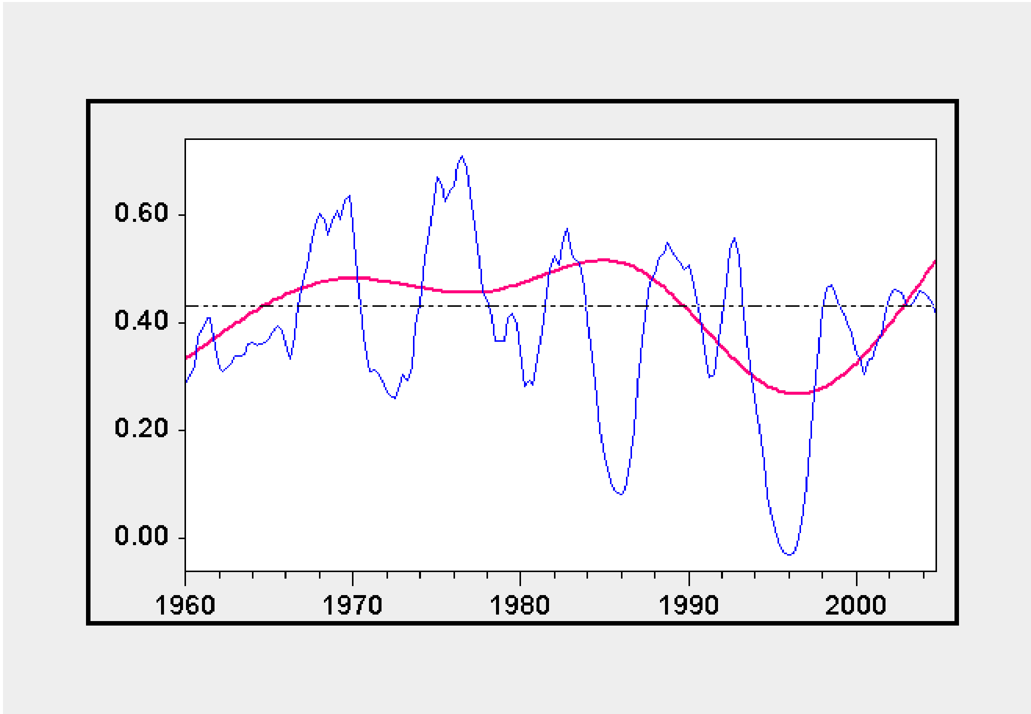

Figure 4: Coefficient β_t in (6) from spline and random walk estimation (bold and thin line, respectively).

On the other hand, the fit implied by the random walk raises R^2 to 0.95. Figure 4 makes it clear that this soaking up of the residual variation is achieved by suitable ups and downs, within short intervals of time, of the slope coefficient β_t (the thin line). Obviously, worries that the "pile-up problem" might carry over are not validated. Right on the contrary, the volatile series in Figure 4 will probably not even match Gordon's informal 'smoothness' criterion.

The short-term variation in the coefficient β_t in Figure 4 is, in fact, typical for the outcome of estimations employing the random walk hypothesis. Hence, if economic theory suggests a low variability in the time-varying coefficients, or if in a brief and atheoretical description of the relationships between the variables we wish to concentrate on the main and outstanding tendencies, then we should either use the deterministic spline approach or the random walk method with an exogenous prespecification of the variances of the coefficients. The latter, however, is not easily standardized since different orders of magnitude of the coefficients and different variances of the regression residuals u_t will require different "suitable" variances for the stochastic components $v_{j,t}$ of the time-varying coefficients. Moreover, our idea of a "suitable" variability in the time paths of a coefficient may not be very different from the outcome of a (likewise "suitable") spline estimation of deterministic coefficients. Therefore, we will decide in favour of the deterministic spline approach.

Having made this decision, still the number of segments has to be chosen. Figure 5 juxtaposes the evolution of β_t for the spline functions composed of 3, 5, 7, and 10 segments, respectively. The bold line is based on 5 segments and is reproduced from Figure 4. The non-oscillating dashed line, obviously, is obtained on the basis of 3 segments, the dotted line from 7, and the thin solid line from 10 segments. The main difference between the four specifications is their ability to let β_t decrease in the mid-1990s, where the segments must be as short as $45/10 = 4.5$ years to get a more pronounced minimum than that of the bold line. Note also that Figures 4 and 5 coincide in scale, which shows that even spline functions based on 10 segments have significantly less variability than the coefficients deriving from the random walk estimation.

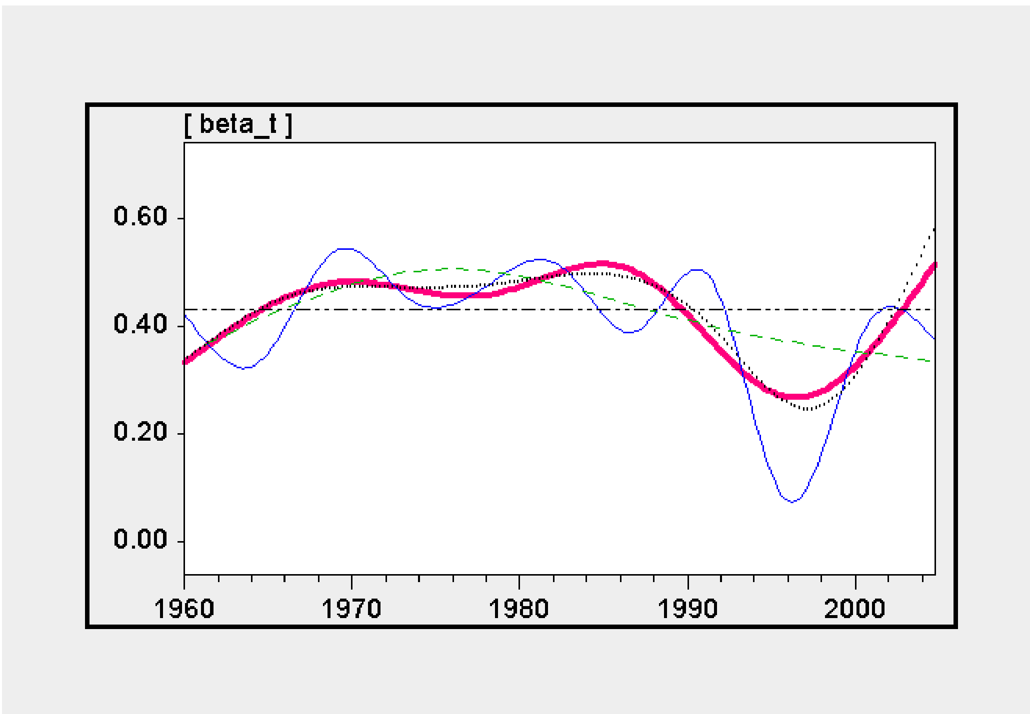

Figure 5: Coefficient β_t in (6) from splines based on 3, 5, 7, and 10 segments, respectively.

An additional criterion besides a desired, or acceptable, degree of variability is the goodness-of-fit achieved thereby. It may thus be asked if the fit brought about by 10 segments is so much better than that from 3 or 5 segments. Table 1, which in its first two rows reports the standard error of the regression and the sum of squared residuals for the four specifications, shows that the improvement by a higher number of segments is rather limited. Comparing the 7-segment to the 5-segment specification, already the standard error indicates that the better fit is not worth the increase in the number of parameters. According to the more elaborated measure of the Bayesian information criterion (BIC, to be minimized), the parsimony of the 3-segment specification outweighs the better fit of all the other specifications, while the Akaike information criterion (AIC), which penalizes additional parameters less heavily, would prefer the other extreme of the 10-segment specification. The ambiguity of AIC and BIC is a further indicator that the four alternatives are not essentially different.

	Segments			
	3	5	7	10
SER :	0.436	0.430	0.432	0.417
SSR :	34.000	32.224	32.132	29.409
AIC :	218.8	213.2	216.7	206.7
BIC :	231.6	232.3	242.2	241.8

Table 1: Fit of (6) under alternative number of segments for the spline function.

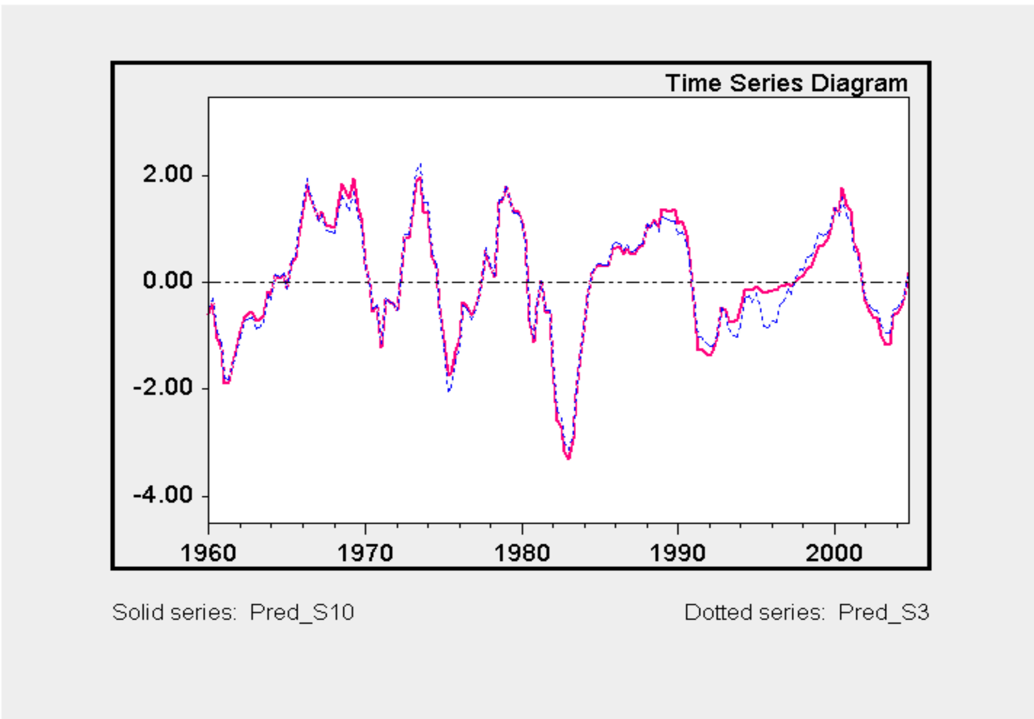

Figure 6: Predictions of e_t^{dev} in (6) from 10-segment (solid line) and 3-segment (dotted line) spline functions.

Figure 6 depicts the predictions of e_t^{dev} in (6) that result from the estimations with a 10-segment (solid line) and a 3-segment (dotted line) spline function. There is only one time interval where the two visibly differ from each other, which are the years between 1994 and 1997. A look at Figure 3 explains why: in this episode the employment rate returns earlier to normal than output returns to its trend. In the simple regression approach (6) this calls for lower values of the coefficient β_t, and the 10-segment specification has greater potential for that.

There may, however, be two different interpretations of this phenomenon. First, one views the basic pattern of output as not much different from the employment rate, but its return to normal in these years was two times set back by adverse shocks, which employers considered to be temporary (and possibly not so severe) and so did not cut back on employment. This would indeed be captured by a weaker reactions β_t. On the other hand, the employment rate itself, which after all is a composed variable, was subjected to special influences in the episode. In this case the decrease in β_t may not be overinterpreted (as suggested by the 10-segment spline). As long as this question is not investigated in greater detail, the bold line in Figure 5 from the 5-segment spline estimation is perhaps an acceptable compromise between these two points of views.

Also apart from the specific context, the 5-segment spline generates a motion that exhibits some variability in the coefficient but not "too much". For the remainder of the report, these experiments lead us to begin our investigation of time-varying coefficients with a 5-segment spline. Subsequently we will check this result with shorter and longer segments and report the modifications thus brought about if they appear important; otherwise the presentation will be confined to the 5-segment choice.

3 Okun's law and the natural rate of unemployment

In this section we take up an idea from the literature that exploits Okun's law and the purported relatively stable link between output and employment to obtain an alternative estimation of a natural rate of unemployment, which avoids the inflation context from which this concept, in the form of the NAIRU, is usually derived. However, because of the central role of the output gap, which it plays not only here but also in other parts of the report, we have first to discuss the topic of how to detrend a growing time series. We will actually find that the familiar recipes in this field should be seriously reconsidered.

3.1 The problem of detrending

3.1.1 Stochastic and deterministic trends

A fundamental issue in the analysis of systematically growing time series (or nonstationary series in general) is the notion of the trend. In this respect we are here interested in methods whose detrending procedures generate fluctuations that can be interpreted as forming part of the business cycle. Again, we are facing the alternative between deterministic and stochastic approaches.

Beginning with Nelson and Plosser (1982), it has been argued that the trends in macroeconomic time series are stochastic, so that much of the variation that used to be considered as business cycles would actually be permanent shifts in trend. While this stochastic view of the world soon became predominant, the pendulum has, in the meantime, swung back from that consensus. In a succinct summary of recent research on this issue, it can be concluded that "at the very least there is considerable uncertainty regarding the nature of the trend in many macroeconomic time series, and that, in particular, assuming a fairly stable trend growth path for real output—perhaps even a linear deterministic trend—may not be a bad approximation" (Diebold and Rudebusch, 2001, p. 8).[12] Against this background, we feel generally legitimated to work with the notion of a deterministic trend.

Because of its high flexibility, a widespread deterministic concept is (still) the Hodrick-Prescott filter.[13] Having made this decision, it remains to deal with the one degree of

[12]This short paper is a slightly revised version of the introductory chapter of their comprehensive book on business cycles (Diebold and Rudebusch, 1999).

[13]In recent times, the band-pass (BP) filter developed by Baxter and King (1996) has gained in popularity. This procedure rests on spectral analysis and so is mathematically more precise about what constitutes a cyclical component. The BP(6,32) filter preserves fluctuations with periodicities between six quarters and eight years while eliminating all other fluctuations, both the low frequency fluctuations that are associated with trend growth and the high frequencies associated with, for instance, measurement error. More exactly, with finite data sets the BP(6,32) filter approximates such an ideal filter. As it turns out, for the time series with relatively low noise (little high frequency variation) the outcome of the BP(6,32) filter is almost

freedom of the filter, that is, with the choice of the smoothing parameter λ. Whereas the familiar value for quarterly data is $\lambda = 1{,}600$, we will cast doubt on that convention.

Before turning to this problem, it should be outlined that applications of the stochastic approach are not foolproof and have their arbitrary elements, too. As an example, consider Gordon's (2003, pp. 219ff) Kalman filter approach to detrend labour productivity. An important first observation is the limited scope of the approach: it may be useful for some time series but not for others. For example, after estimating the stochastic trend *growth rate* of productivity, Gordon (p. 223, fn 16) points out that despite considerable effort, estimations of a corresponding log-level model failed because of implausibly low variation in the implied trend growth rates.With respect to the employment rate, Gordon (p. 224) points out that "no smoothing parameter of the Kalman filter [which amounts to setting the random walk variance; RF] was found to achieve the desired degree of stability". In this case he even resorts to a completely different detrending procedure, also distinct from the Hodrick-Prescott filter as his complementary option (see below).

Regarding the specification of the stochastic approach in the present context, denote labour productivity as z_t and let $x_t \in \mathbb{R}^m$ be a suitable set of m explanatory variables. Then Gordon estimates the trend growth rate as the time-varying intercept γ_t in the equations

$$d \ln z_t \;=\; \gamma_t \;+\; \beta' x_t \;+\; u_t \tag{7}$$

$$\gamma_t \;=\; \gamma_{t-1} \;+\; v_t \tag{8}$$

where u_t and v_t are assumed to be normally distributed with variances σ^2 and σ_v^2. As in previous work mentioned above, Gordon treats s_v^2 as an exogenous parameter at the researcher's command. He probably requires the variance to meet his smoothness criterion for the trend deviations (see Section 2.1.1). In addition, he makes explicit reference to another *a priori* judgement when he discards one value of s_v^2 with an otherwise reasonable outcome because of a high terminal growth rate of $\gamma_t = 3.38\%$ in 2003:2, which is not compatible with the view that the 2002 – 03 surge in actual productivity was an ephemeral event (Gordon, 2003, p. 222).

While the Hordrick-Prescott filter is a univariate procedure that examines a time series on its own, eq. (7) also uses outside information. Gordon (p. 219) considers it an advantage of the Kalman filter that the equation may specify any additional number of variables (x_t) to control for determinants of actual changes of the dependent variable that do not represent fundamental causes of changes in the trend. As for the changes in productivity, the x variables "could include changes in unemployment or the output gap, or dislocations caused by short-run events such as strikes or temporary changes in oil prices" (p. 219).

Certainly, the selection of the explanatory variables x_t in (7) is based on judgement, which may not always be unanimous. The differences caused by including or omitting

identical to Hodrick-Prescott filtering under the usual smoothing parameter $\lambda = 1{,}600$. For real national US output, this is exemplified in King and Rebelo (1999, p. 933, Figure 1).

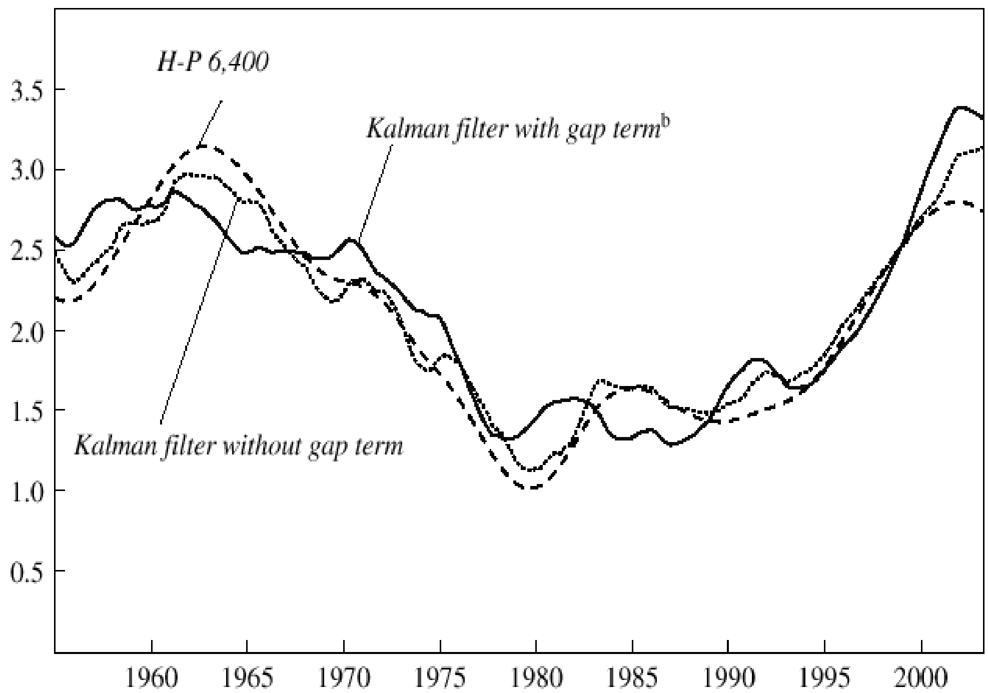

Figure 7: Trend productivity growth rates obtained from Hodrick-Prescott and two specifications of the Kalman filter.

Note: Reproduced from Gordon (2003, p. 220; bottom panel of Figure 1).

a specific variable can be illustrated by Figure 7, which reproduces the bottom panel of Figure 1 in Gordon (2003, p. 220). The dashed line is a Hodrick-Prescott trend of $d \ln z_t$ (at annualized growth rates) with a larger smoothing parameter than usual, $\lambda = 6{,}400$. The dotted line results from estimating (7), (8) without any variables x_t. As the variance σ_v^2 is chosen by Gordon, this kind of trend growth rate comes fairly close to the Hodrick-Prescott outcome.[14] If the current and four leading (not lagged) values of the change in a specifically constructed output gap are included, a piece of information unavailable to the other two filters, several distinct features are obtained. The solid line of the with-gap Kalman trend has a smoother profile over the 1978 – 88 decade; it registers a slower trend in 1962 – 68 (when the output gap was rising); and it registers a faster trend in 1968 – 76 (when the output gap was declining).

