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Estimated Non-linearities and Multiple Equilibria in a
Model of Distributive-Demand Cycles

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Abstract

We introduce the results of a non-parametric estimate of the US wage-Phillips Curve into a simplified version of the model of the wage-price spiral by Flaschel and Krolzig (2008). Making use of Okun's law, the non-linearity in the wage inflation-employment relation translates into a non-linearity in the so-called *distributive curve* of the economy. We then provide a dynamical analysis both in wage-led and profit-led effective demand regimes. In a profit-led scenario, shown to be the empirically relevant case for the US economy, there are 2 stable equilibria of Goodwin (1967) growth cycle type, identified as a stable depression and a stable boom, and a saddle-path stable equilibrium in between them. Both stable steady states are surrounded by trajectories that cycle counterclockwise around their basins of attraction. The obtained type of growth fluctuations can be verified by a long phase cycle estimation for the US economy using a method developed by Kauermann, Teuber and Flaschel (2008).

Keywords: profit-led demand, profit squeeze, long phase cycles, multiple steady states, booms and depressions.

JEL CLASSIFICATION SYSTEM: E24, E31, E32.

1 Introduction

It is standard in applied macro literature to express the labor market and goods market dynamics by a single Phillips curve, in which the cost pressure on the two markets is working on a single inflation rate. A single Phillips curve for the two markets requires the simplifying assumption that prices are a constant mark-up on wages.

On the other hand, Post-Keynesian macroeconomic models have recently been considering two separate Phillips curves, one for the labor market (the *wage*-Phillips curve) and one for the goods market (the *price*-Phillips curve), in order to analyze the interacting dynamics of the adjustment processes (Proaño *et al.*, 2007, Flaschel and Krolzig, 2006). Such adjustments are usually referred to as *wage-price spiral*, a well-known process in macroeconomics having to do with opposite sides facing off in wage-setting and trying to maintain their respective purchasing power in view of price variations. The distributional conflict that is at the source of such adjustments is of obvious interest for Post-Keynesian macroeconomists, and the consideration of two separate Phillips curves provides additional insights in the analysis of the interrelation between income distribution, inflation, and growth. A recent example is the model studied by Flaschel and Krolzig (2006), which constitutes the starting point of our analysis.¹ From a structural form involving both a wage and a price Phillips curve, they derive a dynamic equation for the wage-price spiral as the corresponding reduced form, in which the growth rate of the wage share is affected by the employment rate, the rate of capacity utilization, and the growth rate of labor productivity.

In this paper, we are interested in the reduced form wage-price spiral that arises from the consideration of two separate Phillips curves in order to extend the now pretty standard (at least for the readership of a journal such as *International Review of Applied Economics*) structuralist macroeconomic model of demand-led growth and income distribution (Bhaduri and Marglin, 1990a, 1990b, Taylor, 2004, Barbosa-Filho and Taylor, 2006 are just a few examples) and study how this extended model constitutes an improvement towards our understanding of the US economy in the post-WWII era. The aim of structuralist models is to provide a description of the macroeconomy that focuses on the interaction between the rate of capacity utilization and one distributive variable, say the wage share.² Such an analysis is meant to be alternative to, and more truthful to Keynes' ideas and the Post-Keynesian tradition than the standard AD-AS framework. The demand side of the structuralist model is obtained from the equilibrium between savings and investment, and leads to a reduced form equation in which the rate of capacity utilization depends on income distribution. Such equation has been called alternatively 'IS', or 'effective

¹See also Flaschel, Kauermann and Semmler (2007).

