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Empty Sources of Growth Accounting, and Empirical Replacements à la Kaldor with Some Beef

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Abstract Standard sources of growth accounts are empty of content because they rely on neoclassical production theory. Rather, analysis can be based on productivity growth equations derived from NIPA accounting conventions and a helpful algebraic identity. These schemes impose valid restrictions on growth rates of the wage rate, profit rate, capital, labor, and their respective average productivities. One states that the output growth rate equals employment growth plus productivity growth. The standard "convergence" model basically adds accumulation dynamics to this identity. Replacing the aggregate production function with proper accounting restrictions gives a growth model with detailed results that differ markedly from those of the standard model. Alternative, essentially Kaldorian supply-and demand-based alternatives to sources of growth based on a familiar output growth vs. productivity growth diagram with constant employment growth contours added in look like a useful alternative to the mainstream models. With distributive dynamics added in, the model would also generate Goodwin-style cycles.

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By now, it should be widely recognized that aggregate production functions have negligible empirical content. As Felipe and Fisher (2003) observe, "… the *relationship* $GDP = F(K,L)$ between aggregate output (*GDP*) and aggregate inputs (K,L) used in theoretical and applied macroeconomic work does not have, in general, a meaningful interpretation. This

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implies that the statement that there must be some connection between aggregate output and aggregate inputs, and that this is what the aggregate production function shows, has no theoretical basis." (p. 248, emphasis in original).

In this note, we take this point for granted, and illustrate it in terms of growth accounting. We use *accounting decompositions* from the national income and product accounts (NIPA) and an *algebraic identity* to investigate how average labor and capital productivity growth rates change over time, along with convergence properties of growth models based on such indicators.

As the paper is organized, section 1 reviews growth accounts based on NIPA decompositions while section 2 takes up algebraic identity accounting. Section 3 is devoted to some implications. Section 4 looks at sources of growth and section 5 analyzes convergence of an accounting-based version of the Solow (1956) growth model. Sections 6 through 9 present supply- and demand-based Kaldorian growth models which to our eyes are more realistic than the Solow formulation preferred by the mainstream. Goodwin-style cycles would be a natural extension.

1. NIPA-based Growth Accounting

We assume that real output or value-added (X) and capital stock (K) measures exist, the former constructed by double deflation of GDP from current value national income and product accounts, and the latter generated by perpetual inventory procedures from real gross fixed capital formation estimates in the NIPA. Employed labor (L) estimates are also assumed to be available.[1]

For present purposes, we also assume that real value-added (at factor cost) can be decomposed in the form

$$X = wL + rK \qquad (1)$$

with w as an index of the real wage and r the ex post rate of profit. In practice, decompositions of the form (1) can be difficult to construct from available data, especially in developing countries. Real value-added is ideally estimated by a double deflation technique which says nothing directly about distribution between wages and profits. In poor countries the dominant flow in value-added

[1] Felipe and Fisher (2003) argue that estimates of quantities like K and L can also be quite problematical but for any sort of quantified macroeconomics it seems impossible to work without them.

is “proprietors’ incomes” (peasants, small informal urban enterprises) which does not naturally split into labor and capital components. In a country with a real GDP per capita on the order of a thousand dollars the wage bill might be 20-30% of GDP and profits of “formal sector” enterprises 10-20% at most. Different procedures are used to allocate the remainder between labor and capital, without being entirely convincing.

Setting such data problems aside until section 3 and using continuous time for simplicity, let the growth rate “X-hat” of X be given by $\hat{X} = (dX/dt)/X = \dot{X}/X$, etc. We define average levels of labor and capital productivity as

$$e_L = X/L \tag{2L}$$

and

$$e_K = X/K \tag{3L}$$

with respective growth rates

$$x_L = \hat{e}_L = \hat{X} - \hat{L} \tag{2G}$$

and

$$x_K = \hat{e}_K = \hat{X} - \hat{K} \quad . \tag{3G}$$

The observed labor share of output at any time is $y = wL/X = w/e_L$; the capital share is $1-y$. One can logarithmically differentiate (1) to get

$$\hat{X} = y(\hat{w} + \hat{L}) + (1-y)(\hat{r} + \hat{K}) \quad . \tag{4}$$

Let

$$x(y) = \hat{X} - y\hat{L} - (1-y)\hat{K} \tag{4G}$$

be total factor productivity growth (TFPG) as usually defined. Then (4) and (4G) show that

$$x(y) = y\hat{w} + (1-y)\hat{r} \quad . \tag{5}$$

At any time $x(y)$ is a flow of “surplus” (or real output growth) that must be split between growth of real wages and profits (recall the profit surge accompanying the speedup of productivity growth in the American economy in the late 1990s). One can also use (4) to show that

$$x(y) = yx_L + (1-y)x_K \tag{6}$$

so that TFPG is a weighted average of the growth rates of average labor and capital productivities. Subtracting (6) from (5) gives

$$y\,(\hat{w}-x_L)+(1-y)(\hat{r}-x_K)=0 \qquad , \tag{7G}$$

a cost-side restriction on observed growth rates of average productivities which we make use of below.