In justifying a choice of one trend over the other, Gordon makes explicit reference to "visual inspection" (p. 221), which is used together with some basic *a priori* ideas of how the trend line should, or should not, look like. Nevertheless, accepting the general necessity of judgement and specifically the with-gap Kalman trend in Figure 7, it may be asked whether

[14] Incidentally, having the variance σ_v^2 endogenously determined as described in Section 2.1.2 causes the series γ_t to coincide with the dependent variable.

similar outcomes could be achieved by a more parsimonious univariate method, simply by exploiting the degree of freedom inherent in it. This brings us back to the Hodrick-Prescott filter and a discussion of its smoothing parameter.

3.1.2 The output gap and the smoothing parameter in the Hodrick-Prescott filter

The popularity of the Hodrick-Prescott (HP) filter to detrend a time series is certainly due to the fact that it is easy to understand and to use in estimation. It is, in any case, more transparent than the Kalman filter. Nevertheless, HP detrending requires the specification of one smoothing parameter λ. At one extreme, the choice of $\lambda = 0$ yields a trend that exactly tracks every value of the series being detrended. At the other extreme, a parameter of infinity yields a straight line.

As is well-known, Hodrick and Prescott themselves endorsed the benchmark $\lambda = 1{,}600$ for quarterly data. It has not only become the default value in econometric software, but it is also hard to find an empirical study of the business cycle working with the HP filter that does not follow this recommendation. As a rule, the matter is not even discussed (if $\lambda = 1{,}600$ is made explicit at all). The profound skepticism by Gordon (2003) against $\lambda = 1{,}600$ is really an exception, when he characterizes this value as implying "implausibly large accelerations and decelerations of the trend *within* each business cycle (p., 218, emphasis added). He illustrates this property by quoting Hodrick and Prescott's (1997, p. 9) conclusion that the entire economic boom of the 1960s resulted from an acceleration of trend, rather than a deviation of actual output above trend. This evaluation, he incriminates, ignores outside information, "such as the fact that the unemployment rate was unusually low and the capacity utilization rate was unusually high" (Gordon, 2003, p. 218).

This criticism again indicates that a trend, as output of an econometric procedure, is only accepted if it satisfies some (informal) criteria. As a consequence of the observation of excess sensitivity of the 1,600 parameter when it is applied to the growth rates of labour productivity, Gordon (p. 221) goes on and tries the higher values $\lambda = 6{,}400$ and $\lambda = 25{,}600$. This choice indeed reduces the flexibility in the trend series, such that in this respect these filters would become more satisfactory. But now a more detailed element of judgement comes in: "the 25,600 parameter has the disadvantage that its inability to 'bend' causes it to date the beginning of the productivity growth revival of the 1990s well before 1995, and it measures the productivity growth trend in 2002–03 at a relatively low 2.35 percent a year".

The consequence that Gordon draws from all these advantages and disadvantages is creative but unusual. He decides not to rely exclusively on one of the trends examined. Specifically, he takes both the HP 6,400 trend (6,400 since its end-of-period growth rate is somewhat higher than the low 2.35 percent just mentioned) as well as the with-gap Kalman filter and constructs as his trend the average of the two series (i.e., the average of the solid and dotted line in Figure 7).[15]

This solution may be condemned as completely arbitrary (especially if one postulates that all time series in an investigation should be subjected to the same detrending procedure and the principle cannot be overall maintained). On the other hand, one may appreciate that not all is left to a technical mechanism but that, from case to case, specific outside information or *a priori* beliefs or postulates are invoked to select among a number of different options. Actually, this is what the expression "*ad-hoc*" literally means.

Back to the present report, because of the high variability of the estimated trend we do not expect that the Kalman filter with endogenous determination of the random walk variances will be of much help, while treating the variances as exogenous and setting them ourselves would result in trend lines that similarly could also be obtained by HP filtering with a suitable smoothing parameter.[16] For this reason we concentrate on the HP filter right away, explore alternative values of the smoothing parameter λ, and choose a value that generates an "acceptable" outcome. Should no parameter value be able to achieve this, we might discuss some, admittedly *ad-hoc*, "corrections" of the trend line.

As it turns out, detrending the level of output of firms, Y, does not run into such problems.[17] Figure 8 shows the alternative outcomes of the annualized trend growth rates that are implied by the HP trend of $\ln Y$ (obtained by multiplying the first differences of the latter by 400). Here and in all what follows, the trend itself is computed over a longer span of time than shown in the diagram (over the period 1952:1 – 2005:2); so at least to the left there are no end-of-period problems. The thin solid line is based on the familiar value $\lambda = 1{,}600$. This series still exhibits such distinct within-business cycle fluctuations that it could hardly be sold as representing the growth of potential output. According to this kind of evidence, the absence of any discussion on the appropriateness of the parameter value is indeed somewhat astonishing. In any case, the thin solid line in Figure 8 leads us to reject $\lambda = 1{,}600$ as a reasonable parameter.

The other three lines demonstrate how an increase in the smoothing parameter dampens the within-cycle variation. The dashed, dotted and bold lines are generated by $\lambda =$ 6,400, 25,600 and 102,400, respectively. Even a value as high and unfamiliar as $\lambda = 25{,}600$ implies a trend growth rate that is not entirely convincing. Hence, we dare to settle down

[15] If one wishes to avoid a premature rise of the trend growth rate in the 1990s, a later increase might also be enforced by a segmented linear trend line with suitable break points. In fact, the Kalman filter may not be the only reasonable alternative.

[16] In some cases the Kalman filter might produce (desirable) effects over some episodes of the sample period that cannot be obtained by HP. But then it would be poorly understood why the Kalman filter is here more "successful"—whereas in other cases it is not.

[17] The firm sector comprises nonfarm nonfinancial corporate business, nonfarm noncorporate business, and farm business.

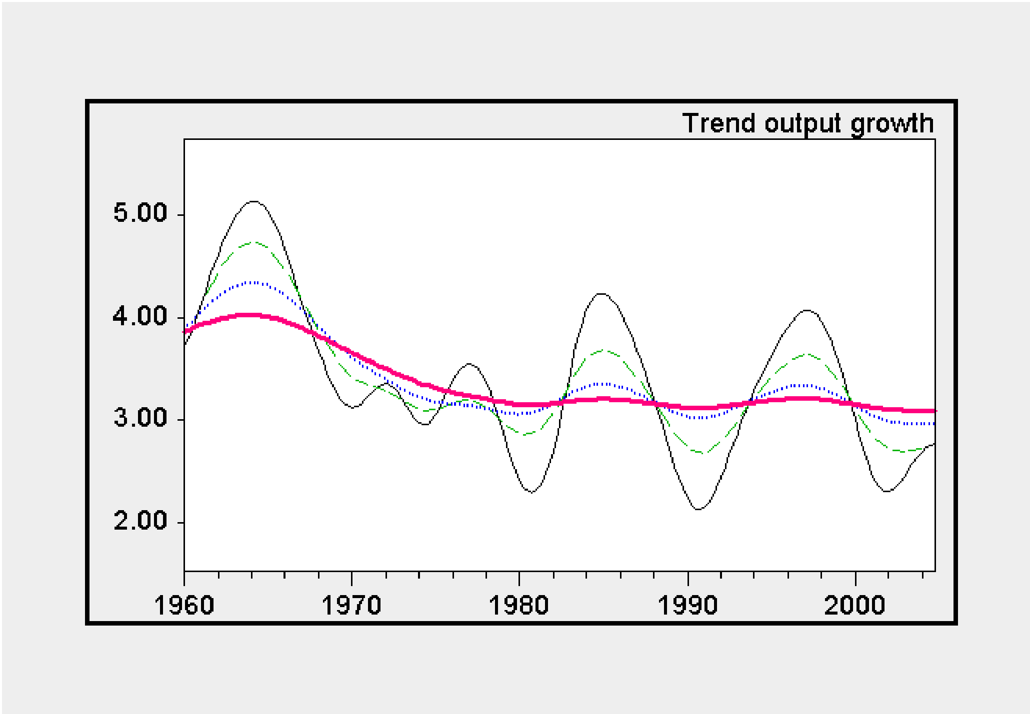

Figure 8: Growth rates of trend output obtained from HP filtering of the levels under alternative smoothing parameters (see text).

on the bold line in Figure 8 and the underlying $\lambda = 102{,}400$. This concept suggests that potential output grows at 4 percent per year in the mid-1960s and that its growth rate then steadily declines to 3.10 percent in recent times, which is a feature that seems to make good economic sense.

The solid line in Figure 9 displays the percentage deviations of output Y from the HP 102,400 trend. The series constitutes the output gap y_t, which will be underlying all of the following investigations ($y_t = 100 \cdot (\ln Y_t - \ln Y_t^\star)$, if $Y^\star$ denotes the trend level). It is contrasted with the well-known outcome of filtering by HP 1,600. While most of the qualitative features are not essentially altered, two differences are noteworthy: the first transitory peak in 1984 resulting for the HP 1,600 output gap disappears in our notion of y_t; and in the 1990s our y_t passes the zero level at a much later date.

Regarding the quantitative features it is obvious that the HP 1,600 gap series must yield a lower variability. Especially some (but not all) of the turning points move closer to the zero level. Although these differences do not seem too pronounced, the overall variability of the HP 1,600 gap as measured by the standard deviation is considerably smaller than that of the HP 102,400 gap; with 1.61 percentage points it indeed amounts to only two thirds of the 2.42 percentage points that we compute for y_t. Besides, the latter also exhibits stronger persistence; its first-order autocorrelation is 0.94 versus 0.87 for the HP 1,600 gap.

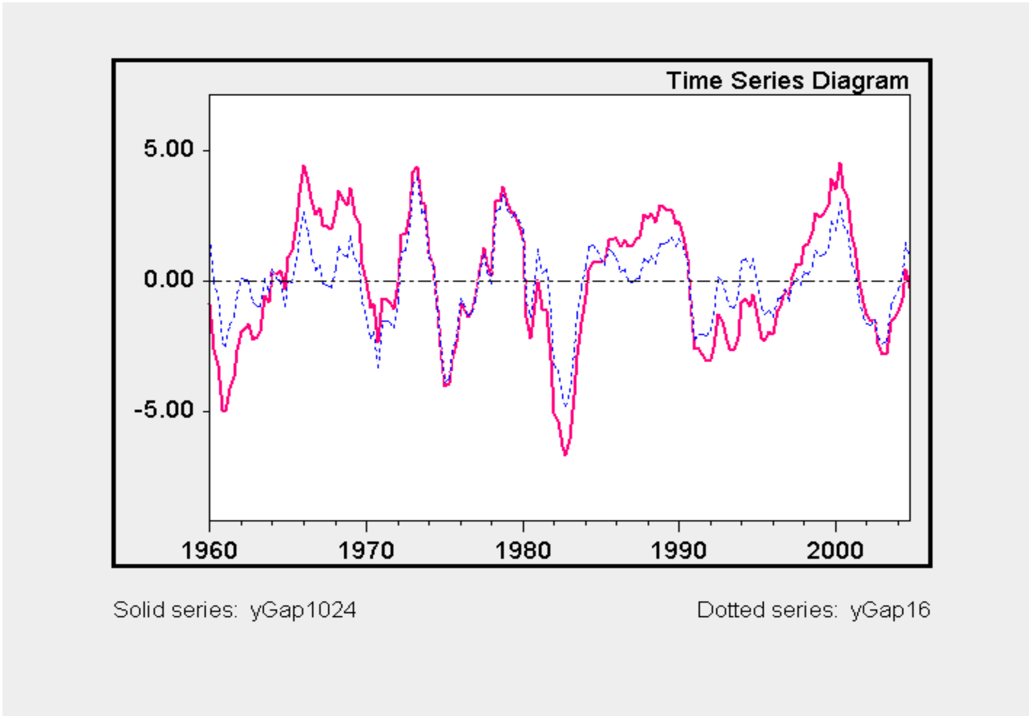

Figure 9: Output gap from HP filtering under $\lambda = 102{,}400$ (solid line) versus $\lambda = 1{,}600$ (dotted line).

Having thus decided the problem of output detrending, the results in Figures 8 and 9 may be finally compared to the outcome that Gordon (2003) obtains when he applies his method to the growth rate of GDP (instead of our levels and the output of the firm sector). Figure 10 reproduces his Figure 3 (p. 227). The trend line in the upper panel is the average of HP 6,400 and the Kalman filter with the exogenously chosen random walk variance. It is quite similar in its variability to the bold line in Figure 8. It differs, however, in its moderate upward tendency from 1993 on, which is not present in Figure 8. The resulting percentage trend deviations in the level of GDP in the lower panel of 10 is in many features similar to the solid line in Figure 9, except that the GDP gap series crosses the zero line later than our output gap, and that its peak in the mid-1960s and its trough in 1982 are more pronounced.

3.2 Deriving the natural rate of unemployment from the data

Although the idea of a "natural rate of unemployment" is an omnipresent reference in macroeconomic theory, there is little agreement as to what precisely the natural rate is or how it is to measured. Consulting Rogerson (1997), one finds nine different definitions in the literature. An often quoted reference is the presidential address by M. Friedman

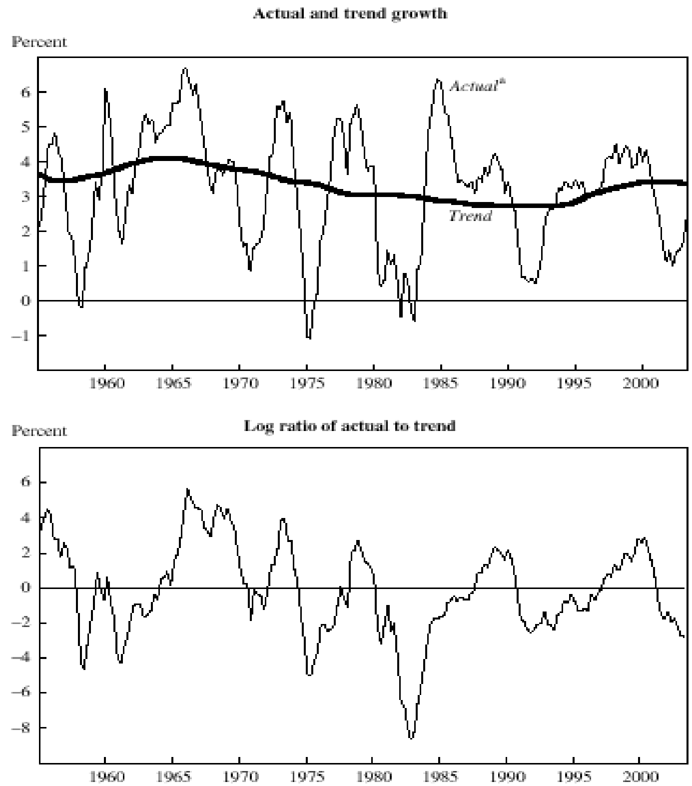

Figure 10: Real GDP: trend growth and deviations of levels from trend as estimated by Gordon (2003).

Note: Reproduced from Gordon (2003, p. 227). "Actual" in upper panel is eight-quarter change of GDP.

(1968) to the American Economic Association, where he characterizes the natural rate as being "ground out by the Walrasian system of general equilibrium equations ... embedded in them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availabilities, the costs of mobilities, and so on". Thus, as a minimum, the natural rate is conceived as an equilibrium rate of unemployment, which depends critically on the institutional characteristics of the economy and may vary depending on demographics and institutions. As the latter change slowly over time, the natural rate of unemployment (NRU henceforth) should be a smoothly evolving time series.

3.2.1 Atheoretical trend lines

Whatever concepts may have been put forward in economic theory to define a NRU, for measurement purposes it is a reasonable idea to proxy it by the trend component of some time series filter. For us this will most conveniently be the HP filter. Figure 11 displays the actual unemployment rate (the thin solid line) and lays the HP trend lines trough it that arise from three different values of the smoothing parameter λ.

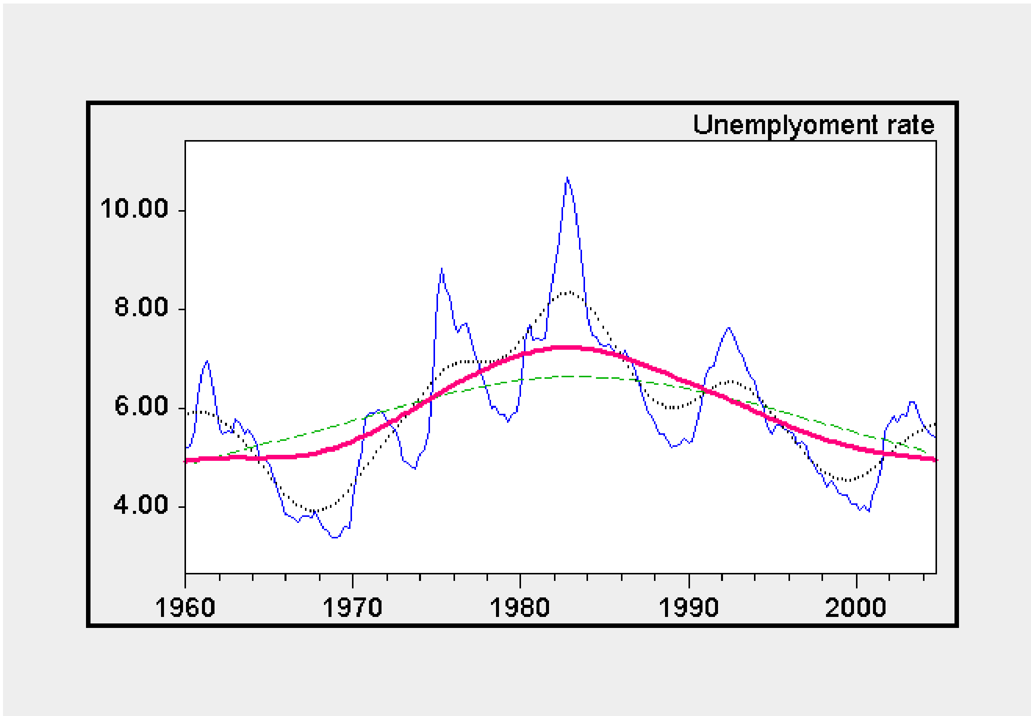

Figure 11: Rate of unemployment and HP trend generated by different λ.

The dotted line is generated by λ = 1,600. The standard parameter again causes considerable trend variability. A movement within 15 years from 3.9% in 1967/68 up to 8.3% in 1982/83 does not appear very "natural", quite apart from the within-cycle variation. Both features are smoothed out if λ = 102,400 is adopted, as shown by the bold line. Here the trend steadily rises from 5.0% at the beginning of the 1960s to a high of 7.24% in 1982:4, and then steadily declines to the original 5.0% at the end of the sample period. The dashed line with almost identical levels at the beginning and end of the period, exhibits an even lower amplitude; it only rises to 6.64% in 1983. With λ = 1,000,000, the underlying value of the smoothing parameter is, however, unprecedently high.

Despite the unusual HP smoothing, the flatness of the bold and the dashed lines are not too implausible to proxy for a natural rate of unemployment. This is exemplified by comparing them with the current estimate (as of 2002) of the NAIRU from the Congressional Budget Office, which is the dotted line in Figure 12 (the diagram has been extracted from Orphanides and Williams, 2005, p. 1931, Figure 2). The CBO concept yields an even flatter line than its counterparts in Figure 11.