²Bhaduri and Marglin (1990a, 1990b) reason in terms of the profit share, but being distributive shares the complement of one another there is no harm in considering the wage share instead, as we do in this paper.

demand regime' in the literature, and its slope depends on the structural characteristics of the economy. In the felicitous terminology coined by Bhaduri and Marglin (1990a), the demand regime is *profit-led*, or *exhilarationist* when capacity utilization reacts negatively to the wage share, whereas a positive impact on the wage share on capacity utilization implies that the demand regime is *wage-led*, or *stagnationist*. The supply side of the model has to do with the behavior of firms, and determines how output is distributed among wage and profit earners. In particular, the interest is in how variations in income distribution (i.e. the wage share) over time are affected by the rate of capacity utilization. The resulting long-run relation is usually called 'producer's equilibrium' (PE), or 'distributive schedule', and again its slope depends on the structural features of the economy. So far, the literature has been considering linear distributive schedules only. An upward sloping distributive schedule means that the economy displays a *profit squeeze* since the profit share is eroded as capacity utilization approaches its maximum level. Conversely, a downward sloping distributive schedule is traditionally referred to as exhibiting *forced saving*, because usually profit earners have higher propensity to save than wage earners, and therefore the closer the economy gets to full utilization the more the economy is pushed toward higher overall savings by the increase in the share of profits.

The elements of novelty we introduce in this paper concern the distributive schedule, which we modify relative to the existing literature in view of both theoretical and empirical arguments. First, we provide a non-parametric estimation of the wage-Phillips curve for post-war United States, resulting in a robust non-linear relation between wage inflation and employment rate. Second, we notice somewhat trivially that one can make use of Okun's law, which links variations in capacity utilization to changes in the employment rate, to convert the wage-price spiral in reduced form into a dynamic equation that it is closely related to the distributive schedule. This combination of empirically-based and *a priori* restrictions allows us to study the implications of the estimated non-linearity for the shape of the distributive curve of the economy. An important feature of our distributive curve is that it displays profit squeeze at low and high levels of capacity utilization, and forced savings corresponding to an intermediate region.

Putting together the non-linear distributive schedule with the effective demand schedule, we are able to analyze in depth the dynamic properties of the economy, both in profit-led and wage-led effective demand regimes. Despite the very elementary modeling, the type of non-linearity we introduce gives rise to multiple equilibria in the distributive-demand framework, and these equilibria have different stability properties corresponding to different demand regimes. The possibility of multiple equilibria due to the shape of the distributive curve in a Post-Keynesian demand-distribution model is so far unexplored in the literature, at least to the best of our knowledge.

The key finding of our model is that a profit-led demand regime, which we show to be the empirically relevant case for the US economy, leads to three equilibria

corresponding to different values of the wage share and of the rate of capacity utilization. The two ‘extreme’ equilibria (E_1 and E_3 in Figure 3) are locally asymptotically stable, and display counterclockwise transitional dynamics. These features are qualitatively consistent with the available evidence on post-war United States (Barbosa-Filho and Taylor, 2006). In view of such stability properties, we call ‘stable recession’ the equilibrium featuring low capacity utilization (and ‘high’ wage share), and ‘stable boom’ the long-run position involving a high capacity utilization rate. The key force determining the stability properties of both these equilibria, given the profit-led demand, is an upward-sloping distributive curve in their respective regions of the phase space, that is a profit-squeeze effect. The stabilizing (destabilizing) effects of profit-squeeze (forced savings) are already known in the Post-Keynesian literature. The novelty in this contribution is that, differently from the existing investigations on the subject (Taylor, 2004) in which a linear distributive curve can either be downward or upward sloping, the non-linearity in our distributive curve allows for both profit-squeeze and forced saving, corresponding to different ranges of capacity utilization.

The fact that a profit-led economy such as the US can have stable recessions and stable booms separated by an intermediate, mostly unstable region, has important implications for demand policies aimed at stimulate or contract the macroeconomy. In fact, fighting a stable recession through expansionary demand policy requires strong, other than well-targeted measures, whereas cooling down an economy that is deemed to be overheated may result into forcing the system into a slump that will be hard to contrast. An intermediate equilibrium that is only saddle-path stable means indeed that there is only one stable dynamical path (and therefore only one initial condition) leading to it, from which the need of a very well-targeted measure aimed at reaching such equilibrium. The implication is that a weak measure will not be able to push the economy out of the basin of attraction of the stable recession. Conversely, too strong of a restriction to cool down an overheated system may push the dynamics toward the low-capacity equilibrium, which is dynamically stable and therefore hard to turn around.