Note that (7G) implies that

$$x_K-\hat{r}<0, x_L-\hat{w}>0 \qquad \text{or} \qquad x_K-\hat{r}>0, x_L-\hat{w}<0 \tag{8}$$

when the economy is not at a steady state with constant factor shares. One factor share must be rising while the other falls. But growth of a real factor payment cannot differ from its corresponding productivity growth rate forever. Either distributive growth cyclicality along Goodwin's (1967) lines or endogenous cessation or reversal of the divergent trends in (8) is required from the accounting. Such restrictions are often ignored in growth theory.

2. A Useful Algebraic Identity

Consider the algebraic identity $K/L = K/L$. Dividing numerator and denominator on the left-hand side by $X \neq 0$ and substituting from (2L) and (3L) gives a level equation of the form

$$e_L/e_K = K/L \quad . \tag{9L}$$

The growth rate version

$$x_L - x_K = \hat{K} - \hat{L} \tag{9G}$$

follows directly.

Equations (9L) and (9G) impose one restriction on four variables. In a system with three degrees of freedom they basically say that all four variables *L, K,* e_L, e_K along with their four growth rates cannot vary independently.[2] Also, average productivity of labor will rise in comparison to average productivity of capital when the capital-labor ratio goes up.

3. Applying the Accounting

As noted above, there is a real data problem posed by the fact that GDP (at factor cost) in practice splits into three components: labor remunerations, profits of incorporated enterprises,

[2] When *L, K,* e_L, and e_K satisfy (9L), X will follow from (2L) or (3L).

and incomes of the "self-employed," "entrepreneurs," "proprietors," or some other equally ill-defined group. In his magisterial study of *Modern Economic Growth*, Kuznets (1966) was well aware of the issue. Gollin (2002) rather belatedly goes over some of the same ground. Some sort of imputation is needed if incomes of the self-employed are to be split between returns to "capital" and "labor" Both Kuznets and Gollin in effect suggest imputing average employee compensation to the self-employed labor force to get an overall labor share.[3]

There are at least two problems with any such correction. One is that it can alter labor shares by a substantial amount. In a study discussed in more detail below, Young (1995) presents share data for four rapidly growing Asian economies in the period 1966-1990. Meanwhile, Soon and Ong (2001) give shares of labor remuneration in value-added at factor cost in the late 1990s. The numbers go as follows:

	Soon-Ong	Young
Hong Kong	0.52	0.63
Korea	0.47	0.70
Singapore	0.50	0.51
Taiwan	0.55	0.74

The remuneration shares were presumably lower (and not constant) in the period Young considers, so the corrections are large, and moreover subject to fairly arbitrary adjustments in their computation. It is hard to be convinced.

Second, Kuznets was careful to embed his imputations within the income-generation side of the national accounts. Mainstream studies rarely do that, preferring to focus only on equations like (4G). Without an appropriate cost-side correction redefining labor force and capital stock

[3] An alternative is to impute a rate of return to "entrepreneurial capital" and derive self-employment labor income as a residual. Kuznets does it both ways for a small sample of developed countries, and finds large differences in results but says the labor allocation makes more sense because "… the predominant majority of entrepreneurs are self-employed workers … and the major portion of their income is derived from labor" (p. 180). But the fact that both imputations give dissimilar results is disquieting: "….differences among countries and periods may well reflect genuine differences in the labor-property income composition of entrepreneurial income…." (p. 179).

growth to fit the imputations, just re-labeling some part of self-employment as "wages" is not very helpful.

The conclusion, perhaps, is that in setting up growth models one should work with the "observed" labor share y and carry along its distributive implications. But in work on empirical growth analysis, it is essential to recognize that estimating y is subject to potentially large errors.

4. Sources of Growth

We can use the foregoing results to investigate sources of growth using equations (4G) and (6). There is *no* presumption that an aggregate production function and associated marginal productivity conditions the analysis.