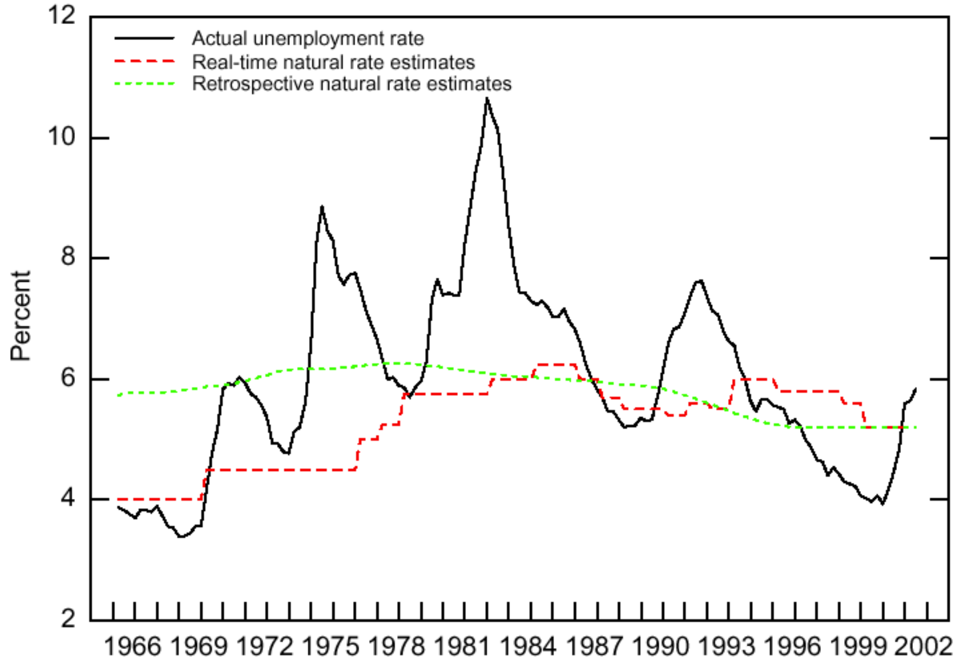

Figure 12: NAIRU estimate by CBO (as of 2002; dotted line).

Note: Reproduced from Orphanides and Williams (2005, p. 1931, Figure 2). The solid line is the actual unemployment rate, the dashed line is a real-time series, i.e., values of the NAIRU as it was estimated in that particular point in time, on the basis of the then available data.

On the other hand, the two NRU proxies in Figure 11 behave in a similar way as the NAIRU time path by Staiger et al. (1997) in Figure 2 above, especially if the latter's confidence band is taken into account. Interestingly, Gordon's (1997) estimates from Figure 1 move in a narrower corridor but display more within-cycle variability. Contrasting the bold and dotted lines from our HP trend in Figure 11 with the alternatives in Figures 1, 2 and 12, there is no reason that would disqualify the result of the atheoretical approach *a priori*. The two lines may even be conceived as sort of a compromise.

3.2.2 Estimation of the NRU by Okun's law

In the discussion of the HP trends in Figure 11, the natural rate of unemployment has implicitly been identified with the NAIRU. In fact, the interchangeable use of the terms is common practice in the literature. Grant (2002, pp. 96f) insists that these are two different theoretical concepts that should be more carefully distinguished. The natural rate of unemployment is a microeconomic Walrasian equilibrium outcome in which labour markets are cleared by wages and prices, whereas the NAIRU is fundamentally a disequilibrium, or Keynesian, macro outcome where "the inflationary forces of the excess demand markets balance the disinflationary forces of the excess supply markets" (p. 97).

We can here leave it open whether this is an accurate characterization of the two notions. More interesting is Grant's idea of how (in lack of suitable microeconomic data) to estimate the NRU at the macro level. Instead of referring to the inflation context, he proposes to exploit the relatively stable link between output and employment that has already been observed by Okun (1962). As Grant (2002, p. 98) writes, "Okuns conceptual framework of this employment-output link lends itself nicely both to Friedman's intuitive exposition and to econometric specification. Unemployment can be considered to be the sum of three components. Frictional unemployment and structural unemployment may exist due to microeconomic market imperfections which inhibit fluid job search and matching. Both may exist alongside unfilled labor demand. A cyclical unemployment gap exists due to deficiencies or excesses in aggregate product demand from the economy's sustainable potential."

Now, the latter are conveniently captured by the output gap y_t as we have discussed it in Section 3.1.2.[18] If we let UR_t designate the actual unemployment rate, Okun's link between the gap in unemployment and the output gap can then be specified econometrically as

$$\mathrm{UR}_t \;=\; \gamma_{0,t} \;-\; \gamma_{y,t}\, y_t \;+\; u_t \tag{9}$$

Apparently, $\gamma_{0,t}$ is the econometric estimate of unemployment that would exist when the economy was running at the level of capacity given by the estimated potential output. The time path of $\gamma_{0,t}$ is Grant's output-based estimate of the NRU (Grant, 2002, p. 98). In contrast to the NAIRU relationship between the labour market and general price inflation, $\gamma_{0,t}$ in (9) is an estimate of the relationship between labour and product markets (p. 108).

In estimating eq. (9), Grant follows the usual procedure and assumes that the coefficients $\gamma_{0,t}$ and $\gamma_{y,t}$ evolve as random walks, for which he employs the Kalman filter (p. 104). However, he makes no mentioning of the random walk variances, whether they are part of the estimation or set exogenously. The output gap he uses refers to GDP and is based on a HP trend (either HP 1,600 or HP 10,000; Grant does not make clear which one). The time path of $\gamma_{0,t}$ that Grant obtains is reproduced in Figure 13 (taken from Grant, 2002, p. 107, Figure 4); it is the solid line in the diagram (HPNRU). For a better evaluation of its properties, Grant contrasts this NRU with a NAIRU estimate by Gordon (the dashed

[18] To check the robustness of his results, Grant considers several detrending procedures, including HP filtering. Regarding the NRU estimates he concludes that they are of secondary importance. More precisely, the results of a subgroup of detrending procedures that contain two HP filters are fairly similar. This finding allows us to concentrate on our HP 102,400 trend right away, also when we compare our results to Grant's results.

line TVNAIRU), which appears to be a slightly smoothed version of the NAIRU that was presented in Figure 1 (the solid line there).

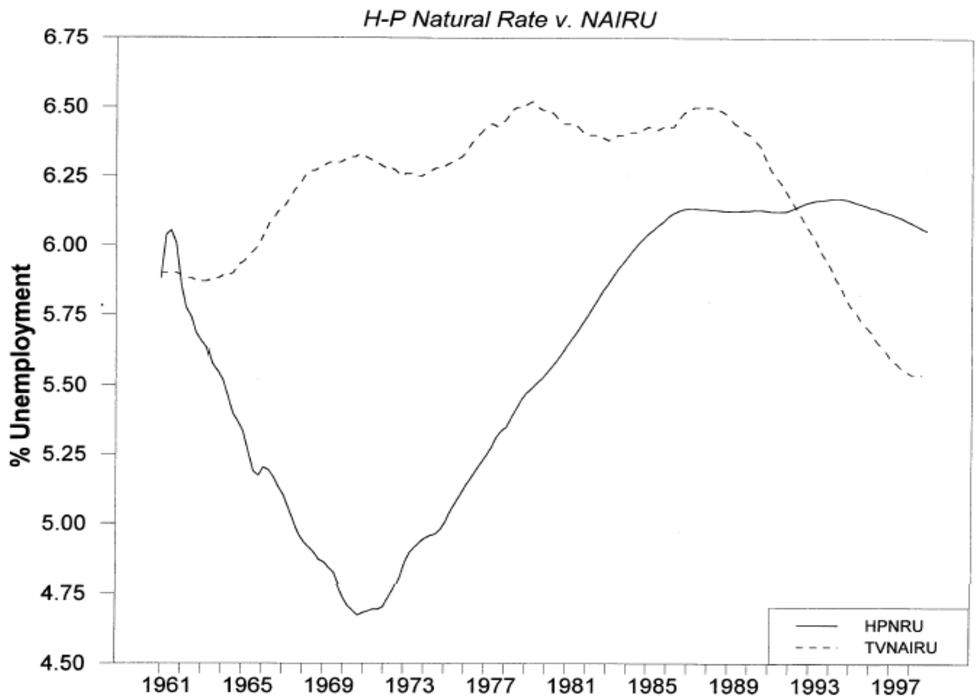

Figure 13: Grant's NRU estimation from eq. (9) (solid line).

Note: Reproduced from Grant (2002, p. 107, Figure 4). The dashed line is a NAIRU estimate by Gordon.

The differences between the two time series are striking. First, while Gordon's NAIRU (or at least its 'trend') increases over the 1960s and 1970s by about 0.5 percentage points, the NRU decreases by about 1.2 percentage points until 1970 and then again increases up to its initial level, which it reaches another 15 years later. Equally remarkable is the second difference. In contrast to the decline of the NAIRU from the mid-1980s until 1997, the NRU stays constant over these years. In these respects, the differences of Grant's NRU estimate from the NAIRU estimates by Staiger et al. (1997; see Figure 2), the CBO (see Figure 12), or from our HP trends in Figure 11 are all qualitatively the same.

Do we get similar results when we now estimate eq. (9) by specifying the time-varying coefficients as discussed in Section 2.1? The output gap y_t for such a regression is obtained from detrending $\ln Y_t$ by HP 102,400, which results in the solid line in Figure 9 (we have checked that the results do not change much under filtering output by HP 1,600). The sample period is again 1960:1 − 2004:4.

Consider first the outcome of $\gamma_{0,t}$ when we likewise adopt the random walk hypothesis for the coefficients, and the variances are endogenously determined. The resulting time path is drawn as the thin solid line in Figure 14 (note that the diagram has the same scale as Figure 11). The series maintains the main cyclical pattern of the actual unemployment rate, so that it already looks like a compressed image of it. This estimate of the NRU is, in particular, very different from Grant's application of the Kalman filter. The diagram points this out by the dashed line, which with its three linear segments is a stylized reproduction of his estimate in Figure 13 (the solid line there).

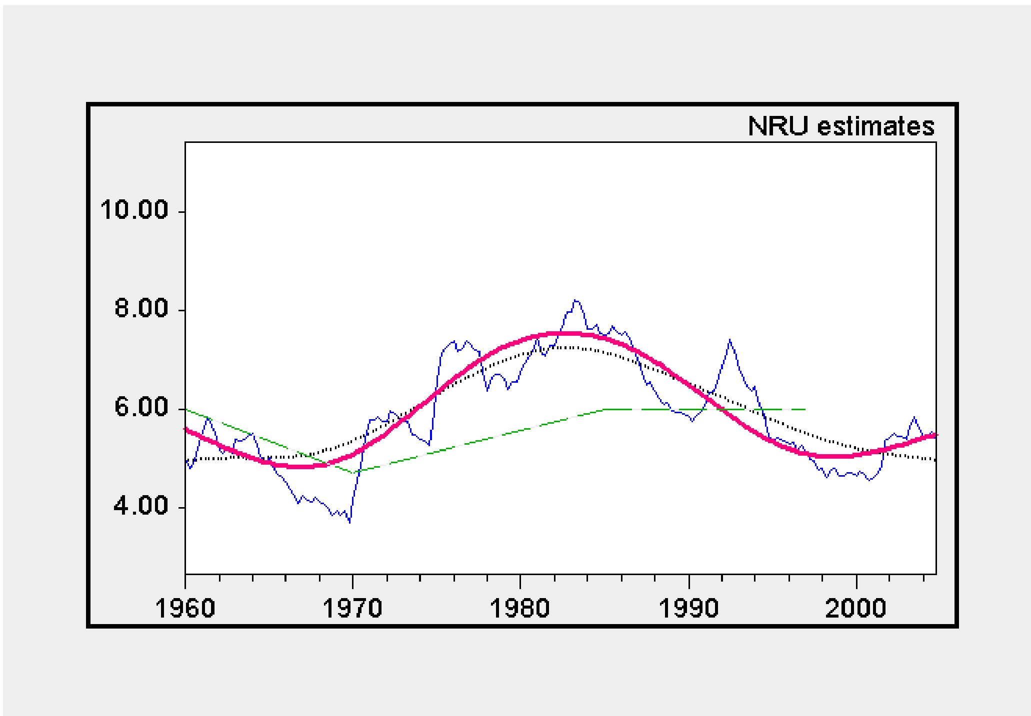

Figure 14: NRU estimates of eq. (9): random walk (thin solid line) and spline function (bold line).

Note: The spline function is based on five equally spaced segments. Dotted line is the HP 102,400 trend of the unemployment rate (see Figure 11), dashed line is a stylized reproduction of Grant's estimate in Figure 13.

Actually, the systematic deviations of Grant's NRU from the thin solid line seem somewhat surprising. As a possible explanation, we can think of an extremely low signal-to-noise ratio that Grant may have fixed from the outside, though this issue should have certainly been discussed. In this respect one might suspect that his kind of estimation comes close to detrending the unemployment rate by a segmented linear trend. However, computing such a trend of three equally spaced segments over the shortened sample period 1960:1 – 1997:4 yields a different picture from the dashed line in the diagram: in the first third of the time span this trend slightly rises from 5.0 to 5.2 percentage points, then increase up to 8 points, and in the last third returns to the initial five percentage points. So, a puzzle remains.

Alternatively to the stochastically varying coefficients $\gamma_{0,t}$ and $\gamma_{y,t}$ in (9), we should also adopt the deterministic approach of a spline function. As indicated above, five segments seem a reasonable compromise between too much and too little variability (we have checked that it is). The result is the bold line in Figure 14. The feature that it is much smoother than the random walk estimate will have been expected and conforms to what has been discussed in Section 2.3. It is also interesting to compare it with the atheoretical HP 102,400 trend of unemployment, which is represented by the dotted line (this is just the bold line in Figure 11). Although the spline function estimation of the natural rate of unemployment contains more structure, the outcome comes fairly close to the HP trend.

To sum up, four conclusions emerge from this investigation: (*i*) Grant's estimation of the NRU is (presently) not convincing. (*ii*) The NRU time path of the random walk estimation with endogenous variances is too variable. (*iii*) The spline function estimation of the NRU appears reasonable, and is also quite in line with the more involved NAIRU estimates that one finds in the literature. (*iv*) The HP 102,400 trend does a good job, too.

4 Okun's law as a time-varying statistical regularity

4.1 Different specifications of the relationship

The negative relationship between unemployment and output as it is summarized by Okun's law is usually specified in two different versions, where one is based on the levels of the two variables and the other on their rates of change. If we refer to the rate of employment rather than unemployment and generally allow for time variability in the relationship, the two versions read

$$e_t - e_t^\star \;=\; \beta_t\,(Y_t - Y_t^\star)/Y_t^\star \;+\; u_t \tag{10}$$

$$e_t - e_{t-1} \;=\; \beta_t\,(gY_t - gY_t^\star) \;+\; u_t \tag{11}$$

The letter e denotes the employment rate, Y is an output variable, gY its rate of growth, and u_t are the residuals in a regression (as in eq. (1) above). The underlying time unit is a quarter and here and in the following all rates of change are annualized percentage numbers (thus, $gY_t = 400 \cdot (Y_t - Y_{t-1})/Y_{t-1}$). A star symbol indicates trend values. The coefficient β_t will be referred to as the "Okun coefficient", where however a word of caution has to be added since occasionally this expression is used for the reciprocal $\tilde{\beta}_t = 1/\beta_t$ in the reversed equation, such as $gY_t - gY_t^\star = \tilde{\beta}_t\,(e_t - e_{t-1}) + \tilde{u}_t$, for example.

In the present research context it is particularly interesting to relate our discussion to (selected parts) of two recent papers by Semmler and Zhang (2005) and Hemraj, Madrick and Semmler (2006). There a third specification is investigated, namely (in our notation),

$$e_t - e_{t-1} \;=\; \beta_t\,(gY_t - gY_{t-1}) \;+\; u_t \tag{12}$$

(see Semmler and Zhang, 2005, p. 4, fn 1). Although this relationship looks a lot like eq. (11), it introduces an accelerationist element that is not present in (11). It is thus an open question whether (11) and (12) will lead to similar results. Unfortunately, the authors present their new specification without comment, so the reader is left with no hint as to what may have motivated this choice.

Let us begin with a constant Okun coefficient for the US economy. Using quarterly real GDP from the International Statistical Yearbook over the period 1961 − 2000, Semmler and Zhang (2005, p. 4) estimate eq. (11) with a highly significant $\beta_t = \beta =$ const. and a constant trend rate of growth as[19]

$$\Delta e_t \;=\; 0.364 \cdot (gY_t - 3.3) \;+\; u_t\;, \qquad\qquad R^2 = 0.734 \tag{13}$$

The value for R^2 is surprisingly high given that a quarterly rate of change is regressed on just one explanatory variable. So we should first check this result with our quarterly data

[19] Hemraj, Madrick and Semmler (2006, Table 1 on p. 4)) want to begin their discussion with the same result but present the t-ratios instead of the β-coefficient.

from the Fair-Parke software package for the firm sector.[20] In fact, on this basis we get for the same sample period a considerably lower goodness-of-fit, and also the (still highly significant) Okun coefficient is much smaller:

$$\Delta e_t \;=\; 0.232 \cdot (gY_t - 3.36) \;+\; u_t \;, \qquad\qquad R^2 = 0.394 \tag{14}$$

Despite these differences it is remarkable that the trend rates of growth in (13) and (14) are practically identical.

Taking the (at least theoretical) notion into account that causality runs from output to employment and that the adjustments of the latter may take place with some delay, we should try several lags of output growth on the right-hand side of (11). Indeed, including three lags of gY, all of the coefficients come out as distinctly significant and they decline with increasing lags even more nicely than could have been expected. The Okun coefficient furthermore rises from 0.232 to a more familiar order of magnitude like 0.424. On the other hand, the goodness-of-fit improves but still falls short of that in (13). In sum, the estimation yields

$$\Delta e_t \;=\; 0.424 \cdot (0.42 \cdot gY_t + 0.26 \cdot gY_{t-1} + 0.17 \cdot gY_{t-2} + 0.15 \cdot gY_{t-3} - 3.46) \;+\; u_t \;, \tag{15}$$
$$R^2 = 0.612$$

(Note that the coefficients on the growth rates add up to unity.)

The accelerationist version of eq. (12) proves to be an unsuitable alternative to (11). An estimation of (12) actually shows no relationship between Δe_t and ΔgY_t; we get an insignificant coefficient $\beta = 0.020$ and $R^2 = 0.004$ indicates the absence of any fit. A similar finding for their data might be the reason that for a constant coefficient estimation, Semmler and Zhang (2005) prefer specification (11), too. However, one wonders why (12) should then be a better basis for their subsequent estimation of time-varying coefficients. At least, a discussion of this point would have been helpful.

For the complementary level specification (10) of Okun's law we first need to decide on the trend. On the basis of the discussion in Section 3 and in order to have a uniform concept of detrending, we settle down on the Hodrick-Prescott procedure with the high smoothing parameter $\lambda = 102,400$. Thus, we define

$$e_t^{dev} \;:=\; e_t - e_t^\star \;, \qquad e_t^\star \text{ the HP 102,400 trend of } e_t \tag{16}$$
$$y_t \;:=\; \ln Y_t - \ln Y_t^\star \;, \qquad \ln Y_t^\star \text{ the HP 102,400 trend of } \ln Y_t \tag{17}$$

y_t is the output gap, which is also used in many other applications (except for our smoothing parameter), and e_t^{dev} may correspondingly be called the employment gap.

[20] To be precise, Y is real output of the firm sector, whereas for lack of more detailed data e is 1 minus the economy-wide rate of unemployment.

Because of the adjustment lags in employment already mentioned above and since it gives a better fit, we regress the employment gap not on the contemporaneous output gap but on one lag of y_t. In this way we obtain for the period 1961 – 2000,

$$e_t^{dev} \;=\; 0.407 \cdot y_{t-1} \;+\; u_t \;, \qquad\qquad R^2 = 0.840 \tag{18}$$

(The result is, however, not essentially different from the unlagged regression, which yields an Okun coefficient 0.396 with $R^2 = 0.786$).) The coefficient $\beta = 0.407$ is quite in line with the coefficient 0.424 in eq. (15) and suggests that, when dealing with a growth rate specification of Okun's law, we should better include the three lags of the output growth rate. In any case, for the firm sector an Okun coefficient

$$\beta \;\approx\; 0.40 \tag{19}$$

is still a good summary to describe the response of employment to changes in output.