The paper is organized as follows. We first present some empirical evidence of demand-distribution cycles in the United States (1956-2004). Then, we review the baseline model by Flaschel and Krolzig (2006), and we introduce a non-linearity consistent with our estimation results into the wage-Phillips curve to derive a non-linear distributive curve for the economy. Combining this long-run relation as the only departure from the existing literature with a standard demand regime borrowed from Barbosa-Filho and Taylor (2006), we present our full model of wage share-capacity utilization dynamics in section 6, discussing both profit-led and wage-led scenarios. All these conclusions are drawn assuming an exogenous growth rate of labor productivity. As an extension, we study the implication of endogenizing labor productivity growth in the model, and show that the main conclusions are mostly robust to this generalization in Section 7. We then discuss the empirical

relevance of profit-led versus wage-led demand regimes for the US economy according to the available evidence in Section 8. Section 9 concludes. The Appendix to this paper provides derivations of the relevant equations, and an explanation of the methodology used for estimation.

A notational convention, used to facilitate the exposition, is that functional relations are denoted by square brackets. For instance, the expression $\gamma[u - \bar{u}]$ denotes $\gamma[\cdot]$ being a *function* of the difference between u and \bar{u} . Conversely, round parentheses are used for multiplication purposes: an expression like $\delta(e - \bar{e})$ denotes that δ is any variable multiplying the difference $e - \bar{e}$. Also, following the Post-Keynesian literature, we will use the terms *capital utilization* and *capacity utilization* alternatively.

2 Demand-Distribution Cycles in Post-WWII United States

Demand-distribution cycles in post-war United States have been the subject of several important studies in the Post-Keynesian tradition (Barbosa-Filho and Taylor, 2006, Bhaduri and Marglin, 1990a, 1990b, Hein and Vogel, 2007, Stockhammer and Onaran, 2004). The available evidence points toward Goodwin-style cycles in the employment rate and the labor share. In the empirical plots shown in Figure 1, we depict an estimated long phase cycles as against the six business cycles that were observed in the US economy from 1956Q1 to 2004Q4 (bottom right and left panels).³ As shown in Kauermann *et al.* (2008), all business cycles have by and large the same counterclockwise orientation, and so does the long-phase cycle. Such paths in macroeconomic fluctuations point toward an explanation that emphasizes the distributional conflict between capital and labor, and the role of unemployment as a discipline mechanism on wage demands by workers. High employment generates wage inflation which, as long as real wages increase more than labor productivity, increases the wage share in output. The resulting decrease in the profit share, in Kaleckian fashion, will however reduce future investment and output. Lower output will in turn reduce labor demand and employment and consequently lead to lower wage inflation or even deflation and thus reduce the labor share. But a higher profit share will produce a surge in investment. This will lead to greater employment and thus improve the bargaining power of workers and consequently wages in Phillips curve fashion. At this point, the wage share in output has increased, and the cycle can repeat itself.

[FIGURE 1 ABOUT HERE]

³Table 1 in the Appendix provides a brief description of the data used for these plots. See also Kauermann *et al.* (2008) for the econometric methodology that allows to separate endogenously long phase cycles from cycles occurring at higher frequency and for empirical applications that are closely related to the ones shown in Figure 1.

The bottom right panel in Figure 1 suggests that the depressed cycles are at most two in number.⁴ Nevertheless, the presence of business cycles around low and high equilibrium levels of capacity utilization is hard to justify in models, such as the ones already in the literature, that are capable to produce a single dynamic equilibrium. In what follows, we will see that the inclusion of empirically-based non-linearities in the model can account for persistent periods of booms as well as recessions, and for transition dynamics that cycle for considerable time around either of these equilibria.