A first observation is relevant to questions of model causal structures or "closure." For example the "Harrod-Domar" model of the 1960s and the "AK" model of the 1990s (the same wine in different goatskins as pointed out by Kurz and Salvadori, 1998) both set *K* and e_K exogenously. As a consequence either employment or labor productivity has to be endogenous (endogeneity of the latter is of course characteristic of new growth theory). Similarly if *L* and e_L are exogenous (or determined by additional equation(s) as discussed below) then either capital or capital productivity has to be endogenous.

Most sources of growth studies treat *L and K* as pre-determined in an econometric sense. They wrap e_L and e_K into $e(y)$ for some value of y and thereby try to apportion growth between factor accumulation and "technical progress." A famous example is Young's (1995) study of four East Asian "Dragons" which argued that their rates of TFPG were not high in comparison to other economies. The inference drawn was that their rapid output expansion was basically due to high rates of employment and capital stock growth, especially the latter. Young's numbers are summarized in Table 1.

Table 1 here

Evidently such conclusions will be sensitive to the choice of y. The "observed" labor share is the usual selection but as discussed above there is a high degree of arbitrariness in assigning it a value. Are Young's numbers for y any more plausible than something 0.1 higher or lower? Why should they remain stable over a quarter-century of rapid structural change?

Moreover, Young's results are basically driven by (9G). All four dragons had quite respectable rates of labor productivity growth and their capital stock grew more rapidly than employment. As a consequence, their growth rates of capital productivity *had to be negative* as shown in the table. Their modest values of $x(y)$ are a direct consequence of definitional identities as combined in (6) and (9G) for the values of y presented in the table. Other values between zero and one would make $x(y)$ either strongly positive or negative. A similar observation applies to the commonly held view that Asian growth has been "inefficient" because it is associated with falling average capital productivity. The purported inefficiency is the consequence of a "theorem of accounting," neither more nor less.

More generally, (9L) and (9G) capture the fundamental neoclassical notion that an increase in the capital-labor ratio "should be" associated with a rise in labor productivity in comparison to capital productivity. One is reminded of Becker's (1962) ancient observation that demand curves tend to have negative slopes because that is the slope of the budget constraint.

With regard to non-mainstream economics, if one were to write (9L) in the form $e_L = e_K (K/L)$ and *assume* that $e_K = \bar{e}_K (K/L)^{-h}$ with h between zero and one and $\bar{e}_K$ as a scaling factor, then one gets

$$e_L = \bar{e}_K (K/L)^{1-h} \qquad (10)$$

which is basically Kaldor's (1957) "technical progress function." In effect, Kaldor was presenting a slightly disguised identity.

An immediate question is how well it fits the data. Arc-elasticity calculations for three countries in Table 1 suggest that h is around 0.45 or 0.5 with the fourth (Hong Kong) giving a value of 0.15. On the other hand, Table 2.8 in Foley and Michl (1999) presents growth rates for labor, capital, and their respective productivities for 20 selected periods in six countries since 1820 based on data from Maddison (1995).[4] Growth of the capital-labor ratio is positive in all

[4] Needless to say, the numbers satisfy (9G).

periods. However, capital productivity growth is positive in eight of them, implying a negative h (or a positive trend in $\bar{e}_K$). When $x_K < 0$, arc-elasticity values for h all lie below 0.5. Insofar as h can be viewed as a proxy for the exponent on labor from a Cobb-Douglas production function (a common neoclassical reinterpretation of Kaldor), it appears closer to the share of remunerations in GDP than to revised estimates of the labor share à la Kuznets, Gollin, and Young.

5. Convergence

Unsurprisingly, our identities also shed light on the steady state analysis that informed the "convergence debates" of the 1990s. The crux of this section is that convergence in savings-driven growth models à la Solow (1956) easily occurs when there is no aggregate production function – a fact that is well-known (Taylor, 2004) but ignored by the mainstream. Proper identity accounting is all that is required. It gives results that differ from those from models with production functions.

For all practical purposes, the standard model gets output growth from (2G) with exogenous growth rates of employment and labor productivity and tells a complicated story about accumulation dynamics which concludes with the output-capital ratio at a steady state. The usual specification (Mankiw, 1995) incorporates a stock of "effective" labor which at time t can be expressed as $Z(t)L(t)$. Both K and $Z(t)L(t)$ are assumed to be fully employed, determining the level of output and wage and profit rates from an aggregate production function $X = F(ZL, K)$ with the usual marginal productivity conditions.

Capital accumulation follows from national saving, set as a fraction of output. One can thereby set up a differential equation for $k = K / ZL$ which converges to a steady state under the usual assumptions. At the steady state the profit rate and the output-capital ratio e_K are constant so that technical progress is Harrod-neutral.