4.2 Time variations in the Okun coefficient

The Okun coefficient depends (in part) on how firms adjust the number of jobs in response to temporary deviations in output from "normal". This adjustment depends in turn on such factors as the international organization of firms and the legal and social restrictions on hiring and firing. It is well-known that therefore the coefficient is quite distinct across different countries; and in conformity with everyday economic intuition the United States, where the labour market institutions are relatively flexible, exhibit the largest coefficient among the major industrialized countries (see, e.g., Blanchard, 2003, p. 185, and Semmler and Zhang, 2005, p. 4).

Blanchard (and others, of course) furthermore argues that the coefficient is also likely to change over time. Increased competition in goods markets has led firms in most countries to reconsider and reduce their commitment to job security (in exchange for the workers' loyalty), and their pressure on the government has resulted in a weakening of the legal constraints on hiring and firing. From this one expects a larger response of employment to fluctuations in output and thus a larger value of β, and indeed this holds true in many Western countries.

For the United States, however, the picture is less clear. In an estimation of the constant coefficient version of eq. (11) with annual rates of change, Blanchard (2003, p. 185) subdivides the forty years from 1960 to 2000 in two equally long samples and obtains for each subperiod the same coefficient $\beta = 0.39$ (in stark contrast to Germany, UK and Japan, where the coefficient for the second half is considerably higher). As shown in Table 2, a very similar outcome is obtained if the same procedure is applied to our quarterly data from the firm sector. Re-estimating eqs (15) and (18) correspondingly, the coefficient is even lower in the second period 1981 – 2000, although very slightly so.

	1961 – 2000	1961 – 1980	1981 – 2000
β from Blanchard	0.390	0.390	0.390
β from eq. (15)	0.424	0.444	0.413
β from eq. (18)	0.407	0.411	0.402

Table 2: Estimations of the Okun coefficient β over two subperiods.

Besides, the simultaneously estimated trend rate of growth in eq. (15) is $gY^\star = 3.91\%$ for 1961 – 1980 and $gY^\star = 3.00\%$ for 1981 – 2000, which confirms the confidence in these estimations.

We can extend the idea of comparing different subsamples by estimating the regression over consecutive intervals of time and plotting the values of the resulting Okun coefficients as a (presumably) rather smooth time series. For simplicity, let us consider the level version of Okun's law, the regression approach of eq. (18) over a shorter period of time. Choose the length of the rolling sample period shorter than the 20 years of Table 2 but not too short; 10 years, say. We also exploit the full sample of data that we have available from the Fair-Parke package, whose final quarter is 2005:2. The outcome of this battery of regressions is shown in Figure 15, where at time t the coefficient β is plotted that results from an estimation over the *past* 10 years.

Figure 15 gives a more pronounced picture than Table 2. The most conspicuous feature in the diagram is the decline (rather than an increase) of the coefficient from 1990 on. Note, however, that the coefficient at t = 1995:1, for example, does not capture a connection prevailing in this quarter, but summarizes the relationship between output and employment from 1985:1 until 1995:1.[21] Hence, the reasons for the decline of the coefficient in the 1990s have already to be sought in the 1980s. It may also be observed that though β is consistently falling over the 1990s, the reduction is not overly dramatic (the scale on the y-axis exaggerates the phenomenon a bit).

The advantage of the method of the rolling sample period is its simplicity; its meaning as well as its limitations are immediately clear. On the other hand, each $\beta = \beta_t$ is separately estimated from the others and only uses the information of the short sample. Therefore integrated methods are considered to be preferable, where the whole series of the β_t is

[21] This could perhaps be emphasized by plotting the coefficient at the mid-point of the rolling sample interval, at t = 1990:1 in this case.

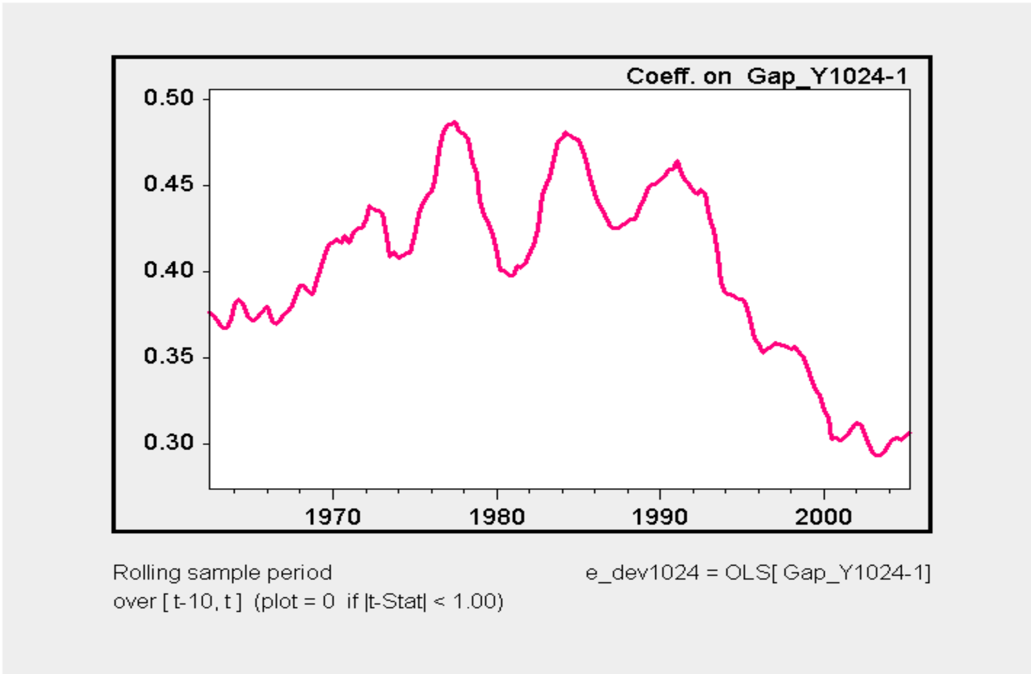

Figure 15: Estimation of the level version of Okun's law with a rolling sample period of 10 years.

estimated in a joint effort.[22] In any case, it is interesting to compare Figure 15 to a readily available plot of a time-varying coefficient from the literature that is based on such an integrated approach. To this end we reproduce in Figure 16 the panel for the US data of Figure 3 in Semmler and Zhang (2005, p. 7; which is one of four such panels for the US and other countries).[23] This diagram plots the deviations of the time-varying β_t from the full sample estimate of a constant $\beta = \bar{\beta}$.

The common feature of Figures 15 and 16 is the decline of the Okun coefficient that sets in around 1990, and that the coefficient tends to increase in the decades before. The main difference is the relative size of the changes in the two periods before and after 1990. In Figure 16 the increase of β_t in the first stage is roughly twice as strong as the subsequent decline, whereas in Figure 15 it is almost the other way round.

Apart from this asymmetry, it is remarkable that the changes of the Okun coefficient are of a similar order of magnitude. Unfortunately, Figure 16 does not indicate the levels of the β_t (the original paper by Semmler and Zhang does not make them explicit, either). Moreover, the estimations of these coefficients are based on the accelerationist version (12) of Okun's law, where in lack of a more detailed presentation in Semmler and Zhang (2005)

[22] Although in our opinion this argument is not fully convincing. Part of the search for technical refinements may well be an end in itself (or convention or just an exhibition of technical skills).

[23] The (4-panel) diagram can also be found in Hemraj, Madrick and Semmler (2006, p. 5).

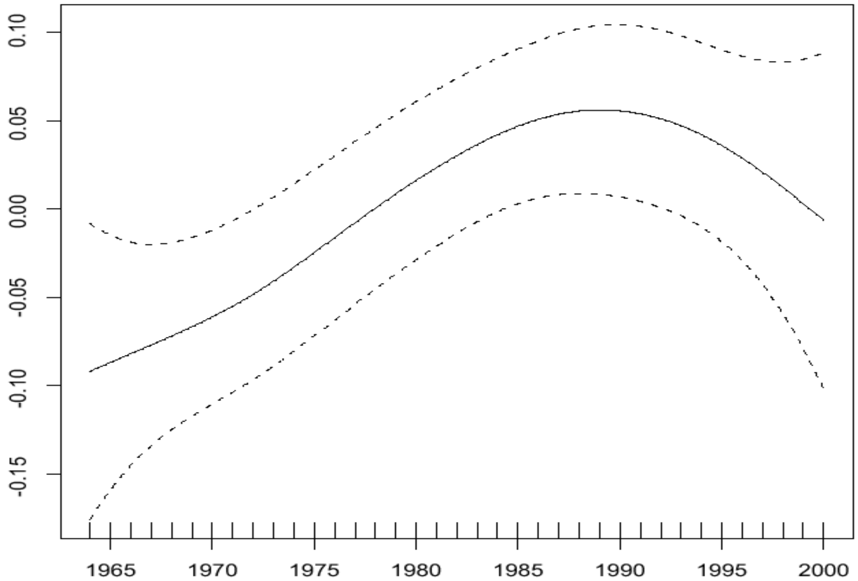

Figure 16: The time-varying Okun coefficient from Semmler and Zhang (2005).

Note: Reproduced from Semmler and Zhang (2005, p. 7, Figure 3). The y-axis gives the deviations of β_t from the estimate of a constant $\bar{\beta}$ over the full sample period. The dotted lines are confidence bounds.

we have already expressed our doubts whether this specification can provide a sound basis. Nevertheless, Figure 16 prompts us to estimate a time path of the Okun coefficient with the spline method described (and selected) in the methodological Section 2.2 above, using our firm sector data.[24]

As before, we should try both the level and the first-difference specification of Okun's law. Beginning with the latter, which is closer to the accelerationist version underlying Figure 16, we include the three lags of eq. (15). Regarding the trend growth of output, the constant rate of 3.46% in (15) is replaced with a variable HP 102,400 trend $gY_t^\star$. Thus, we estimate

$$e_t - e_{t-1} \;=\; \sum_{k=0}^{3} \gamma_{k,t}\,(gY_{t-k} - gY_{t-k}^\star) \;+\; u_t \qquad (20)$$

[24] Semmler and Zhang make use of an estimation approach that is substantially different from all of the methods mentioned in Section 2. It is somewhat reminiscent of the general Hodrick-Prescott idea, where, however, the smoothing parameter is now endogenously determined; see Semmler and Zhang (2005, pp. 5f) for a rough sketch.

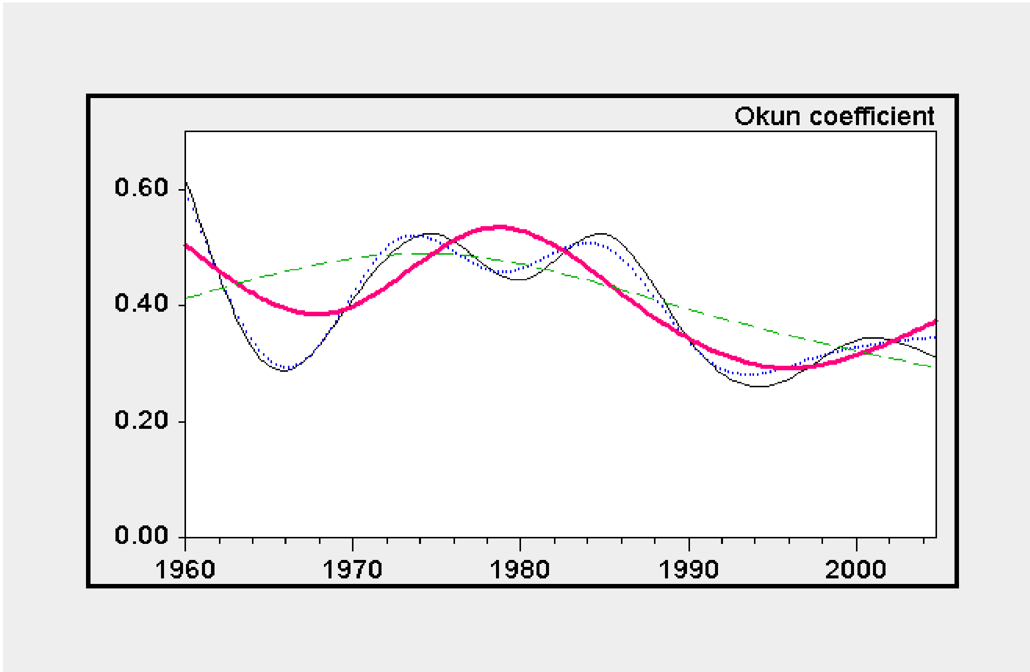

Figure 17: The time-varying Okun coefficient from eqs (20), (21).

Note: The estimations are based on the following number of segments for the spline functions: 3 (dashed line), 5 (bold), 7 (dotted), and 10 (thin solid line).

with the spline method. Our time-varying Okun coefficient β_t is then given by the sum of the coefficients on the single output growth rates,

$$\beta_t \;=\; \sum_{k=0}^{3} \gamma_{k,t} \tag{21}$$

Using 3, 5, 7, and 10 segments over the sample period 1960:1 − 2004:4, the result is shown in Figure 17. Generally the time patterns of the β_t are closer to Figure 15 than to Figure 16 from Semmler and Zhang. This even holds true for the least variable dashed line in Figure 17 that is generated by the 3-segment spline version, where the decline of β_t begins much earlier and is proportionally much than in Figure 16.

The more variable time paths in Figure 17 arising from the 5-, 7-, and 10-segment splines share the property of a decline sometime in the second half of the sample period. Because of the different degrees of variability it begins at different dates, but it always sets in at least five years earlier than in Figure 16. This finding calls into question the interpretation of Figure 16 that is given in Hemraj, Madrick and Semmler (2006, p. 5), accroding to which the diagram "shows that the response of unemployment to growth rates steadily moved down since the beginning of the 1990s. The US case clearly shows a decline of the response of employment to economic growth—thus, a jobless recovery, as some have called it." The changes indicated by the Okun coefficient are apparently less unique than that.[25]

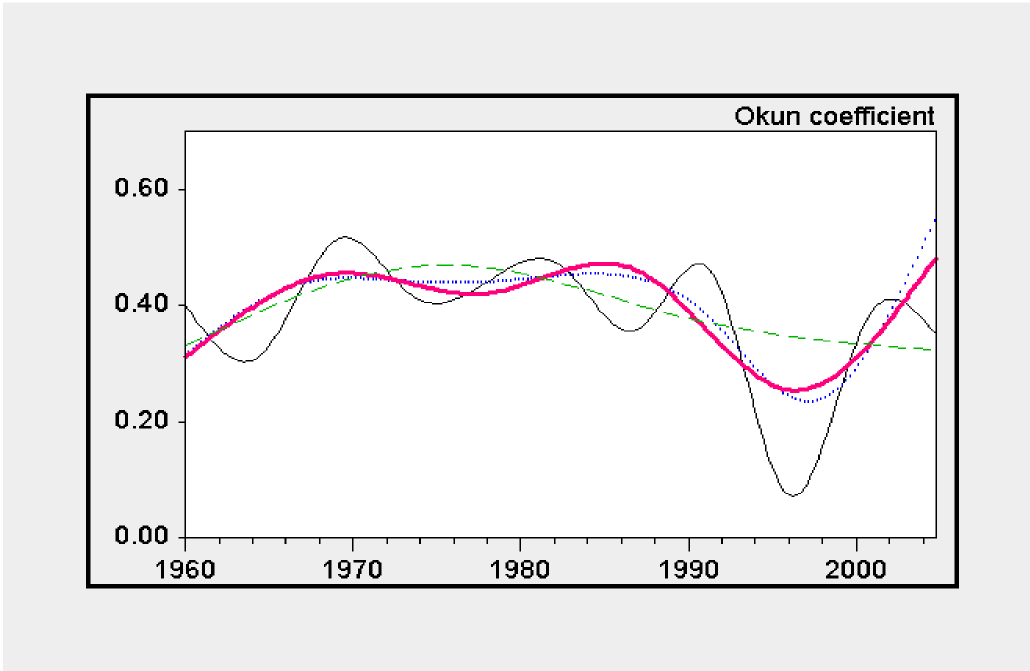

Figure 18: The time-varying Okun coefficient from eq. (10).

Note: The estimations are based on the following number of segments for the spline functions: 3 (dashed line), 5 (bold), 7 (dotted), and 10 (thin solid line).

Figure 18 checks the results of Figure 17 by employing the level version (10) of Okun's law, where again β_t is determined by deterministic splines on the basis of 3, 5, 7 and 10 segments, respectively. This approach, too, yields falling values of the Okun coefficient, though the decline begins later than in Figure 17. The diagram also reveals a feature that was only relatively weakly indicated in Figure 17, namely, from the mid-1990s on the coefficient begins to rise again—in Figure 18 to previous or even higher levels.

As discussed in the methodological section, if asked for a final decision regarding the number of the underlying segments for the splines, we would settle down on the 5-segment versions that are plotted as the bold lines in Figures 17 and 18. For a better comparison, they are reproduced together in an extra time series diagram. Figure 19 shows that the coefficients arising from the two approaches of levels and first differences to Okun's law vary over a similar range. Also their troughs in the 1990s are not too widely apart. The earlier variations, however, are "out of phase". This indicates that these motions should not be overinterpreted, or the interpretation must take the specific version into account to

[25]We do not wish to deny that the recovery after the 1991 recession was of a jobless nature. But this has to be shown by more detailed methods—and the authors in fact do this on the subsequent pages.

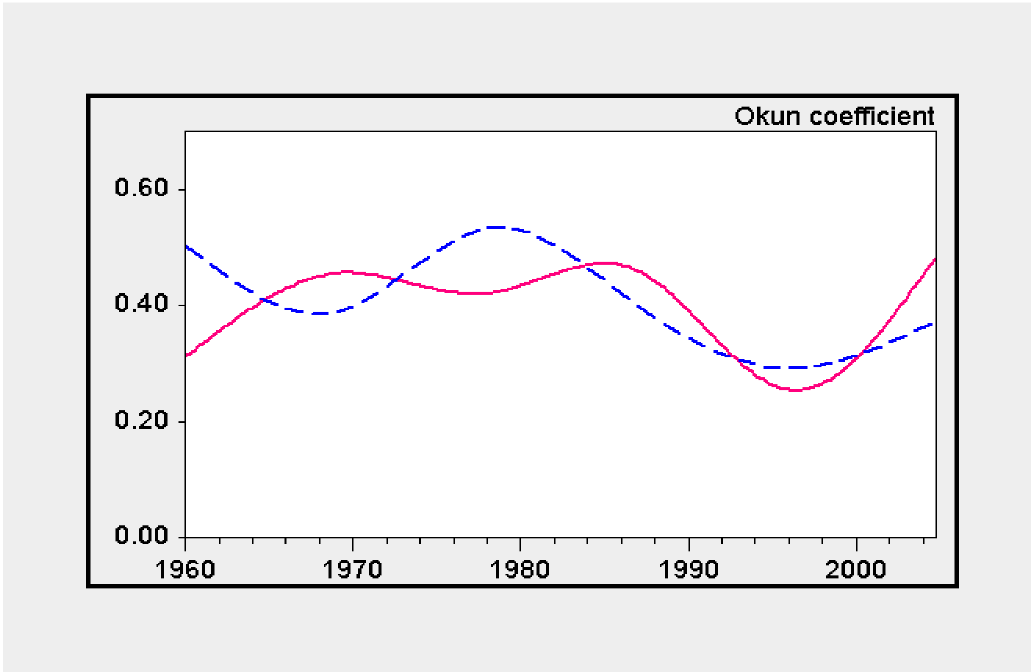

Figure 19: Time-varying coefficients β_t from the level (solid line) and first-difference (dashed line) specification of Okun's law.