3 Cross-over Wage-Price Dynamics

The starting point for our analysis is a simplified version of the model of the wage-price spiral estimated by Flaschel and Krolzig (2006), which we modify to allow for labor productivity growth. The structural form is given by:⁵

$$\begin{aligned}\hat{w} &= \beta_w (\bar{U}^l - U^l) + \kappa_w (\hat{p} + n_x) + (1 - \kappa_w)(\pi + n_x), & \hat{w} &\equiv \dot{w}/w & \text{wage inflation} \\ \hat{p} &= \beta_p (\bar{U}^c - U^c) + \kappa_p (\hat{w} - n_x) + (1 - \kappa_p)\pi, & \hat{p} &\equiv \dot{p}/p & \text{price inflation}\end{aligned}\tag{1}$$

where w denotes the nominal wage, p the aggregate price level, U^l is the rate of unemployment of labor, U^c the rate of unemployment of capital, the bars indicate inflationary barriers for the two variables, n_x stands for the exogenous rate of Harrod-Neutral technical change, and π is a term that captures expectations about inflation, which we will refer to as *inflationary climate* in what follows. The wage share, denoted by ψ , is the ratio of the real wage w/p over labor productivity x . Define furthermore the labor employment rate as $e \equiv 1 - U^l$, $\bar{e} \equiv 1 - \bar{U}^l$, and similarly the rate of capital utilization as $u \equiv 1 - U^c$, $\bar{u} \equiv 1 - \bar{U}^c$. In Appendix A, we show that from (1) the following reduced-form equation for the evolution of the wage share can be derived:

$$\hat{\psi} = \kappa ((1 - \kappa_p)\beta_w(e - \bar{e}) - (1 - \kappa_w)\beta_p(u - \bar{u}))\tag{2}$$

where $\kappa \equiv (1 - \kappa_p\kappa_w)^{-1}$. Equation (4) shows that the growth rate of the wage share responds to both utilization rates in labor and capital inputs, other than of course to labor productivity growth.⁶ Assuming linear coefficients and Okun's law

⁴In the figures top right and left we show in addition (by dots) the time series that was used in the estimations as well as the long phase fluctuations and the business fluctuations around them.

⁵In their model, the authors consider, in both equations, error-corrections for the deviation of the wage share from a certain level ψ_0 . For reasons of expositional simplicity, we do not analyze in this paper the consequences of this augmentation in both the money wage and the price Phillips curve.

⁶Flaschel, Kauermann and Semmler (2007) provide estimates suggesting that κ , κ_p , κ_w are all positive. See also fn. 8. The growth rate of labor productivity is included in the definition of $\hat{\psi} \equiv \dot{w} - \dot{p} - n_x$.

$e - \bar{e} = \sigma(u - \bar{u})^7$ it is easy to find that, since $\kappa > 0$ the labor share adjustment responds positively (negatively) to the rate of capacity utilization if and only if

$$\alpha \equiv (1 - \kappa_p)\sigma\beta_w - (1 - \kappa_w)\beta_p > 0 \quad (\text{respectively } < 0). \quad (3)$$

Proaño *et al.* (2007) denoted the case of a positive response of $\hat{\psi}$ on economic activity ($\alpha > 0$) as *labor market-led* wage-adjustment process, and defined the wage-adjustment to be *goods market-led* when $\alpha < 0$ holds. The intuition is that, when $\alpha > 0$, it is the labor market that drives the adjustment process since the effect of higher employment on wage share growth dominates the effect of higher capacity utilization. Conversely, when $\alpha < 0$ it is the latter effect that dominates in determining the growth rate of the labor share.

In Post-Keynesian macroeconomic modeling, it is common to consider not only a relation like (2) in which changes in the wage share respond to the rate of capital utilization, but also the way in which changes in wage share affect the capital utilization rate. Such relation determines the so-called *effective demand regime* of the economy. If capital utilization reacts positively to the wage share, the demand for goods is said to be *wage-led*, while if variations in the wage share cause changes in capital utilization rate of opposite sign the demand regime is said to be *profit-led*.

The sign of the parameter α , combined with the characteristics of the demand regime of the economy, determines whether real wage adjustments have stabilizing or destabilizing effects. Wage adjustments will have stabilizing effects if the negative response of investment to changes in real wages outweighs the positive response of consumption, and if wages are more flexible to labor demand pressures than prices to goods market pressures (or both v.v.). Conversely, if investment reacts less than consumption to a change in real wages and wages remain more flexible than prices (in terms each of their own demand pressures), or both v.v., then real wage adjustments will show destabilizing effects. The four possible scenarios are presented in Table 1, taken from Proaño *et al.* (2007), although a deeper discussion of such effects is not within the scopes of this paper.