The Felipe-Fisher critique assures us that this model makes no sense. What we can do is use NIPA-based accounting conventions to replace it with something empirically defensible. Because there is no other source, in practice the time trend of the augmentation factor Z must be inferred from the observed growth rate of average labor productivity x_L. Mainstream models always say that $\hat{Z} = x_L$, meaning that Z itself must be proportional to e_L. Without loss of generality the constant of proportionality can be set equal to one. Then $k = K/ZL = K/e_L L = K/X$, or

$$ke_K = 1 \quad . \tag{11}$$

Convergence analysis can be done in terms of either k or e_K. We choose the latter because it fits better with models to be developed below.

Under typical Say's Law assumptions, capital accumulation is driven by available saving in a closed economy,

$$\dot{K} = sX - dK$$

or

$$\hat{K} = se_K - d \tag{12}$$

with s as the national saving rate and d the depreciation rate.

The next step is to turn (12) into an equation for $\dot{e}_K$. That can be done by bringing in labor and output growth rates in a rearranged version of (2G),

$$\hat{X} = \hat{L} + x_L \quad . \tag{13}$$

We can incorporate accumulation dynamics by adding and subtracting the term $se_K - d$ on the right side of (13). Rearranging using (12) gives

$$\hat{x_K} = -se_K + (d + x_L + \hat{L})$$

or

$$\dot{e}_K = -e_K[se_K - (d + x_L + \hat{L}) \tag{14}$$

with a steady state at

$$e_K = (d + x_L + \hat{L})/s \quad . \tag{15}$$

or

$$s = (d + x_L + \hat{L})k$$

from (11). The expression from the standard Solow model is of course

$$sf(k) = (d + x_L + \hat{L})k$$

where $f(k)$ is the intensive form of a neoclassical aggregate production function.

Linearizing around (15) gives another contrast. Convergence of e_k to a steady state follows from the condition

$$d\dot{e}_K / de_K = -se_K = -(d + x_L + \hat{L}) < 0 \quad .$$

The speed of convergence is relatively fast for values of the parameters postulated in the literature.[5] This finding is in conflict with mainstream econometric results which purport to demonstrate "slow" convergence of about two percent per year. However, given the way that economies work in practice the dynamics in (15) make sense. For example as Young (1995) and other data sources show, East Asian economies including Japan since the 1950s and more recently China have had falling capital productivity with $se_K > d + x_L + \hat{L}$ consistently for decades. Their *lack* of convergence is due to rising saving rates which offset falling output-capital ratios. In the US on the other hand, the output/capital ratio is stable across business cycles, signaling that convergence is "fast."

What happens to distribution? Using (15), (7G) can be restated as

$$\psi(\hat{w} - x_L) + (1-\psi)\{\hat{r} - [se_K - (d + x_L + \hat{L})]\} = 0 \qquad (16)$$

In East Asia with real wage growth roughly equaling labor productivity growth and capital productivity falling, there would appear to be strong tendencies toward a falling rate of profit. In the US on the other hand (16) more or less takes the form

$$\psi(\hat{w} - x_L) + (1-\psi)\hat{r} = 0 \ ,$$

suggesting the possibility of a stable or cyclical profit rate satisfying the inequalities (8) above.

6. Supply-Driven Models

The favored empirical approach to economic growth treats employment as predetermined and derives capital accumulation from either an aggregate saving equation like (12) or a Ramsey-style dynamic optimization exercise (which we omit). The existence of an aggregate production

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[5] Typical values might be $d = 0.03$ and $x_L = \hat{L} = 0.02$, making $d\dot{k} / dk = 0.07$.

function is taken for granted. The foregoing arguments suggest that this line of analysis leads nowhere.

Nevertheless, supply-based studies of growth are likely to remain of interest, not in a never-never land of steady states but rather over periods of a few decades as in Young's work and Table 1. Here we suggest two specifications to pursue, respectively incorporating Kaldor's models of technical change circa late 1950s and late 1960s.

The first starts out from the Harrod-Domar-AK model mentioned above, with predetermined $\hat{L}$ (and thereby L at any point in time) and accumulation equation (14). Instead of holding the output/capital ratio e_K constant, however, we may just as well follow Kaldor (1957) in setting $e_K = \bar{e}_K (K/L)^{-h}$ so that "in the medium run" labor productivity comes from (10). Our rough-and-ready calculations reported above suggest that this may be a useful approximation – at least when capital productivity is not trending upward!