Note: The diagram reproduces the bold lines of the 5-segment estimations from Figures 17 and 18.

which it refers. If we do not go into these details, the differences in the oscillation of β_t can be reconciled by concluding that until the early 1980s the Okun coefficient did not vary too systematically. By contrast, the subsequent decline of the coefficient with a more or less strong "recovery" appears to be a more universal phenomenon.

4.3 Comovements of the components of output and employment

Interpretations of changes of the Okun coefficient in the literature, or at least their formulations, tend to identify employment with the employment rate, thus speaking of the "response of employment" to the variations in output or output growth. This habit neglects that the employment rate is a composite variable, or it assumes that there are no significant changes in the evolution of the other variable(s) involved. In this subsection we want to shed some light on this issue, where we go beyond the simple definition of the employment rate as a ratio of labour demand and supply. The light will perhaps not be very bright but it will still suffice to make it clear that an explanation of the changes in the Okun coefficient over the last 10 or 15 years must take more variables into account than e and Y.

To this end we decompose total output into labour productivity, hours per job, the employment rate, and the labour force. We define:[26]

E	employment, i.e. number of jobs
H	total hours worked per quarter
L	the labour force (number of heads)
e	employment rate, $e = E/L$
h	hours per job, $h = H/E$
z	output per hours (labour productivity), $z = Y/H$

Similarly as in Gordon (2003, p. 212), for example, we consider the output identity,[27]

$$Y \;=\; \frac{Y}{H} \cdot \frac{H}{E} \cdot \frac{E}{L} \cdot L \;=\; z \cdot h \cdot e \cdot L \tag{22}$$

In studying phenomena like the abovementioned jobless growth, the increase of trend productivity growth over the past 15 years has certainly a role to play. In this report, however, we limit ourselves to changes in the relationship between output and employment that abstract from this kind of structural change. That is, we are looking for possible changes in factors that are more directly related to a business cycle frequency. Accordingly, we begin by first detrending the variables. As before, the trend is uniformly given by the HP 102,400 filter, from which we define for an arbitrary dynamic variable $x = x_t$ and its trend $x_t^\star$,

$$x_t^{dev} \;=\; (x_t - x_t^\star)/x_t^\star \tag{23}$$

Although somewhat clumsy, we will maintain the superscript '*dev*' to avoid any confusion on that. By logarithmic differentiation of (22) we then see that the output gap y_t is composed of the following sum of percentage deviations,[28]

$$y_t \;=\; z_t^{dev} \;+\; h_t^{dev} \;+\; e_t^{dev} \;+\; L_t^{dev} \tag{24}$$

[26] The empirical time series entering here are readily available from the Fair-Parke database. E is the series they call JF (number of jobs in the firm sector, in mill.); hours H are obtained from their series HF (number of hours paid per job in the firm sector, per quarter), thus $H = \text{HF} \times \text{JF}$; the ratio E/L for the firm sector is identified with the economy-wide employment rate, which is $1 - \text{UR}$ (UR the unemployment rate); consequently, the somewhat artificial variable labour force 'in the firm sector' is constructed by dividing the number of jobs by the employment rate, $L = \text{JF}/(1 - \text{UR})$. Incidentally, Fair-Parke's own series PROD for labour productivity in fact coincides with the series Y/H.

[27] Gordon (2003) works with a more detailed decomposition and relates output, not to the labour force L, but to the working-age population N, so that L in (22) becomes $(L/N) \cdot L$ and he can also consider the labour force participation rate. Apart from that, his output measure is GDP, whereas quarterly productivity data are only available for the firm sector. In this way he additionally introduces (*i*) a so-called 'mix effect', which he defines as the ratio of output per payroll employee in the total economy to that in the firm sector; and (*ii*) the ratio of total employment in the payroll survey to that in the household survey (cf. Gordon, 2003, p. 212). For our purposes we can neglect this detailed differentiation and soak up all these effects in the labour force variable L.

[28] To be exact, e_t^{dev} as defined by (23) now differs from the definition as a pure difference $e_t - e_t^\star$ in eq. (16). Of course, the numerical differences are only minor.

This output gap decomposition can also be solved for the employment gap, which gives

$$e_t^{dev} \;=\; y_t \;-\; z_t^{dev} \;-\; h_t^{dev} \;-\; L_t^{dev} \tag{25}$$

Comparing this identity to eq. (10), it is seen that the regularity of Okun's law, with a stable coefficient β, is dependent on labour productivity and hours per job moving in a procyclical fashion (and a smaller amplitude than the output gap itself), while the cyclical variations of the labour force are preferably small or at least unsystematic. We are now going to check if, or how far, this view is (still) warranted.

The dotted in each of the six panels in Figure 20 displays the output gap y_t as the measure of the business cycle. It is contrasted with the four components of y_t in (24). The two panels at the bottom add the trend deviations of jobs E_t^{dev} and hours H_t^{dev}. The latter two variables are in fact strongly procyclical, where hours move roughly one-to-one with output and perhaps somewhat surprisingly, there are longer spells of time where also the amplitude of employment is not much smaller than that of output. We can conclude from the latter that the main reason for a coefficient β_t in a relationship like $e_t^{dev} = \beta\, y_t + u_t$ to be around 0.40 derives from the fact that the labour force is not growing at a nearly constant rate but shows some variation around its trend path, too. In fact, given that E_t and Y_t move relatively closely together so that for illustrative purposes one may write $E_t^{dev} \approx \gamma\, Y_t^{dev} = \gamma\, y_t$, where γ is not much less than unity, the equation

$$e_t^{dev} \;=\; E_t^{dev} \;-\; L_t^{dev} \;\;\approx\;\; \gamma\, y_t - \; L_t^{dev} \tag{26}$$

demonstrates that $\beta_t \approx 0.40$ and the more persistent variations of this coefficient are mainly due to the behaviour of the supply variable, i.e., the fluctuations of the labour force. However, L_t^{dev} in the fourth panel of Figure 20 displays no consistent cyclical pattern.

Regarding the labour demand variable it is nonetheless remarkable that while most of the sample period employment moves synchronously with output or lags slightly behind, we observe a certain lead of employment in the mod-1990s. This not only holds true for the number of jobs but also for total hours.

The employment rate itself appears to maintain roughly the same pattern over the whole 45 years of the sample period. In this respect, hours per job (h_t^{dev}) do not seem to be a very dramatic variable, either. In contrast, the cyclical pattern of labour productivity $z_t^{dev} = Y_t^{dev} - H_t^{dev} = y_t - H_t^{dev}$ in the top panel of Figure 20 undergoes a severe change. It is largely procyclical, with a certain lead, until the mid-1980s.[29] But from the end of the 1980s on, this type of behaviour has completely disappeared!

The summary statistics for the amplitudes and comovements of the variables in Table 3 make these qualitative observations on possible "regime shifts" more precise. The table

[29] The author faintly recalls a remark in a working paper (perhaps by O. Blanchard) some 10 or 15 years ago, where it was stated that if there is a stylized fact of the business cycle at all, then it is the procyclicality of labour productivity (which was seen to be closely connected to Okun's law).

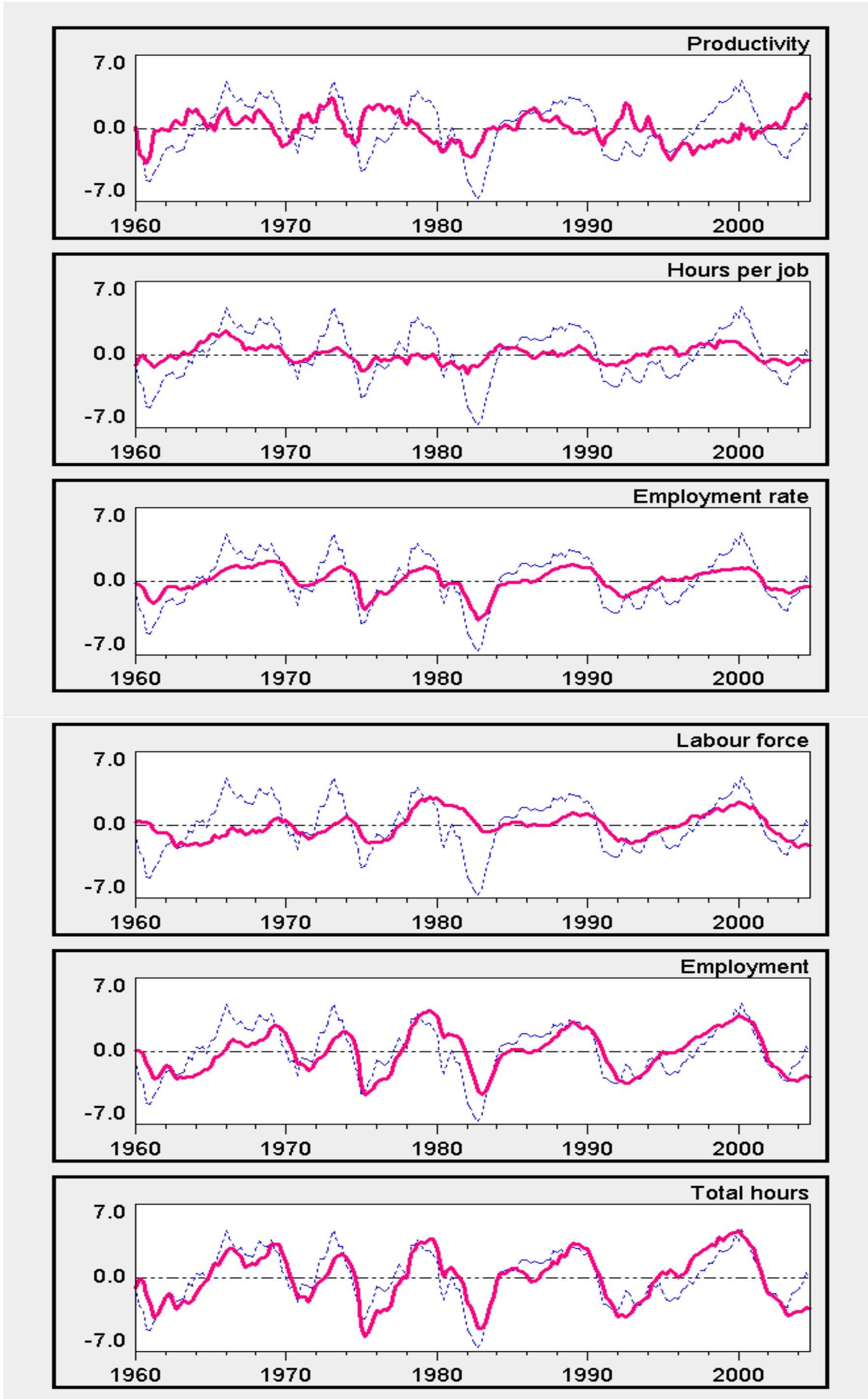

Figure 20: Percentage trend deviations z_t^{dev}, h_t^{dev}, e_t^{dev}, L_t^{dev}, E_t^{dev}, H_t^{dev} (the dotted line is the output gap y_t).

subdivides the 45 years from 1960 to 2004 into the two periods 1960:1 – 1983:4 and 1984:1 – 2004:4; the corresponding figures are given in the first two rows for each variable e_t^{dev}, E_t^{dev}, etc. In addition, special emphasis is put on the 1990s, for which purpose the third row adds

		cross correlations between y at time t and x at time						
Series x	σ_x/σ_y	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$
	0.49	0.47	0.62	0.78	0.90	0.94	0.91	0.83
e^{dev}	0.42	0.63	0.72	0.78	0.84	0.86	0.84	0.77
	0.42	0.61	0.71	0.79	0.85	0.86	0.82	0.73
	0.75	0.13	0.32	0.52	0.69	0.78	**0.82**	0.79
E^{dev}	0.95	0.62	0.71	0.78	0.84	0.86	0.86	0.81
	1.02	0.64	0.72	0.80	0.85	**0.87**	0.85	0.79
	0.88	0.35	0.53	0.70	0.84	**0.89**	0.87	0.80
H^{dev}	1.19	0.69	0.76	0.81	0.83	0.83	0.80	0.73
	1.29	0.72	0.78	0.83	**0.85**	0.84	0.79	0.71
	0.54	**0.67**	0.65	0.58	0.48	0.28	0.09	−0.07
z^{dev}	0.66	−0.03	−0.01	0.01	0.02	−0.05	−0.12	−0.16
	0.70	**−0.32**	−0.24	−0.18	−0.14	−0.21	−0.28	−0.31
	0.33	0.65	0.68	**0.70**	0.69	0.59	0.46	0.32
h^{dev}	0.33	0.68	0.69	0.66	0.61	0.53	0.44	0.32
	0.36	**0.78**	0.77	0.73	0.66	0.57	0.46	0.34
	0.45	−0.30	−0.15	0.00	0.16	0.28	0.37	**0.42**
L^{dev}	0.55	0.59	0.66	0.74	0.78	0.81	0.82	0.80
	0.62	0.63	0.70	0.78	0.81	0.84	**0.85**	0.81

Table 3: Comovements with the output gap.

Note: The first row for each variable is based on the subperiod 1960:1 − 1983:4, the second on 1984:1 − 2004:4, the third on 1990:1 − 2004:4. σ denotes the standard deviation.

the results for the fifteen years 1990:1 − 2004:4. We point out the following features arising from this investigation.

1. The cyclical behaviour of the employment rate does not show great changes over the entire sample period. On average, the employment rate lags one or two quarters behind the output gap, while the amplitude of its variations has slightly decreased (rather than increased). The latter information is given in the second column, which computes the standard deviation of the variables and expresses it as a fraction of the standard deviation of the output gap.

2. Total employment lags one or two quarters behind output. There is a weak indication that the delay in the employment adjustments may have somewhat shortened in the last twenty years, but the evidence with this quarterly data is not yet very conclusive. Much stronger, however, is the evidence of an increasing amplitude in the fluctuations of the number of jobs, from 3:4 in relation to output to almost exactly 1:1 (cf. the bold face figure in the second column).

3. Total hours exhibited a one-quarter lag in former times but now move synchronously with output. Even more importantly, a substantial "overreaction" of hours has in the meantime developed. While until the 1980s hours varied somewhat weaker than output, in a proportion of roughly 9:10, this relationship has reversed and on average, from the 1990s on, a one percent change in output leads to a 1.3 percent change in hours worked.[30]

4. Obviously, the latter feature must affect the cyclical properties of labour productivity $z = Y/H$. Actually, until the 1980s z_t^{dev} was (perhaps not strongly but) distinctly procyclical, with a lead of three quarters, whereas over the last fifteen years productivity moves in a weak but rather countercyclical fashion. In contrast to the changing features of total employment and hours, this phenomenon is more clearly visible in the top panel of Figure 20.

5. The utilization of the workforce as represented by hours per worker has not changed very much. It is nevertheless notable that the previously contemporaneous movements now exhibit a slight lead of one or two quarters and that the connection with output has become somewhat tighter.

6. The labour force as the supply variable has substantially changed its cyclical characteristics. It was weakly procyclical with a one-year lag and a relative amplitude of 0.45 until the 1980s.[31] Over the last fifteen years the amplitude has increased to 0.62. Also, the labour force now follows economic activity much more closely, as indicated by the cross correlation coefficient of 0.85 at a lag of two quarters. Besides, the connection is not much weaker for the contemporaneous movements.

To return to Okun's law, we simplify the comovements of employment and the labour force with output and posit $E_t^{dev} \approx \gamma_E \, y_t$, $L_t^{dev} \approx \gamma_L \, y_t$. This allows us to relate the employment rate to the output gap as

$$e_t^{dev} \;=\; E_t^{dev} - L_t^{dev} \;\approx\; (\gamma_E - \gamma_L)\, y_t \;=:\; \beta\, y_t \tag{27}$$

[30]In a next step it may be interesting to study this relationship separately for booms and recessions.

[31]The cross correlation of L_{t-k}^{dev} with y_t is 0.44 for a lag $k = 4$ over this subperiod, which is slightly higher than the coefficient 0.42 for a three-quarter lag in Table 3.

which is of the same form as eq. (10) above. The coefficient γ_E may be approximated by the ratio of the standard deviations σ_E/σ_y multiplied by the contemporaneous cross correlation coefficient between E_t^{dev} and y_t; and analogously for the coefficient γ_L on the labour force. For the first and third sample period considered in Table 3 we then obtain:

$$\begin{array}{llcccccccc} 1960{:}1-1983{:}4 & : & \beta = \gamma_E - \gamma_L & = & 0.75 \cdot 0.69 & - & 0.45 \cdot 0.16 & = & 0.45 \\ 1990{:}1-2004{:}4 & : & \beta = \gamma_E - \gamma_L & = & 1.02 \cdot 0.85 & - & 0.62 \cdot 0.81 & = & 0.36 \end{array}$$

Hence the proxy for the Okun coefficient from this back-of-the-envelope calculation does not only decline, the numbers are also of a similar order of magnitude to that in Figure 18 for the time-varying level specification of Okun's law.

In this way we can see two different mechanisms acting on β. First the difference of the amplitudes of E_t^{dev} and L_t^{dev}. The amplitudes both increase, and since the amplitude of E_t^{dev} rises more than that of L_t^{dev}, we have a positive influence on β. The second mechanism originates with the contemporaneous cross correlations of E_t^{dev} and L_t^{dev} with output, both of which increase, too. Here, however, the change for L_t^{dev} is much stronger than for E_t^{dev}, so that on the whole the negative influence from the labour force becomes dominant. This stylized explanation underlines the significance of the labour supply variable and identifies the changes in its cyclical behaviour as the most important contribution to the recent variation in the Okun coefficient.

In other words, if one wants to study macroeconomic changes in the employment policy of firms over the business cycle, one should better directly refer to the volume of employment rather than to the rate of employment as a composite variable, even if Okun's law has an honourable tradition. The brief summary in point 2 above is here a first pertinent observation.

5 A model of a simple recruitment policy of firms

This short section is devoted to a simplified determination of hours and employment that would allow an easy integration into a low-dimensional macrodynamic framework. Concerning hours it completely abstracts from cyclical variations of labour productivity. By contrast, as regards the number of jobs, it proposes an active recruitment policy of firms in a straightforward manner, which is based on the following points.

1. The adjustments of employment to the changes in production are not instantaneous but take place in a gradual manner.
2. Firms hire additional workers (above normal growth) if the workforce is currently overutilized, they (relatively) decrease the number of jobs if the average employee works less than normal.
3. Firms pay attention to the situation on the labour market, in the sense that they are willing to operate at higher utilization rates of the workforce, without creating additional jobs, if the labour market tightens, i.e. if the employment rate has risen.

We repeat the notation in order to be self-contained. Thus, to specify the ideas, let H denote total hours, E the number of jobs, or employment, $h = H/E$ the average hours per job, and h^n the normal hours per job. The utilization of the workforce within firms is given by $u_w = h/h^n$, while with L being the labour force, the (outside) employment rate is $e = E/L$. We assume the existence of a so-called natural rate of employment, e^o, whose foundations are not explained within the model and which directly serves as a benchmark in several of the behavioural functions below. Potential output Y^p derives from the current labour force, corrected for the natural employment rate, labour supply and an exogenously given level z of labour productivity (in hours), $Y^p = e^o\, z\, (h^n L)$. Actual inputs of hours are supposed to be governed by solely technological factors, so that actual output and hours are linked by the same number, $Y/H = z$; $H = Y/z$ might be conceived as a short-period production function.[32]

By virtue of the latter assumption, the utilization of the workforce can be simply expressed as the ratio of capacity utilization and the employment rate. Formally, with $y = Y/Y^p$ for capacity utilization we get $u_w = h/h^n = H/h^n E = (H/Y) \cdot (Y/Y^p) \cdot (Y^p/h^n L) \cdot (L/E) = (1/z) \cdot y \cdot e^o z \cdot (1/e)$,

$$u_w \;=\; y\, e^o \,/\, e \tag{28}$$

The utilization of the workforce may play a role in a wage Phillips curve that takes outsider as well as insider effects into account, so that the nominal wage changes depend (positively)

[32] This productivity may be conceived of as growing at some constant rate, but since z cancels out in the following discussion and other parts of a full macro model may be constructed likewise, the precise assumption on the evolution of z does not matter.

on both the employment rate e and the utilization of the workforce u_w. Eq. (28) points out that then, *ceteris paribus*, a wage increase from a rising employment rate is mitigated by the simultaneous reduction in the average hours worked per job. On the other hand, if a positive demand shock is not sufficient for firms to raise employment (relative to the growing labour force), there is nevertheless still a certain pressure on wages through the insider effect, which in (28) shows up directly as a rise in capacity utilization.