[TABLE 2 ABOUT HERE]

4 Non-Linearities in the Wage Demand-Pressure Terms. The Distributive Curve

Consider now the term $\beta_w(e - \bar{e})$ in (2), describing how the wage share reacts to the employment rate level. As opposed to Flaschel and Krolzig (2006), who estimated a VAR for the US economy assuming a constant parameter β_w , in what follows we will

⁷Flaschel and Krolzig (2006) parameterize $\sigma = 1$, an assumption that is confirmed by the estimation results in Proaño *et al.* (2007). Also, Foley and Michl (1999), p.179 can be read as arguing that $\sigma = 1$.

consider instead a p-spline estimation of the US-money wage Phillips curve, where ‘p’ stands for ‘penalized’. The estimation technique is described in Appendix B, and amounts, as it is standard in non-parametric estimation, to assume the functional relation among the dependent and the predetermined variables to be an unknown but smooth function. The outcome of estimation will be a *plot*, as opposed to a vector of parameters which is the outcome of parametric estimation. In particular, we find the non-linear relationship between wage inflation and demand pressure on the labor market shown in the top panel in Figure 2: the curve is increasing up to an employment rate of slightly more than 92%, then has an almost flat or at most slightly decreasing region, and eventually becomes again increasing for values of the unemployment rate smaller than 6%.⁸

[FIGURE 2 ABOUT HERE]

A typical element of interest in spline estimates is also the plot of the first derivative of the function. The first derivative of our wage-employment relation is displayed in the bottom graph appearing in Figure 2. By looking at the two portions of the figure, we see that the curve is increasing but concave, until an inflexion point around a 6.5 % unemployment rate, after which the curve becomes convex, first decreasing then increasing. Eventually, there is another inflexion point around an employment rate of 95.5% or so, and the final portion of the curve is increasing, but concave again. From the bottom graph, we can locate the unconditional mean of the first derivative of β_w around 0.6.

A standard Keynesian economic intuition behind the behavior of the curve will focus on the bargaining power of labor supply. For high levels of unemployment, the workers’ bargaining power is small: they (or the labor union representing them) will accept only small increases, or even resign themselves to small decreases in the nominal wage in order to increase the employment rate. Corresponding to the center of the curve, there is a flat region where labor is resisting wage inflation decreases at the given expected price inflation. Finally, as soon as the unemployment rate is below its inflationary barrier, workers will exercise their increased bargaining power in requiring significantly more than proportional increases in wage inflation (as compared to price inflation). In view of such arguments, let us reconsider equation (2). Given the above non-linearity, it makes sense to consider the term $\beta_w[\cdot]$ as a general, non-linear function of the employment gap, and not a constant parameter.

In order to obtain a distributive curve in reduced form, we make the following assumptions:

1. There is a smooth function $\beta_w[e - \bar{e}]$ whose behavior is depicted in the top panel of Figure 2;

⁸The other estimated p-spline functions are not statistically different from linear ones – including the price Phillips curve – with the exception of the inflation climate which however does not matter for the law of motion of real wages.

2. β_p is a constant coefficient, as in Flaschel and Krolzig (2006);
3. Okun's law holds: $\sigma(u - \bar{u}) = e - \bar{e}$;
4. very much in the spirit of Rose (1967), price flexibility is higher than wage flexibility in the middle range of the money wage Phillips curve that is $(1 - \kappa_p)\sigma \frac{\partial \beta_w}{\partial u} - (1 - \kappa_w)\beta_p < 0$ within the relevant range (note the parallel with $\alpha < 0$ in equation 3);
5. for any value of capacity utilization, there is a negative, linear relation between wage share growth and the difference $\psi - \bar{\psi}$, where $\bar{\psi}$ is a constant parameter representing an inflationary barrier on the wage share.