However, it does have at least one drawback, shared by the traditional production function. For a given rate of accumulation, a higher labor force growth rate means that K/L will rise more slowly, holding down productivity. In other words, an economy with a more rapidly growing population will be poorer. But then *negative* employment growth should work wonders! Because it is hard to find anyone who believes that a shrinking, aging population in, say, Japan will have such beneficial effects a model which makes them a central plank should perhaps be taken with a grain of salt.[6]

The latter-day Kaldorian approach can be seen as giving the labor productivity equation (2G) pride of place in combination with a technical progress function of the form proposed by Verdoorn (1949) and Okun (1962),

$$x_L = \bar{x}_L + g\hat{X} \qquad (17)$$

[6] Of course one could always get around the problem by making e_L an increasing function of both K/L and $\hat{L}$, but this begins to look like adding epicycles.

in which the productivity trend term $\bar{x}_L$ could be affected by human capital growth, industrial policy, international openness, population growth, and other factors. Given $\hat{L}$, $\hat{K}$ from (12), and x_L from (10), the growth of capital productivity x_K follows from (9G).

This model can be analyzed in terms of Figure 1, sketched verbally but not actually drawn by Kaldor in his 1966 Inaugural Lecture (published in Kaldor, 1978). To the traditional diagram we have added "Employment growth contours" with slopes of 45 degrees.[7] Each one shows combinations of the output growth rate ($\hat{X}$) and labor productivity growth rate (x_L) that hold the employment growth rate ($\hat{L} = \hat{X} - x_L$) constant. Employment growth is more rapid along contours further to the SE. As will be seen, the contours can be blended with the other schedules in various ways.

Figure 1 here

The one of interest for now combines a "Kaldor-Verdoorn" schedule representing (17) with a predetermined employment growth rate along one of the contours, as at point A, with $\hat{X}$ determined endogenously (ignore the "Output growth" schedule for the moment). If employment growth were faster, say along the contour passing through point B, then $\hat{X}$ would increase as well.

By how much will $\hat{X}$ go up in response to a higher $\hat{L}$? With $0 < g < 1$, the Kaldor-Verdoorn curve has a shallower slope than the employment growth contour, cutting it from above. An upward shift in $\bar{x}_L$ means that $\hat{X}$ and x_L rise by the same amount. Faster employment growth requires a move to a contour further SE and so both $\hat{X}$ and x_L rise, the former by a larger amount so that $\hat{L}$ can in fact accelerate.[8] But growth in output per worker does *increase* as a function of $\hat{L}$, just reversing the Japan story mentioned above (along with myriad papers in the economic

[7] The 45-degree slope comes from the absence of a "relative price term" between the real indexes X and L in (1L).

[8] In algebraic terms the system solves as $\hat{X} = (\bar{x}_L + \hat{L})/(1-g)$ and $x_L = (\bar{x}_L + g\hat{L})/(1-g)$.

demography literature saying that the way to raise per capita income growth is to reduce population growth).

7. A Demand-Driven Growth Model

For many if not most economies in the recent period, a predetermined "full employment" labor force does not make a lot of sense. This observation suggests another use of Figure 1. We can combine Kaldor-Verdoorn with an Output growth schedule which makes $\hat{X}$ depend on x_L, letting employment growth be determined along one of its contour lines as at point D.[9] As drawn, the schedule presupposes that faster productivity growth stimulates output growth. As will be seen below, this positive association may or may not be supported by the data.

One might reasonably take effective demand or available foreign resources as binding restrictions on $\hat{X}$. With such a growth rate closure, effects on employment of shifts in the two schedules become of interest. The employment growth rate is higher for combinations of $\hat{X}$ and x_L values lying below the contour running through D than at the point itself, and lower for combinations above. As discussed below, faster overall productivity growth in the sense of an upward shift of the Kaldor-Verdoorn schedule could reduce $\hat{L}$ due to "labor-shedding." This case is illustrated in Figure 1, with its relatively steep Output growth schedule which means that $\hat{X}$ is *in*sensitive to x_L. An outward shift in the Output growth curve (for example, due to more rapidly growing aggregate demand and/or more availability of foreign exchange) would speed up job creation.

Insofar as increased employment growth is a policy objective, it may or may not transpire depending on how the schedules shift. Evidence reported in Taylor (2001, 2005) and elsewhere suggests that external liberalization in many developing countries in the 1980s and 1990s was associated with faster productivity than demand growth (especially in traded goods sectors), leading to reductions in $\hat{L}$.