While the changes in capacity utilization would be the subject of another part in a full macro model, the adjustments in employment can here be specified in accordance with the features 1−3 listed above. With respect to a benchmark level of the utilization of the workforce, $\tilde{u}_w$, and the growth rate of the labour force as a benchmark for the normal growth of jobs, the first two points can, in continuous time, be described by an equation like $\hat{E} = \hat{L} + \beta_e (u_w - \tilde{u}_w)$ $(\beta_e > 0)$. The third points suggests making the utilization benchmark an increasing function of the employment rate, $\tilde{u}_w = \tilde{u}_w(e)$ with $d\tilde{u}_w/de \geq 0$. Referring to the changes $\hat{e} = \hat{E} - \hat{L}$ of the employment rate, the recruitment policy of firms is summarized by $\hat{e} = \beta_e [\, u_w - \tilde{u}_w(e)\,]$.

While as a first idea this equation may appear a plausible approach to delayed adjustments of employment, it has to be checked that it is (broadly) compatible with the (quarterly) data. To this end let $\tilde{u}_w$ be a linear function of the employment rate, which with respect to the steady state employment rate e^o can be written as $\tilde{u}_w(e) = 1 + \gamma_o + \gamma_e(e - e^o)$. For the estimation, $h^n = h^n_t$ entering the definition of $u_{w,t}$ and $e^o = e^o_t$ are obtained as a Hodrick-Prescott trend (with a large smoothing parameter $\lambda = 102,400$). The corresponding regression is then given by

$$4\,\Delta e_t = \alpha_o + \alpha_u\,(u_{w,t} - 1) + \alpha_e\,(e_t - e^o_t) + \eta_t \qquad (29)$$

$$\eta_t = \rho\,\eta_{t-1} + \varepsilon_t \qquad (30)$$

Neglecting the (in absolute terms) low variability in e_t and multiplying the continuous-time equation for $\hat{e}$ by the model's steady state value e^o, which is a constant, the coefficients are related by the equations $\alpha_u = \beta_e\, e^o$, $\alpha_o = -\beta_e\, \gamma_o\, e^o$ and $\alpha_e = -\beta_e\, \gamma_e\, e^o$. Accordingly, the coefficient α_u should come out positive and significant, while α_e if it turns out significant should be negative. The autoregressive error terms in (29) are a short-cut to capture the other effects in the employment adjustments.

In all estimations that we performed the constant γ_o proved to be insignificant and the fit was hardly affected when it was excluded. The goodness-of-fit is, however, heavily dependent on including the AR(1) error process; the constraint $\rho = 0$ reduces R^2 to an order of magnitude as low as 0.23 (at most), with a Durbin-Watson statistic less than one.

Apart from the inconsistency problems from residuals that may be correlated with some of the regressors, an OLS estimation does not prove to be very attractive since it yields an undesired positive estimate of α_e. The correct negative sign is obtained if 2SLS or GMM are employed, where with all sets of instrumental variables that we explored, the GMM fits were clearly superior. As instrumental variables we considered Δe_{t-1} and several lags of $(u_{w,t}-1)$ and $(e_t - e_t^o)$. It seems that satisfactory fits require up to eight lags of the latter two variables. For $(u_{w,t}-1)$ we also included the contemporaneous values. As further variations of the lags did not lead to noteworthy improvements, we may settle down on the estimation reported in Table 4 .

$u_{w,t}-1$	$e_t - e_t^o$	ρ	R^2	SER	DW	J
0.85	−0.58	0.56	0.40	1.00	1.93	0.078
(4.31)	(−2.83)	(10.9)				

Table 4: GMM estimation of (2) and (3) over 1961:1 − 2003:1.

Note: t-values in parentheses, and standard errors of ε_t in percentage points; for the instrumental variables see text.

From the J-statistic, the null hypothesis that the overidentifying restrictions regarding the instrumental variables is satisfied can be inferred to be acceptable.[33] Although eqs (29) and (30) do not purport to represent the "true" data generation process, the results of Table 4 appear sufficiently credible to be employed in a small macrodynamic model.

The structural coefficients of the employment module are recovered from the above relationships between the α, β and γ coefficients. Assuming a steady state employment rate $e^o = 0.94$, they result as $\beta_e = \alpha_u/e^o = 0.90$ and $\gamma_e = -\alpha_e/\beta_e e^o = -\alpha_e/\alpha_u = 0.65$ (slightly rounded). In sum, the simple recruitment policy here proposed and numerically specified, is described by the two equations:

$$\hat{e} = \beta_e \, [\, u_w - \tilde{u}_w(e) \,] \, , \qquad \beta_e = 0.90 \tag{31}$$

$$\tilde{u}_w(e) = 1 + \gamma_e \, (e - e^o) \, , \qquad \gamma_e = 0.65 \tag{32}$$

[33]Under the null with $\ell - k = (1+9+8) - 3$ overidentifying restrictions, the J-statistic times the number of observations (169) is asymptotically χ^2-distributed. The resulting p-value is 0.59.

6 Gradual adjustments of hours and employment

In this section, we put forward a model where firms gradually adjust employment as well as hours in response to certain gaps that constitute a disequilibrium for them. While the model does not make any direct reference to Okun's law, it will be seen that the first-differences version of Okun's law emerges as a special case when several reaction coefficients are set to zero.

6.1 Theoretical framework

6.1.1 Preliminaries

For the following theoretical discussion a number of new variables are introduced. They are combined with other variables already known, but for convenience we list the new symbols together with the old ones that are here relevant.

E	employment, i.e., number of jobs (the workforce);
E^d	desired number of workers corresponding to H^d;
e	employment rate; $e = E/L$;
e^o	normal, or 'natural', rate of employment;
g_y^e	rate at which firms expect their output to grow (over the next one or two years, the medium-term);
H	total hours worked;
H^d	desired hours by firms to produce current output;
h	average hours per job, $h = H/E$;
h^n	normal hours per job, which change at the growth rate $g_h = \hat{h}^n$;
L	labour force, which grows at the rate $g_\ell = \hat{L}$;
u	utilization with respect to potential output, $u = Y/Y^p = 1 + y$;
u_w	utilization of the workforce;
Y^p	potential output;
y	the output gap, $y = (Y - Y^p)/Y^p$;
z	actual labour productivity; $z = Y/H$;
z^o	productivity under 'normal conditions' (the trend, representing the state of technology), which grows at rate $g_z = \hat{z}^o$.
ζ	ratio of actual to trend productivity, $\zeta = z/z^o$;

The growth rates g_z and g_ℓ, the normal working time h^n together with its (negative) growth rate g_h, and the normal rate of employment e^o are considered to be exogenous. While in a full-fledged theoretical model they may be supposed to be constant, it here suffices to assume that they are predetermined in the short period, or at the beginning of a quarter.[34] In the empirical work, the trend values of these magnitudes are adopted (again derived from the HP 102,400 trend). Trend and steady state values are designated by a superscript 'o'.[35]

In the formal analysis below we have recourse to the components of potential output and its growth rate. For the exposition of the model it is useful to specify the rates of change in continuous time. Substituting the steady state values in the output decomposition equation (22) and neglecting possible changes in the normal rate of employment, we obtain

$$Y^p = e^o \, z^o \, h^n \, L \tag{33}$$
$$\hat{Y}^p = g^o := g_z + g_h + g_\ell \tag{34}$$

Desired hours H^d by firms to produce their current output correspond to production under normal conditions, which is a state where labour productivity attains its normal level z^o. E^d is the corresponding volume of employment, when workers in that state work normal time. Hence,

$$H^d = Y \,/\, z^o \tag{35}$$
$$E^d = H^d \,/\, h^n \tag{36}$$

The model contains two utilization variables. One refers to output and potential output and represents utilization by $u = Y/Y^p$, which takes the role of the output gap y previously considered in this report ($u = 1 + y$, of course). The other concept refers to the 'stock' of workers currently employed. Here the utilization of the workforce relates actual hours H to the hours that the employed workers would normally work, which are given by $h^n E$. Accordingly,

$$u_w = H \,/\, (h^n E) = (H/E) \,/\, h^n = h \,/\, h^n \tag{37}$$

6.1.2 The two adjustment equations for employment and hours

The modelling of employment and hours is based on the assumption that the production decisions are made first. The determinants of the latter are not discussed within the present framework, so the time paths of output Y, utilization u, and in general also the expected output growth rate g_y^e are treated as exogenous. The two other control variables of the firm (besides prices and investment, which are here completely left aside) are (the flow of) hours H and (the stock of) workers E presently employed. Because of (material and immaterial)

[34] For example, considering the moderate comovements of the output gap and the lagged labour force, the growth rate of the latter might be specified as an endogenous variable that responds (weakly) to the recent output growth. This idea could be conveniently represented by a so-called adaptive expectations mechanism.

[35] In the description of the theoretical model we refer to the steady state rather than the trend values.

adjustment costs, which are not made explicit, E and H are mostly different from their normal or (in some sense) optimal levels.

The number of jobs as well as the total number of hours worked are predetermined in the short period. They adjust gradually over time in response to the disequilibria that the firms perceive, and in order to match up with future growth. Formally, this means that we specify the growth rates of employment and hours ($\hat{E}$ and $\hat{H}$) as functions of several benchmark and gap expressions. In detail, the following components are distinguished, which for simplicity all enter in a linear way.

1. Firms increase both employment and hours to account for the general growth trend. Regarding hours, the growth trend is given by the difference between the steady state output growth rate (g^o) and the rate of technological progress (g_z); regarding employment, the growth rate of normal hours per job (g_h) has to be subtracted in addition.[36] All rates are assumed to be known by the firms.[37]

2. The adjustments may exhibit certain inertia, such that the changes occurring in the previous quarter find some reflection in the present quarter, too. This is most conveniently specified by including the respective growth rates of the previous quarter in the terms determining $\hat{H}$ and $\hat{E}$. To be consistent, the growth terms from this and the preceding point enter as weighted averages with weighting factors for the lagged growth rates β_{hg} and β_{heg}, respectively.

3. Firms seek to gradually bridge the gap between desired (H^d) and actual hours (H), which they do with adjustment speed β_{hh}. Likewise, they seek to close the gap between the number of desired (E^d) and actual jobs (E) with adjustment speed β_{ee}.[38] The gaps are specified as percentage deviations from the current levels, $(X^d - X)/X$ for $X = H, E$, and as before, a one-quarter lag is assumed.

4. Firms increase (decrease) both employment and hours if they expect their output to grow faster (more slowly) than the trend, i.e., if g_y^e exceeds (falls short) of g^o; the corresponding speeds of adjustment are β_{ey} and β_{hy}, respectively.

[36] Consider the identities $H = Y \cdot (H/Y) = Y/z$ and $E = Y \cdot (H/Y) \cdot (E/H) = Y/(zh)$ for the steady state values and subject them to logarithmic differentiation.

[37] Generally, firms may have subjective perceptions of what a proper trend growth rate might be, which they cautiously revise in the light of recent observations. Again these adjustments could be conveniently modelled as a (formally) adaptive expectations mechanism with a (very) slow speed of adjustment. The main dynamic properties of a fully formalized model should remain largely unaffected by this device, which may justify the short-cut (which, after all, is a universal type of simplification in macroeconomic modelling).

[38] One might argue that employment could also respond to the gap in hours, or that this additional possibility should be empirically tested. The two gaps in employment and hours are, however, strongly correlated, so that a distinct influence of the two variables cannot be properly identified.

5. Firms also increase (decrease) the number of jobs if their workforce works on average more (less) than the normal working time, i.e., if hours H exceed (fall short) of $h^n E$; the adjustment speed is designated β_{eh}.

Referring to discrete-time adjustments in a quarterly model, the (annualized) growth rates of hours H and employment E are thus determined by the following equations:

$$\hat{H} \;=\; \beta_{hg}\,\hat{H}_{-1} + (1-\beta_{hg})\,(g^o - g_z) \;+\; \beta_{hh}\Big(\frac{H^d - H}{H}\Big)_{-1} \;+\; \beta_{hy}\,(g^e_y - g^o) \tag{38}$$

$$\begin{aligned} \hat{E} \;=\;& \beta_{eg}\,\hat{E}_{-1} + (1-\beta_{eg})\,(g^o - g_z - g_h) \\ &+\; \beta_{ee}\Big(\frac{E^d - E}{E}\Big)_{-1} \;+\; \beta_{ey}\,(g^e_y - g^o) \;+\; \beta_{eh}\,\frac{H - h^n E}{h^n E} \end{aligned} \tag{39}$$

The equations are also the basis for the estimations below.

The gap between desired and actual hours does not seem to be a familiar variable. It is, however, nothing else than the trend deviations of labour productivity, since $H^d/H = (Y/z^o)\,/\,(Y/z) = z/z^o$. Hence,

$$\frac{H^d - H}{H} \;=\; \frac{z - z^o}{z^o} \;=\; \zeta - 1 \tag{40}$$

The gap terms with H^d and E^d can also be characterized the other way round; $(H - H^d)$ and $(E - E^d)$ are the amount of excess hours currently worked, and the amount of excess labour currently on hand, which firms seek to reduce in a gradual manner.

6.1.3 The adjustments in continuous time and intensive form

A straightforward way to translate (38) and (39) into a continuous-time formulation is to interpret the growth rates on the left-hand and right-hand sides as $\hat{H}_t$ and $\hat{H}_{t-\Delta t}$, respectively, which are based on a fixed time unit ($\hat{H}_t = (H_t - H_{t-\Delta t})/(\Delta t\, H)$) and Δt is the length of the adjustment period (the same applies to the growth rates of employment, of course). Regarding the weights, two polar cases are conceivable. First, we may assume that β_{hg} is multiplied by the length of the adjustment period, so that

$$\hat{H}_t \;=\; \Delta t \cdot \beta_{hg}\,\hat{H}_{t-\Delta t} \;+\; (1 - \Delta t \cdot \beta_{hg})\,(g^o - g_z) \;+\; \text{rest}$$

Letting Δt shrink to zero, the inertia dissolve completely and we obtain

$$\hat{H} \;=\; (g^o - g_z) \;+\; \beta_{hh}\,\frac{H^d - H}{H} \;+\; \beta_{hy}\,(g^e_y - g^o) \tag{41}$$

$$\hat{E} \;=\; (g^o - g_z - g_h) \;+\; \beta_{ee}\,\frac{E^d - E}{E} \;+\; \beta_{ey}\,(g^e_y - g^o) \;+\; \beta_{eh}\,\frac{H - h^n E}{h^n E} \tag{42}$$

On the other hand, the weights β_{hg} and β_{eg} can be supposed to remain unaffected by the length of the adjustment period, that is,

$$\hat{H}_t - \beta_{hg}\,\hat{H}_{t-\Delta t} \;\; = (1-\beta_{hg})\,(g^o - g_z) \; + \; \text{rest}$$

from (38). If here Δt tends to zero, we have $\hat{H}_t - \beta_{hg}\,\hat{H}_{t-\Delta t} \;\; \longrightarrow \;\; (1-\beta_{hg})\hat{H}_t$. The same kind of equations as (41) and (42) are obtained, except that that the original coefficients β_{hh}, etc., are divided by $(1-\beta_{hg})$ and $(1-\beta_{eg})$, respectively. Of course, this presupposes that the estimates of β_{hg} and β_{eg} are less than unity, and do not come close to it, either. On the whole, eqs (41) and (42) are an appropriate continuous-time counterpart of the discrete-time specification (38) and (39).

For a model analysis it is necessary to set up an intensive form of this building block, such that the state variables could remain constant over time in a state of long-run equilibrium. The intensive-form variables corresponding to H and E are the utilization of the workforce, u_w, and the employment rate, e. They, too, are determined in a dynamic way, i.e., they are predetermined in the short period, and in continuous time their changes over time are governed by differential equations that are not too difficult to derive. Besides, of course, the exogenous time path of utilization u, also labour productivity will enter these relationships. This variable is, however, statically endogenous, i.e., the ratio z/z^o can be expressed as a function of the three variables u, e and u_w.

To establish this relationship, rewrite the workforce utilization as $u_w = H/h^nE = (Y/z)/h^nE = (Y/Y^p)\,(Y^p/h^nL)\,(L/E)\;(1/z) = u\,(e^oz^o)\,(1/e)\,(1/z)$; the last equality sign is based on (33). Solving for $\zeta = z/z^o$ gives

$$\zeta \;=\; z/z^o \;=\; \frac{e^o\,u}{e\,u_w} \;=\; \frac{u\,/\,u_w}{e\,/\,e^o} \qquad (43)$$

If labour productivity is expressed this way, it is no longer so obvious why it should be a procyclical variable with a slight lead. The ratio ζ might react in a fairly sensitive way already to small variations in the lag structure of e and u_w versus utilization u, or in their relative amplitudes.

To derive the changes of the employment rate we note that $E^d/E = H^d/h^nE = (H/h^nE)(H^d/H) = u_w\zeta$ by (36) and (40). Then, with (37), $\hat{e} = \hat{E} - \hat{L} = (g^o - g_h - g_z) + \beta_{ee}(u_w\zeta - 1) + \beta_{ey}(g^e_y - g^o) + \beta_{eh}(u_w - 1) - g_\ell$. The growth rates cancel out by (34), and with $u_w\zeta = e^ou/e$ by (43) we have

$$\hat{e} \;=\; \beta_{ee}\,(e^ou/e - 1) \;+\; \beta_{eh}\,(u_w - 1) \;+\; \beta_{ey}\,(g^e_y - g^o) \qquad (44)$$

While the changes in the employment rate are only dependent on the reaction coefficients in the adjustment equation (39), the changes in the utilization of the workforce are more involved and include the coefficients of both equations (38) and (39), since $\hat{u}_w = \hat{H} - \hat{h}^n - \hat{E}$. This gives us $\hat{u}_w = (g^o - g_z) + \beta_{hh}(\zeta - 1) + \beta_{hy}(g^e_y - g^o) - g_h - (g^o - g_h - g_z) - \beta_{ee}(e^o u/e - 1) - \beta_{ey}(g^e_y - g^o) - \beta_{eh}(u_w - 1)$. Again, the exogenous growth rates cancel out, and we arrive at

$$\hat{u}_w \;=\; -\beta_{eh}\,(u_w - 1) \;+\; \beta_{hh}\,(\zeta - 1) \;-\; \beta_{ee}\,(u_w\zeta - 1) \;+\; (\beta_{hy} - \beta_{ey})\,(g^e_y - g^o) \qquad (45)$$

It is thus seen that the changes in the utilization of the workforce do not only depend on its current level u_w, which when above unity firms seek to reduce by employing more workers (see the origin of the coefficient β_{eh}). In addition, they are influenced by the deviations of productivity from trend, ζ, and the mixed term $u_w\zeta$. Whether expected growth enters positively or negatively depends on the relative size of the respective coefficients β_{hy} and β_{ey} in the hours and job adjustments.

6.1.4 Connection to Okun's law

It seems at first sight that the differential equation governing the changes in the employment rate does not have anything more to do with Okun's law. Though capacity utilization, or the output gap for that matter, show up on the right-hand side of (44), it does this as a level variable, u, and not as a rate of change, $\hat{u}$, as it should if we want to relate (44) to the growth rate specification of Okun's law in eq. (11).