To justify our fifth assumption, we can follow Barbosa-Filho and Taylor (2006) in using a combination of two arguments. The first one lays on an upward-sloping relation between the level of the wage share and the rate of growth of labor productivity, known in the literature as *induced technical change* effect. Generally, this effect is positive since higher wages to pay will induce firms in adopting more labor-savings techniques. The second argument is that the bargaining power of the labor force increases with the wage share. Thus, the rate of growth of the real wage \hat{w} should depend positively on the wage share. Assuming both relations to be linear, if the induced technical change effect is higher than the bargaining power effect on the real wage, then the rate of growth of the wage share should be negatively affected by its own level. A different story, which however shares the same ending, is told in Flaschel and Krolzig (2006). They assume that an increasing wage share will dampen the evolution of wage inflation, building on Blanchard and Katz (1999) to 'microfound' this negative relation with a bargaining argument. What matters for the purposes of our analysis here is that a negative relation between $\hat{\psi}$ and ψ is confirmed by the empirical evidence presented in both papers, and these findings provide further support for our assumption.

We claim that the features of our framework, combining elements coming from empirical findings with *a priori* restrictions on the relations between the variables of interest are interesting enough to be exploited in imposing the following dynamic equation for the evolution of the wage share over time:

$$\hat{\psi} = \kappa((1 - \kappa_p)\beta_w[\sigma(u - \bar{u})] - (1 - \kappa_w)\beta_p(u - \bar{u})) - \beta_{\psi\psi}(\psi - \bar{\psi}) \quad (4)$$

where $-\beta_{\psi\psi} < 0$ constant, and $\bar{\psi}$ denotes inflationary barriers on the labor share.⁹ Equation (4) implies that the growth rate of the wage share is increasing in capacity utilization when $\frac{\partial \beta_w}{\partial u} > \frac{1 - \kappa_w}{\sigma(1 - \kappa_p)}\beta_p$, a case which corresponds to the labor-market led

⁹There are multiple equilibria in the model, and that is why we refer to multiple barriers as opposed to a single one.

wage adjustment described above, and that $\frac{\partial \hat{\psi}[\cdot]}{\partial u} < 0$ otherwise.¹⁰ Therefore, the stationary points of this composite function will lay where $\frac{\partial \beta_w}{\partial u} = \frac{1-\kappa_w}{\sigma(1-\kappa_p)}\beta_p$.

We are now ready to characterize the isocline relating wage share to capacity utilization, that is the *distributive curve* of our economy. To do so, it is sufficient to solve (4) for ψ by setting the time-derivative of the wage share equal to zero:

$$\psi[u, \bar{u}, \bar{\psi}, n_x] = \bar{\psi} + \frac{\kappa}{\beta_{\psi\psi}} \left((1 - \kappa_p)\beta_w[\sigma(u - \bar{u})] - (1 - \kappa_w)\beta_p(u - \bar{u}) \right) \quad (5)$$

In Post-Keynesian macroeconomics, it is common to look at the sign of the first partial derivative of the function with respect to u in order to interpret the shape of the distributive curve.¹¹ We say that the economy is ‘Marxist’ or it exhibits *profit-squeeze* if $\frac{\partial \psi}{\partial u} > 0$, given that an increase in capacity utilization will determine a falling profit share. Conversely, $\frac{\partial \psi}{\partial u} < 0$ means that the economy displays *forced saving* along ‘Kaldorian’ lines (Taylor, 2004), given that the increase in the profit share associated with a higher utilization rate translates *à la* Kaldor into higher savings. Our assumptions ensure that the non-linear distributive curve features profit-squeeze at low and high capacity utilization rates, and forced savings corresponding to an intermediate region of u , as shown in Figure 3. In particular, since $\beta_{\psi\psi} > 0$, the isocline (5) will display profit-squeeze for rates of capacity utilization such that $\frac{\partial \beta_w}{\partial u} > \frac{1-\kappa_w}{\sigma(1-\kappa_p)}\beta_p$, and forced saving otherwise. The behavior of the isocline is depicted in Figures 3 and 4.