[9] One could also think of combining Output growth with a given level of $\hat{L}$, dropping the Kaldor-Verdoorn schedule and making x_L "endogenous" as at point C. This specification mimics much New Growth Theory but for brevity we do not elaborate here.

So how does one model the effects of labor productivity growth on aggregate demand?[10] An illustrative specification focuses on changes of the labor share y. In other contexts, some other distributive variable may be more important than y but the principles underlying alternative specifications would be the same as those utilized here.[11]

If we start with an aggregate demand equation of the form $X = C + I + E$ with the new symbols taking their usual meanings, and assume that $C = [1 - s(y)]X$, then we have

$$X = (I + E)/s(y) \quad . \tag{18}$$

As in Kaldor (1957) a higher labor share will reduce the overall savings rate s if saving rates from profit incomes exceed those from wages (an empirical truism).

Evidently,

$$\hat{X} = l\hat{I} + (1 - l)\hat{E} - \hat{s} \tag{19}$$

with $l = I/(I + E)$.

It is reasonable to postulate that investment demand is stimulated by faster output growth and held back by falling profitability if y goes up:

$$\hat{I} = \hat{I}_0 + f_X \hat{X} - f_y \hat{y} \tag{20}$$

with $\hat{I}_0$ as a trend rate of growth of investment demand and both f_X and f_y being positive.

Omitting a trend term (typically for world demand growth), export sales may be cut back by higher domestic demand as well as by higher unit labor costs,

$$\hat{E} = -q_X \hat{X} - q_y \hat{y} \quad . \tag{21}$$

For the reasons mentioned above the saving rate decreases with the labor share,

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[10] External constraints can be modeled in a "gap" model framework (Taylor, 1994), taking into account foreign aid, capital movements, and shifts in the terms of trade. Using gap-based and other counterfactual methodologies, Taylor and Rada (2003) show that output growth rates in the late 20th century in sub-Saharan Africa and Latin America might have been substantially higher if the debt crisis and adverse terms-of-trade shocks "had not happened."

[11] For example, in Russia the relative price of energy is of crucial importance, while in other countries an increase in the agricultural terms of trade may stimulate overall demand via income effects (as in Turkey with its landed peasantry) or hold it down (as in India with its large proportion of impoverished landless laborers from whom food is the predominant component of demand). Both these relative prices will be influenced by sectoral rates of productivity growth.

$$\hat{s} = -s\hat{y} \quad . \tag{22}$$

By definition, $\hat{y} = \hat{w} - x_L$ so that the labor share itself changes in response to trend growth in the real wage $\hat{w}$ and labor productivity growth.

Combining equations (20)-(24) gives a reduced form expression for $\hat{X}$,

$$\hat{X} = \frac{\hat{I}_0}{m} - \frac{n}{m}\hat{y} = \frac{\hat{I}_0}{m} - c\hat{y} \tag{23}$$

in which

$$m = 1 - l f_X + (1 - l)q_X > 0 \qquad \text{and} \qquad n = l f_y + (1 - l)q_y - s \tag{24}$$

and $c = n / m$.

The term $1/m$ shows how output growth affected by feedbacks of $\hat{X}$ into itself via $\hat{I}$ and $\hat{E}$ in (19), and m must be positive for the model to make sense. The term n shows how changes in $\hat{y}$ influence effective demand. Higher labor share growth $\hat{y}$ reduces $\hat{X}$ by cutting into investment and exports and increases it by lowering the overall saving rate and raising the multiplier $1/s$. If $n > 0$ effective demand growth can be said to be profit-led; otherwise it is wage-led.[12]

For a positively sloped Output growth locus in Figure 1, therefore, demand growth has to be profit-led (this condition is relaxed in the following section). If so, we can combine (23) with the Kaldor-Verdoorn equation (17) to determine $\hat{X}$, x_L , and (along a labor contour) $\hat{L}$. Note that the complicated ratio $c = n / m$ can exceed or be less than one. It will be large when investment and exports respond strongly and saving is insensitive to changes in $\hat{y}$, and investment is strongly crowded in and exports are not strongly crowded out by $\hat{X}$. These are basically conditions for effective demand growth to be "strongly profit-led."