The growth rate of u can, however, be re-introduced by splitting up the term with the expected growth rate. Taking account of $g^o = \hat{Y}^p$ from (34), this gives us $(g^e_y - g^o) = (\hat{Y} - \hat{Y}^p) + (g^e_y - \hat{Y}) = \hat{u} + (g^e_y - \hat{Y})$. Furthermore, we decompose $e^o u - e$, which is the numerator of $e^o u/e - 1 = (e^o u - e)/e$, as $e^o u - e = (u - 1)e - (e - e^o)u$. Taken together, eq. (44) can be equivalently rewritten as

$$\hat{e} \;=\; \beta_{ey}\,\hat{u} \quad + \quad \{\,\beta_{ey}\,(g^e_y - \hat{Y}) \;+\; \beta_{eh}\,(u_w - 1) \;+\; \beta_{ee}\,[(u-1)e - (e-e^o)u]/e\,\} \qquad (46)$$

The way in which the terms governing the changes of the employment rate are organized makes it clear that the two adjustment equations (38), (39) for hours and jobs contain Okun's Law as a core. As a discrete-time counterpart, the growth rate specification of eq. (11) is obtained in the special case $\beta_{ee} = \beta_{eh} = 0$ and if also output is expected to continue its growth at the current rate in the near future. The Okun coefficient β would then be directly given by the structural coefficient β_{ey} in the gradual employment adjustments of the firms. Under these circumstances, their recruitment policy takes trend output and productivity growth into account and, apart from that, increases the number of jobs by β_{ey} percent (per year) if current output grows one percent faster than potential output.

On the other hand, note that considerations of desired levels of employment E^d and the speed at which firms seek to close the gap between E^d and E would play nor role, although informal discussions often seem to see these adjustments reflected in the Okun coefficient. Whether hours currently worked per job are above or below normal would not be taken into acount, either..

The curly brackets in (46) point out the influence of additional factors that in each period may distort the simple relationship (11), even if we continue to identify expected and current output growth. Then, the employment rate is comparably higher if the workforce is currently overutilized ($u_w > 1$), or if in a boom output is above its trend and the employment rate is above normal but, as we expect, u has a larger amplitude than e (so that $(u-1)e > (e-e^o)u$). From the latter we could also conclude that in an upper turning point, where $\hat{y} = \hat{u} = 0$, the employment rate would still be rising. Accordingly, eq. (46) would predict that the employment rate lags behind the output gap.

6.2 Estimation

6.2.1 Adjustments of employment

Even though the module of the delayed adjustments of hours and employment may make good economic sense, it still has to be confronted with the empirical data. Only a few time series of raw data are needed for that purpose, which we have extracted from the Fair-Parke database. They are listed in Table 5, and all of them refer to the firm sector.

FP label	Description
HF	hours per job
JF	number of jobs
PROD	output per hour (labour productivity)
Y	real output per quarter

Table 5: Raw data extracted from the Fair-Parke (FP) database (firm sector).

Let us begin with the employment adjustments in eq. (39). Before resorting to more elaborated econometric methods, we should try how far we can get with OLS. This most elementary regression approach can be applied to (39) if the equation is rearranged as

$$\begin{aligned}\hat{E} - (g^o - g_z - g_h) \;&=\; \beta_{eg}\,[\,\hat{E} - (g^o - g_z - g_h)\,]_{-1} \\ &+\; \beta_{ee}\left(\frac{E^d - E}{E}\right)_{-1} \;+\; \beta_{ey}\,(g_y^e - g^o) \;+\; \beta_{eh}\,\frac{H - h^n E}{h^n E} \qquad (47)\end{aligned}$$

All of the variables entering here can be constructed from the data in Table 5. The three trend rates of growth g^o, g_z, g_h are specified by our method of choice in this report, the HP trend line based on the smoothing parameter $\lambda = 102,400$. The gap term $(E^d - E)/E$ is given by $\ln E^d - \ln E = \ln(H^d/h^n) - \ln E = \ln H^d - \ln h^n - \ln E$. The log of normal hours per job, $\ln h^n$, is proxied by the HP 102,400 trend of the log of actual hours per job. The log of desired total hours, $\ln H^d = \ln(Y/z^o)$, according to (35), is given by the difference between $\ln Y$ and the HP trend of log productivity.

Regarding the expected growth rate of output g_y^e, it turned out that we already obtained the best results by simply employing the most recent actual output growth rate.[39] So we limit the presentation to $g_y^e = \hat{Y}$ right away. The details of the construction of the variables in (47) are summarized in Table 6 (the composed variables themselves are indicated by bold face characters). Observe that all of the one-quarter growth rates are annualized.

Table 7 contains the essential results of the OLS regressions that we performed. Behind the table are explorations of a variety of lags and lag combinations for the independent variables, which all proved to be distinctly inferior. Though the data available (at the time the report was written) extended to quarter 2005:2, the sample period was limited until 2003:4 in order to avoid possible end-of-period effects for the trend variables.

A presentable result, which is shown in the first column (of figures) in Table 7, already comes about if lagged employment growth $\hat{E}_{-1}$ is ignored in (47), putting $\beta_{eg} = 0$. The coefficients on the other three independent variables have the correct sign and come out significant, while the fit itself is satisfactory. Only the low Durbin-Watson indicates nonnegligible serial correlation in the residuals, but this problem might be tackled with instrumental variable techniques. In any way, this regression is a first encouraging outcome concerning the validity of the model.

The fit is substantially improved if the lagged growth rate of employment $\hat{E}_{-1}$ is added to the explanatory variables. In this case, however, the coefficient on the utilization of the workforce, i.e. the gap $(H - h^n E)/h^n E$, turns insignificant. So, in the second column in Table 7, it is directly set to zero. As required, the growth rate coefficient β_{eg} is between 0 and 1, and the other two β_{ee} and β_{ey} are both positive. All these coefficients are highly significant. In addition to the smaller standard error of the regression, including lagged employment growth has the merit that the Durbin-Watson statistic moves close to 2. The goodness of the fit is illustrated in Figure 21, which plots the predicted versus the actual values of $\hat{E} - (g^o - g_z - g_h)$.

[39] We experimented with a short 4-quarter (backward-looking) moving average of output growth for g_y^e, which is a reasonable makeshift substitute for sales expectations. This variant, however, notably deteriorated the goodness-of-fit in the regressions.

name in regression	model notation	construction
E	E	JF
Hours	H	HF · JF
z	z	PROD
Ln_X_HP1024		HP 102400 trend of ln(X), X = HF, Y, z
d_Ln_X_HP1024		400 · Δ Ln_X_HP1024, X = HF, Y, z
go	g^o	d_Ln_Y_HP1024
gh	g_h	d_Ln_HF_HP1024
gz	g_z	d_Ln_z_HP1024
go_gz	$g^o - g_z$	go − gz
go_gz_gh	$g^o - g_z - g_h$	go − gz − gh
gE	$\hat{E}$	400 · Δ ln(JF)
gH	$\hat{H}$	400 · Δ ln(Hours)
gY	$\hat{Y}$	400 · Δ ln(Y)
gY_go	$\hat{Y} - g^o = g^e_y - g^o$	gY − go
gE_ggg	$\hat{E} - (g^o - g_z - g_h)$	gE − go_gz_gh
gH_gg	$\hat{H} - (g^o - g_z)$	gH − go_gz
Ln_Hd	$\ln H^d = \ln(Y/z^o)$	ln(Y) − Ln_z_HP1024
Hd_H	$(H^d - H)/H$	100 · [Ln_Hd − ln(Hours)]
Ln_hn	$\ln h^n$	Ln_HF_HP1024
Ln_Ed	$\ln E^d = \ln(H^d/h^n)$	Ln_Hd − Ln_hn
Ed_E	$(E^d - E)/E$	100 · [Ln_Ed − ln(E)]
Ln_hnE	$\ln(h^n E)$	Ln_hn + ln(E)
H_hnE	$(H - h^n E)/h^n E$	100 · [ln(Hours) − Ln_hnE]

Table 6: Time series constructed from the FP raw data.

Note: HP 102400 is the Hodrick-Prescott trend line with smoothing parameter $\lambda = 102,400$, Δ denotes the quarterly difference operator.

We should also point out that setting the coefficient β_{eh} equal to zero may be convenient for the properties, or the analysis, of the theoretical model, since the influence of the workforce (u_w) is cancelled in the differential equation (44) for the employment rate. In a model where the employment rate enters a wage Phillips curve, for example, the complicated equations (43) and (45), therefore, need not be considered. The variables u_w and ζ would, however, reappear if the changes in money wages do not (only) refer to a trend rate

Dependent variable: gE_ggg

coeff.	variable	1960:1 – 2003:4		60:1 – 83:4	84:1 – 03:4	90:1 – 03:4
β_{eg}	$(\text{gE\_ggg})_{-1}$	– –	0.47	0.44	0.61	0.67
			(9.57)			
β_{ee}	$(\text{Ed\_E})_{-1}$	0.44	0.29	0.30	0.25	0.27
		(5.43)	(4.52)			
β_{ey}	gY_go	0.30	0.27	0.30	0.17	0.14
		(9.61)	(10.41)			
β_{eh}	H_hnE	0.55	– –	– –	– –	– –
		(3.24)				
	sd(gE_ggg)	2.16	2.16	2.54	1.60	1.67
	SER	1.41	1.17	1.36	0.89	0.94
	R^2	0.58	0.71	0.72	0.70	0.70
	DW	1.00	2.11	2.01	2.41	2.45

Table 7: OLS regressions of (47).

Note: 'sd' denotes the standard deviation (of the dependent variable). See Table 6 for the acronyms of the variables. t-statistics are given in parentheses.

of productivity growth but (also) to its actual growth; or, of course, if the Phillips curve takes account of insider effects and the utilization of the workforce is introduced through this direct channel.

On the basis of the strong evidence in favour of the model we feel justified in studying possible time variations of the parameters, where it is important to note that they are here no longer of a purely descriptive nature but that they have a direct structural interpretation. Similar to our investigation of cyclical features above, we first subdivide the sample period into the three periods 1960:1 – 1983:4, 1984:1 – 2003:4 and 1990:1 – 2003:4, and re-estimate (47) over these subsamples. The outcome is shown in the last three columns in Table 7. If we consider the coefficient β_{ee} on the employment gap term $(E^d - E)/E$ to be the central responsiveness of firms to the disequilibria that they perceive, then this responsiveness has not changed very much, and if so, it has slightly decreased as compared to the first 24 years of the sample.

A more detailed examination of possible shifts in the reaction coefficient(s) is, however, possible if we conceive them as time-varying coefficients and estimate these time paths by the method of spline functions. This approach gives a more differentiated, if not different,

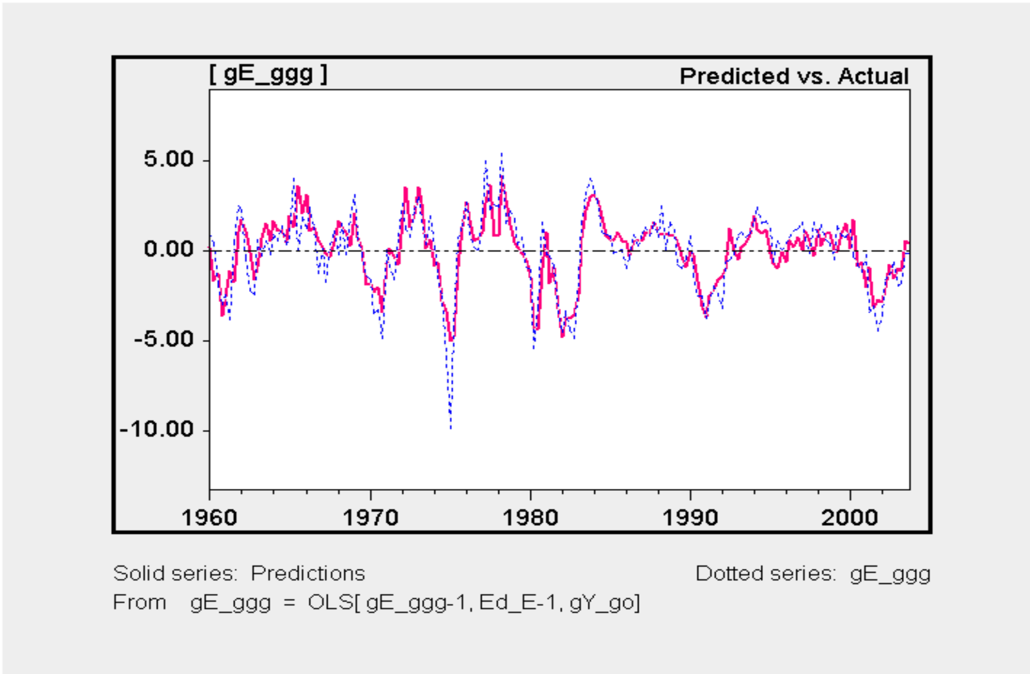

Figure 21: Goodness-of-fit of regression (47), from the second column in Table 7.

picture. We distinguish two cases: (1) only $\beta_{ee} = \beta_{ee,t}$ is time-varying and the other two coefficients are fixed; (2) all three coefficients $\beta_{ex} = \beta_{ex,t}$ are time-varying $(x = g, e, y)$. The top-left panel of Figure 22 shows the result for $\beta_{ee} = \beta_{ee,t}$ in the first case (its estimation is based on five segments). The responsiveness had somewhat increased until the mid-1970s, then it decreased until the end of the 1980s, and from then on $\beta_{ee,t}$ increased to unprecedented levels until the end of the sample period.

It is quite remarkable that, at least from the mid-1970s on, the same behaviour and almost the same figures for $\beta_{ee,t}$ result if all three parameters are free to vary over time. The time path of $\beta_{ee,t}$ thus determined is exhibited in the top-right panel in 22. The time paths of the other two coefficients $\beta_{eg,t}$ and $\beta_{ey,t}$ are shown in the two bottom panels (left and right, respectively). The strong variations of these coefficients makes the close resemblance of the evolution of $\beta_{ee,t}$ in the first two panels all the more noteworthy.

As a special feature in the time path of $\beta_{ee,t}$ it should be noted that this responsiveness is particularly low during the 1991 recession, whose subsequent recovery is often described as jobless growth. It may be interesting to discuss our notion of desired employment E^d and the finding of the slow adjustment toward it at that time in the light of this phenomenon. It may well be that our business cycle perspective would then have to be complemented with a longer-term perspective, where in the first instance one might think of a general increase in labour productivity.

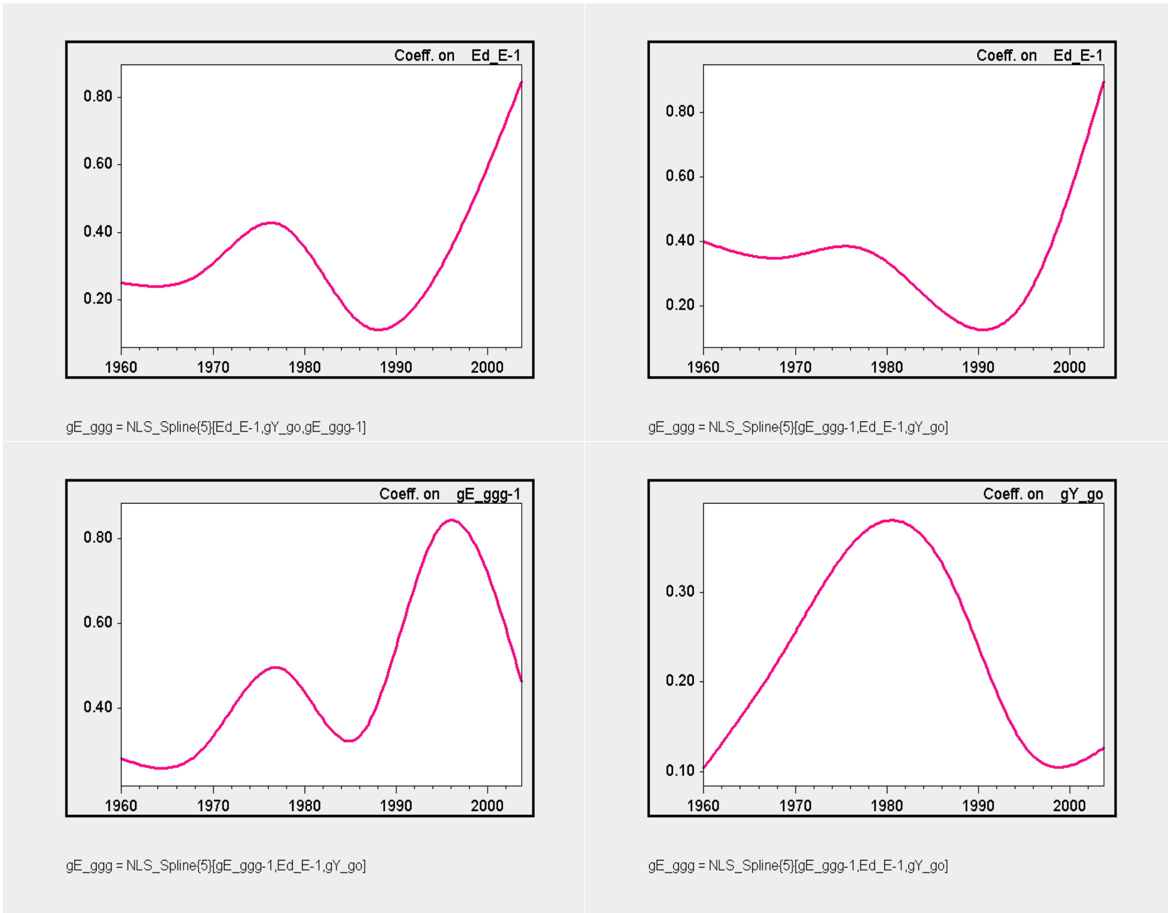

Figure 22: Time-varying coefficients in regression (47).

Note: Estimation with the method of spline functions, subdividing the sample period in five segments.

This, however, would open up new issues and new problems. To illustrate this, we conclude this subsection with a plot of the evolution of the trend rate of productivity growth in Figure 23. This rate has indeed substantially increased in recent times. At the beginning of the 1990s, however, its level was still fairly low.[40]

[40]Incidentally, Figure 23 shows that from from the end of the 1990s it makes quite a difference whether trend productivity growth is obtained from computing the trend of the level of productivity and then taking first differences from it (the series gz as described in Table 6, the solid line in Figure 23); or whether one takes the actual growth rates of productivity and applies the HP filter directly to them (the dotted line).

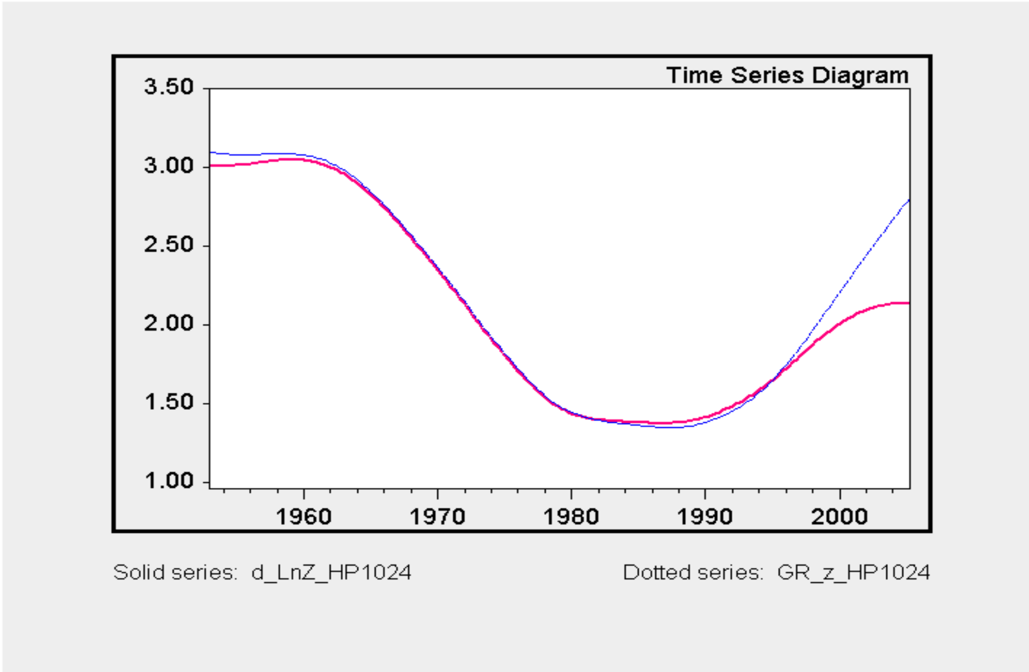

Figure 23: Trend rate of labour productivity growth
(see footnote 40 for an explanation of the difference between the two lines).

6.2.2 Adjustments of hours

The estimations of the hours adjustments equation (38) can proceed analogously to the preceding subsection. For OLS, we first transform (38) to

$$\hat{H} - (g^o - g_z) \quad = \quad b_{hg}\, [\, \hat{H} - (g^o - g_z)\,]_{-1} \; + \; \beta_{hh} \left(\frac{H^d - H}{H} \right)_{-1} \; + \; \beta_{hy}\, (g^e_y - g^o) \qquad (48)$$

The construction of the variables is detailed in Table 6. The regression results for the equation are reported in Table 8. While the regression without the growth rate term ($\beta_{hg} = 0$) is satisfactory, including this inertia again leads to a clear improvement. Though R^2 for the unconstrained version (in column 2) is less than its counterpart in Table 7, the visual impression of the goodness-of-fit is similar; cf. the plot of predicted versus actual values in Figure 24 and compare it to the fit in Figure 21. Regarding the smaller R^2 for (48). note also the greater variability in the dependent variable of Table 8: a standard deviation of 2.93% in contrast to the 2.16% in Table 7.

Dependent variable: gH_gg

coeff.	variable	1960:1 – 2003:4		60:1 – 83:4	84:1 – 03:4	90:1 – 03:4
β_{hg}	$(\text{gH\_gg})_{-1}$	– –	0.31 (5.84)	0.24	0.43	0.51
β_{hh}	$(\text{Hd\_H})_{-1}$	0.71 (6.04)	0.55 (4.96)	0.71	0.36	0.33
β_{hy}	gY_go	0.49 (10.70)	0.43 (9.87)	0.44	0.33	0.27
	sd(gH_gg)	2.93	2.93	3.40	2.26	2.31
	SER	2.08	1.90	2.06	1.67	1.72
	R^2	0.50	0.58	0.64	0.47	0.47
	DW	1.39	2.18	2.03	2.49	2.62

Table 8: OLS regressions of (48).

Note: 'sd' denotes the standard deviation (of the dependent variable). See Table 6 for the acronyms of the variables. *t*-statistics are given in parentheses.

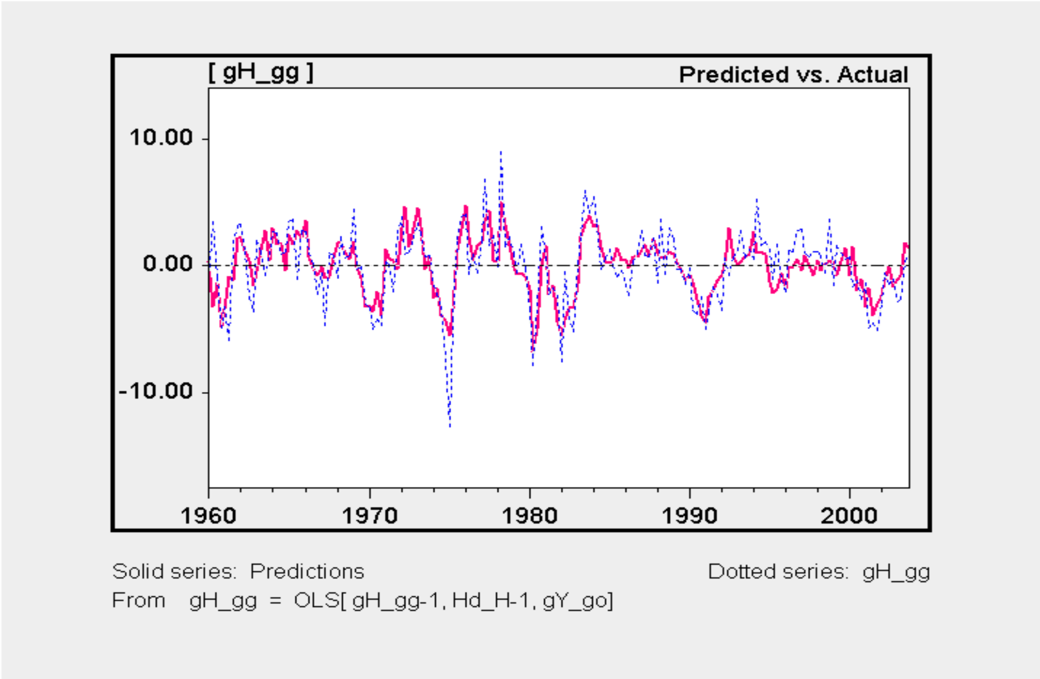

Figure 24: Goodness-of-fit of regression (48), from the second column in Table 8.

The coefficient that represents the speed at which firms adjust actual hours to desired hours is β_{hh}. The last three columns in Table 8 with our three subperiods show that the responsiveness has decreased rather than increased. However, this does not necessarily mean that firms are less concerned about the gap in hours, since the growth rate coefficient β_{hg} has changed in the opposite direction and to similar amounts. Therefore, the two effects may be "substitutes": the lower responsiveness to the gap in the level of hours may just offset the increased inertia in the hours growth rates, where the growth rate of the previous period proves to be a good guideline in the present period, too.

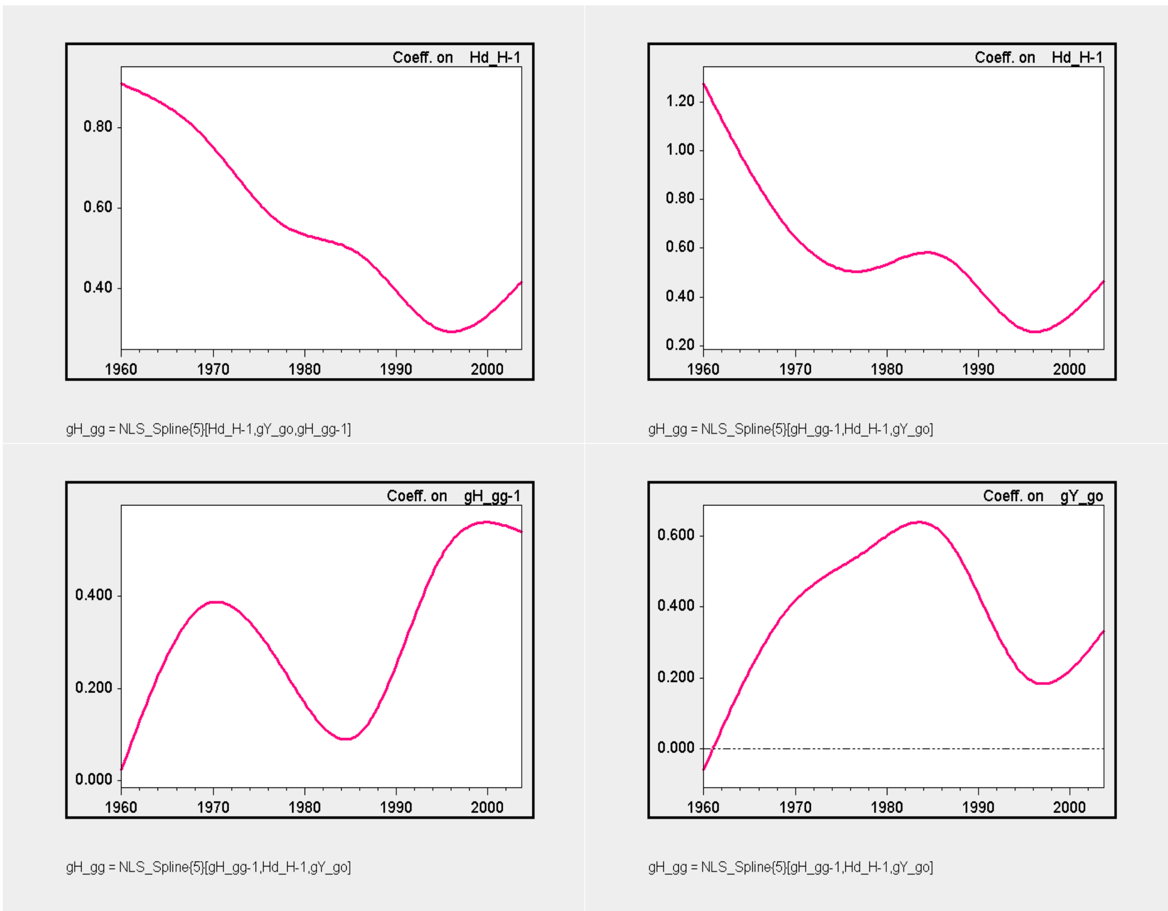

Figure 25: Time-varying coefficients in regression (48).

Note: Estimation with the method of spline functions, subdividing the sample period in five segments.

Computing the time-varying coefficients $\beta_{hh} = \beta_{hh,t}$ and $\beta_{hg} = \beta_{hg,t}$ in the same way as for the employment adjustments in the preceding subsection yields a similar picture to the subsample regressions, although with more medium-term variation; see Figure 25 with $\beta_{hh,t}$ in the two top panels ($\beta_{hh,t}$ as the only time-varying coefficient in the left panel, and $\beta_{hh,t}$ when all three coefficients are free to vary in the panel to the right; the lower two panels display the time paths of $\beta_{hg,t}$ and $\beta_{hy,t}$, respectively). The lower-right panel also shows the changing influence of expected output growth on the growth rate of hours.

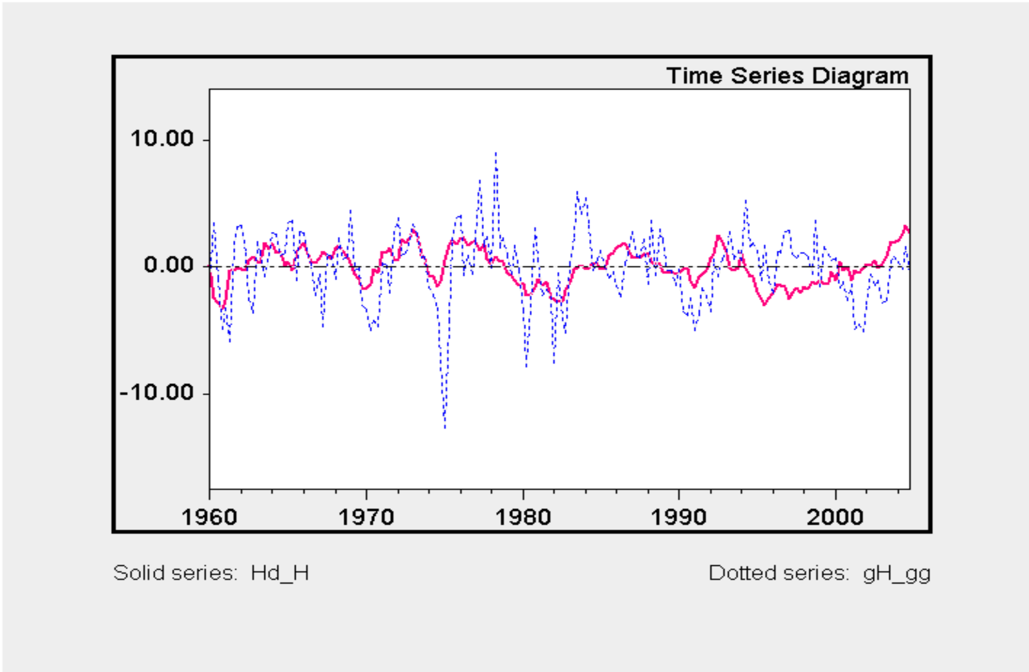

Figure 26: Comovements of $(H^d - H)/H$ (solid) and $\hat{H} - (g^o - g_z)$ (dotted line).

On the whole, the subsample regressions and the time paths of the coefficients in Figure 25 may be indicative of systematic changes in the reaction pattern of firms. They are, however, not easily characterized and identifying deeper reasons for them is even harder. A discussion approaching this subject might take into account that until the mid-1990s the gap $(H^d - H)/H$ moved more or less together with the growth rate term $\hat{H} - (g^o - g_z)$ (apart from the obvious fact that the growth rates are much noisier, which favours estimation), whereas possibly something has changed in the second half of the 1990s. In any case, going beyond a mere description and attaching economic sense to the estimated time paths in Figure 25 is still an open issue.

6.2.3 The adjustment equations ready for use

Apart from the perspective on Okun's law and the (possibly time-varying) macroeconomic relationships underlying it, it was our aim in this section to put forward a theoretical building block for the determination of hours and employment, and thus for the utilization of the workforce (giving rise to insider effects) and of the labour force (giving rise to outsider effects). The module was meant to be incorporated into broader dynamic models of the macro economy. Since these models will be definitely larger than just two or three dimensions, they have to resort to computer simulations and so have to be specified numerically. For a basic or benchmark version of such a model, our estimations over the entire sample period are sufficiently reliable. For discrete-time models with a quarterly adjustment period we can summarize our main result from Tables 7 and 8 as follows:

$$\hat{H} \;=\; 0.31 \cdot \hat{H}_{-1} + 0.69 \cdot (g^o - g_z) \;+\; 0.55 \cdot \left(\frac{H^d - H}{H}\right)_{-1} \;+\; 0.43 \cdot (g_y^e - g^o) \qquad (49)$$

$$\hat{E} \;=\; 0.47 \cdot \hat{E}_{-1} + 0.53 \cdot (g^o - g_z - g_h) \;+\; 0.29 \cdot \left(\frac{E^d - E}{E}\right)_{-1} \;+\; 0.27 \cdot (g_y^e - g^o) \qquad (50)$$

These equation are now ready for use. (For a continuous-time version the remarks at the beginning of Section 6.1.3 may be taken into account.)

7 Conclusion

This report has examined the output-employment nexus from different perspectives. On the one hand, it was concerned with an atheoretical summary of the data for the US firm sector that usually goes under the heading of Okun's law. In particular, apparently systematic variations of the famous Okun coefficient over time could here be identified, which partly turned out to be different from other results in the literature. Complementary to that, we considered the cyclical behaviour of various macroeconomic variables that contribute to the connection between output and the employment rate and found significant changes over the last 15 years for some of them, too.

On the other hand, the report put forward two theoretical models that determine the adjustments of employment and hours by firms in response to expected growth and certain gaps between desired and actual values. The two modules were validated by estimation of the structural parameters, which came out very satisfactorily. In addition, for the more ambitious model regressions with time-varying coefficients were performed. These results allow a more differentiated view on possible "regime shifts" than the variations of the Okun coefficient. Independently of this issue, the two modules and their estimated numerical coefficients can be readily incorporated into larger dynamic macro models.

A side result of the investigations was that the supply side effects, i.e. the cyclical behaviour of the labour force, have gained greater importance in the fluctuations of the employment rate. As we have put forward two building blocks determining the demand side in this respect, it would now be desirable to develop one or two small-scale models that could explain adjustments in the labour force along similar lines.

Apart from that, our empirical investigations used US data only so far. In a next step, the significance of our results and their interpretation may be tested with data and economic reasoning from other industrialized countries.

8 References

Baxter, M. and King, R.G. (1995), "Measuring business cycles: Approximate band-pass filters for economic time series", NBER Working Paper 5022.

Blanchard, O. (2003), *Macroeconomics.* Prentice Hall (3rd edition).

Diebold, F.X. and Rudebusch, G.D. (1999), *Business Cycles: Durations, Dynamics, and Forecasting.* New Jersey: University Presses of California, Columbia and Princeton.

Diebold, F.X. and Rudebusch, G.D. (2001), "Five questions about business cycles", Federal Reserve Bank of San Francisco, *Economic Review*, pp. 1–15.

Franke, R. (2006), "AELSA–FP: Advanced Econometric Least Squares Analysis With the Fair–Parke Database — Manual", Bremen.

Friedman, M. (1968), "The role of monetary policy", *American Economic Review*, 58, 1–17.

Gordon, R.J. (1997), "The time-varying NAIRU and its implications for economic policy", *Journal of Economic Perspectives*, 11:1, 11–32.

Gordon, R.J. (2003), "Exploding productivity growth: Context, causes, and implications", *Brookings Papers on Economic Activity*, no. 2, 207–279.

Grant, A.P. (2002), "Time-varying estimates of the natural rate of unemployment: A revisitation of Okuns Law", *The Quarterly Review of Economics and Finance*, 42, 95–113.

Hamilton, J. (1994), *Time Series Analysis.* Princeton: Princeton University Press.

Harvey, A. (1989), *Forecasting, Structural Time Series Models and the Kalman Filter.* Cambridge: Cambridge University Press.

Hemraj, T., Madrick, J. and Semmler, W. (2006), "Okun's law and jobless growth", The New School, SCEPA, Policy Note, no. 3.

Hodrick, R.J. and Prescott, E.C. (1997), "Postwar business cycles: an empirical investigation", *Journal of Money, Credit, and Banking*, 29, 1–16.

King, R.G. and Rebelo, S.T. (1999), "Resuscitating real business cycles", in J.B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics, vol. 1B.* Amsterdam: Elsevier; pp. 927–1007.

Laubach, T. (2001), "Measuring the NAIRU: Evidence from seven economies", *Review of Economics and Statistics*, 83, 218–231.

Llaudes, R. (2005), "The Phillips curve and long-term unemployment", European Central Bank, Working Paper Series No. 441.

Nelson, C.R. and Plosser, C.I. (1982), "Trends and random walks in macroeconomic time series: Some evidence and implications", *Journal of Monetary Economics*, 10, 139–162.

Okun, A. (1962), "Potential gnp: its measurement and significance", *Proceedings of the Business and Economic Statistics Section.* American Statistical Association, 98–103.

Orphanides, A. and Williams, J.C. (2005), "The decline of activist stabilization policy: Natural rate misperceptions, learning, and expectations", *Journal of Economic Dynamics and Control*, 29, 1927–1950.

Press, W.H. et al. (1986), *Numerical Recipes: The Art of Scientific Computing.* Cambridge: Cambridge University Press.

Rogerson, R. (1997), "Theory ahead of language in the economics of unemployment", *Journal of Economic Perspectives*, 11, 73–92.

Schlicht, E. (2005), "Software: VC—A program for estimating time-varying coefficients", downloadable from `http://epub.ub.uni-muenchen.de/view/subjects/software.htm`.

Schlicht, E. (2003), "Estimating time-varying coefficients with the VC program", mimeo; `www.semverteilung.vwl.uni-muenchen.de/mitarbeiter/es/paper/vc-info.pdf`.

Schlicht, E. (1989), "Variance estimation in a random coefficients model", paper presented at the Econometric Society European Meeting, Munich 1989; `www.semverteilung.vwl.uni-muenchen.de/mitarbeiter/es/paper/schlicht_variance-estimation-in-random-coeff-model.pdf`.

Schlicht, E. and Ludsteck, J. (2005), "Variance estimation in a random coefficients model", *mimeo*, Department of Economics, University of Munich.

Semmler, W. and Zhang, W. (2005), "The impact of output growth, labor market institutions, and macro policies on unemployment", Working Paper, The New School, SCEPA.

Staiger, D., Stock, J.H. and Watson, M.W. (1997), "The NAIRU, unemployment and monetary policy", *Journal of Economic Perspectives*, 11:1, 33–49.

Stock, J. (1994), "Unit roots, structural breaks and trends", in R. Engle and D. McFadden (eds.), *Handbook of Econometrics, Vol. 4.* Amsterdam: Elsevier; pp. 2739–2841.

Stock, J. and Watson, M. (1998), "Median unbiased Estimation of coefficient variance in a time-varying parameter model", *Journal of the American Statistical Association*, 93, 349–358.