Detailed comparative dynamic results when (17) and (23) are solved together are:

[12] See Taylor (2004) for more details. The inner workings of Kaldorian growth models very often hinge on whether effective demand is wage- or profit-led.

$$\hat{X} = \frac{1}{1-gc}[\frac{\hat{I}_0}{m} - c(\hat{w} - \bar{x}_L)] \quad , \tag{25}$$

$$x_L = \frac{1}{1-gc}[g(\frac{\hat{I}_0}{m} - c\hat{w}) + \bar{x}_L] \quad , \tag{26}$$

and

$$\hat{L} = \hat{X} - x_L = \frac{1}{1-gc}[(1-g)(\frac{\hat{I}_0}{m} - c\hat{w}) + (c-1)\bar{x}_L] \quad . \tag{27}$$

We have already assumed that $g < 1$ and for the Output growth and Kaldor-Verdoorn curves to cross in the stable configuration of Figure 1 it must be true that $1-gc > 0$. Faster trend growth of investment increases all three growth rates.[13] By reducing profitability faster real wage growth makes them fall. The effect of a faster trend rate of productivity growth $\bar{x}_L$ is ambiguous. From (27), it only stimulates employment growth when $c > 1$ or the economy is strongly profit-led. In terms of Figure 1, $dx_L / d\hat{X} = 1/c < 1$ along the output growth schedule so that it is less steep than the employment growth contours. Insofar as demand is wage-led, this result can be problematical in terms of providing enough jobs to satisfy potential labor force growth.

More generally, looking at the institutional and historical forces underlying shifts in these curves can be an illuminating method for studying medium-term growth. A long-run caveat is that eventually real wage growth is likely to keep up with labor productivity growth so that the term $x_L - \hat{w}$ in (17) vanishes and $\hat{X}$ will not be affected by productivity increases, but over periods of a decade or three this reservation is not likely to be relevant. Convergence dynamics may well be of interest in themselves, since in light of (7G) they are likely to be cyclical, especially when y affects both demand injections like investment and exports as well as saving and import leakages. Taking these effects formally into the analysis would obviously push it in the direction of Goodwin-style cyclical growth. Barbosa-Filho and Taylor (2006) present a simple econometric model based on US experience.

[13] Faster trend growth of exports and slower trend growth of saving would have the same effects.

8. Effects of Real Wage Growth on Productivity Growth

Are there obvious extensions to the present model? One would be to make investment growth in (20) respond directly to productivity growth, with an elasticity presumably exceeding f_y . Such a linkage has been raised in the literature but is not explored here.

On another front, Naastepad (2006) introduces an interesting macro twist on the old idea that real wage growth may stimulate firms to innovate to raise labor productivity.[14] The outcome can have consequences for whether the economy is wage- or profit-led.

To work things out, assume that the technical progress function (17) now takes the form

$$x_L = \bar{x}_L + b\hat{w} + g\hat{X} \qquad (28)$$

with $b > 0$ gauging the impact of real wage growth on productivity growth[15]. With this new specification equations (25)-(27) for growth rates become

$$\hat{X} = \frac{1}{1-gc}\{\frac{\hat{I}_0}{m} - c[(1-b)\hat{w} - \bar{x}_L]\} \qquad , \qquad (29)$$

$$x_L = \frac{1}{1-gc}[g\frac{\hat{I}_0}{m} + (b-gc)\hat{w} + \bar{x}_L] \qquad , \qquad (30)$$

and

$$\hat{L} = \frac{1}{1-gc}\{(1-g)\frac{\hat{I}_0}{m} - [(1-g)c + b(1-c)]\hat{w} + (c-1)\bar{x}_L\} \quad . \qquad (31)$$

Broadly speaking introducing the wage growth effect on productivity growth pushes the system toward being wage-led. In (29) for example if $c > 0$ the negative effect of wage growth on demand growth is attenuated by higher values of b and reverses if $b > 1$. If $b > gc$ a similar statement applies to productivity growth in (30). Finally, the effects of $\hat{w}$ on $\hat{L}$ in (31) are complicated and can go either way depending on parameters.

[14] The notion goes back to Marx at least. It was reintroduced to the mainstream literature by Hicks (1932) and promulgated by Kennedy (1964) among many others.

[15] In the context of developing economies the positive effect of wage on labor productivity is better understood using the efficiency wage mechanism where higher wage allows one to achieve better education or health care (see Ros 2000).

9. Some Numbers

To get a feel for this model it makes sense to look at "representative" parameters. We can work through some numbers quickly. They are drawn from Taylor (2004) and Naastepad (2006).

The overall saving rate can be written as $s = y s_w + (1 - y) s_r$ with respective rates s_w and s_r from wage and profit income. The elasticity of s with respect to y becomes $-(y / s)(s_r - s_w)$. For a "typical" advanced economy and taking into account tax leakages, s_r might be 0.4 and s_w might be 0.2 If $y = 0.6$ then s becomes 0.28. The elasticity s in (22) takes the value 0.4286. Distributive effects on the multiplier turn out to be significant!

In the investment function (20) f_X might be in the neighborhood of unity, and f_y about 0.4. A more complete model would have separate export and import functions (with imports most naturally treated as a function of output and the real exchange rate). With $\hat{E}$ in (21) referring to growth of *net* exports, plausible values of q_X and q_y might be 0.1 and 0.2 respectively. With $l = 0.8$, we get $m = 0.202$ and $n = -0.0686$. The economy turns out to be weakly wage-led, but small parameter changes could reverse the result and make the economy weakly profit-led. Regardless of its slope, it seems likely that Output growth curve in Figure 1 will be steep.

The traditional value for g in (28) is 0.5. Naastepad (2006) quotes a similar value for b. One implication is that the effect of $\hat{w}$ on $\widehat{X}$ in (29) will be attenuated. In (30) c is a ratio of small numbers and hard to pin down. If it is negative and demand is wage-led, faster real wage growth will stimulate x_L. The profit-led case is ambiguous. The effect of real wage growth and employment growth in (31) is similarly hard to call.

Evidently, the demand-driven model provides a range of potential outcomes which deserve to be explored. Since it is fundamentally based on identities it is bound to fit the data!

10. Conclusions

In summary:

Standard sources of growth accounting is empty of content because it depends upon neoclassical production theory. Rather, growth analysis must be based on productivity equations that can be derived either from NIPA accounting conventions or algebraic identities. These complementary schemes do impose valid restrictions on growth of the wage rate, profit rate, capital, labor, and their respective average productivities.

One restriction states that the output growth rate equals employment growth plus productivity growth. The standard Solow-style “convergence” model basically adds accumulation dynamics to this identity. Replacing the aggregate production function with proper accounting restrictions gives a growth model with detailed results that differ markedly from those of the standard model.

Alternative, essentially Kaldorian supply-and demand-based alternatives to sources of growth based on a familiar output growth vs. productivity growth diagram with constant employment growth contours added in look like a useful alternative to the mainstream models.

The demand-driven variant can give rise to a number of outcomes, which it would be of interest to explore empirically.

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Table 1: Output, Factor Input, and Productivity Growth Rates in East Asia, 1966-1990

(%/year)

Country

	Hong Kong	Singapore	South Korea	Taiwan
$\hat{X}$	7.3	8.7	10.3	8.9
$\hat{K}$	8.0	11.5	13.7	12.3
$\hat{L}$	3.2	5.7	6.4	4.9
x_K	-0.7	-2.8	-3.4	-3.4
x_L	4.1	3.0	3.9	4.0
$x(y)$	2.3	0.2	1.7	2.1
Memo item (%)				
a	62.8	50.9	70.3	74.3

Source: Young (1995)

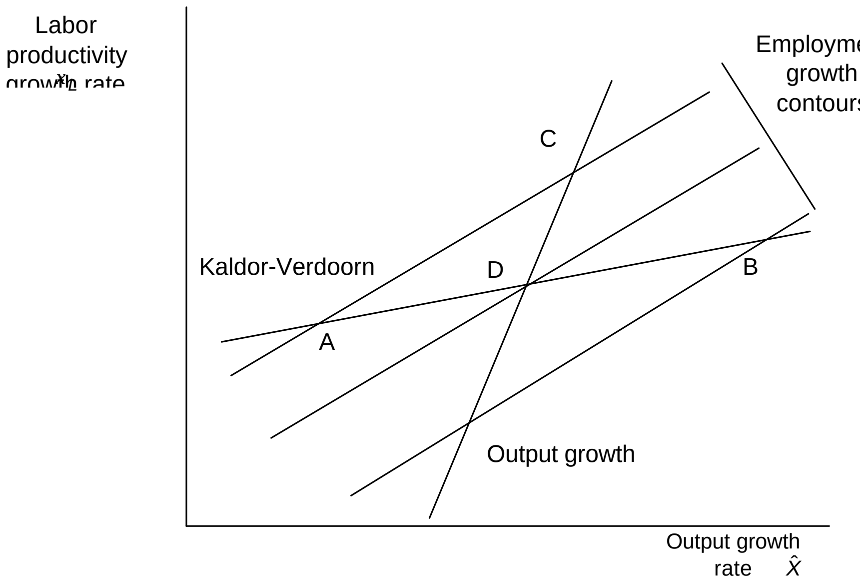

Figure 1: Joint determination of output, labor productivity, and employment growth rates