5 The Demand Regime

The characterization of the demand regime is completely standard in our model. Consider first how the adjustment process of capital utilization rate is affected by variations in the wage share, or equivalently the effective demand regime adjustment process. Departing only slightly from textbook Post-Keynesian macroeconomics (Taylor, 2004), assume that there is a linear relation, captured by a constant parameter $\beta_{u\psi}$, between the growth rate of capacity utilization and the difference $\psi - \bar{\psi}$, where $\bar{\psi}$ is a constant inflationary barrier on the wage share. A profit-led economy corresponds to the case in which $\beta_{u\psi} < 0$, whereas if $\beta_{u\psi} > 0$ we say that the economy is wage-led.

Then, consider the impact of capacity utilization on its growth rate, recalling that capacity utilization is defined as the ratio between output (say X) and installed

¹⁰Flaschel, Kauermann and Semmler (2007), specifying the inflationary climate as 12 quarter moving-average, obtain estimates of $\kappa_w = .4464$, $\beta_p = .0026$ and $1 - \kappa_p = .6859$ so that, given the traditional $\sigma = 1/3$, the composite parameter $\frac{1-\kappa_w}{\sigma(1-\kappa_p)}\beta_p$ is roughly equal to .0063. Clearly, different specifications for the inflationary climate may lead to different results. It is worth to keep in mind, however, that recent estimations of the Okun’s coefficient, such as the one provided in Proaño *et al.* (2007), point toward σ not being significantly different from 1.

¹¹Clearly, $\bar{u}, \bar{\psi}$ are shift parameters in the model.

capacity, so that the growth rate of u equals the difference between output growth and the growth rate of installed capacity. Two arguments support a negative relation between \hat{u} and u . The first one is the general consensus on the basic Keynesian stability condition, according to which $\partial\dot{X}/X < 0$.¹² One has, however, to consider also the effect of an increase in the capital utilization rate on the productive capacity of the economy. Generally, capital formation responds positively to the level of economic activity (which can be interpreted as an acceleration principle). It follows immediately that the rate of growth of capacity is negatively affected by the level of capacity utilization.

In view of such arguments, we have the following dynamic equation for the evolution of capacity utilization:

$$\hat{u} = \beta_{u\psi}(\psi - \bar{\psi}) - \beta_{uu}(u - \bar{u}), \quad \beta_{uu} > 0, \quad \beta_{u\psi} \begin{cases} > 0 & \text{if the economy is wage-led} \\ < 0 & \text{if the economy is profit-led} \end{cases} \quad (6)$$

The isocline $\dot{u}[\cdot] = 0$ will be of the form:

$$u[\psi, \bar{\psi}, \bar{u}] = \bar{u} + \frac{\beta_{u\psi}}{\beta_{uu}}(\psi - \bar{\psi}) \quad (7)$$

Under the above assumptions, the demand regime of our simple economy has a positive (negative) intercept and negative (positive) slope if the economy is profit-led (wage-led).¹³

6 The Dynamical System

We are now able to study qualitatively the dynamics of the economy described by the system of equations formed by (4) and (6). Such dynamics takes place in the phase space (u, ψ) , where the isoclines describing long-run relations between the two variables of interest are represented by (5) and (7).

First of all, due to the non-linear shape of the distributive curve, there are likely multiple equilibria in this model, corresponding to different points at which the isoclines intersect. Furthermore, the stability properties of such steady-states depend on the features of the demand regime of the economy.

In order to get started in our stability analysis, let us evaluate the Jacobian matrix of the dynamical system at the steady states (u_0, ψ_0) :

$$J[u_0, \psi_0] = \begin{pmatrix} -\beta_{uu}u_0 & \beta_{u\psi}u_0 \\ \kappa(\sigma(1 - \kappa_p)\frac{\partial\beta_w}{\partial u}[u_0 - \bar{u}] - (1 - \kappa_w)\beta_p)\psi_0 & -\beta_{\psi\psi}\psi_0 \end{pmatrix}$$

¹²Skott (1989, Chapter 6) is an authoritative dissenting voice on such stability condition, and especially about its plausibility in the long-run.

¹³We rule out as uninteresting the case of a demand regime laying entirely in the orthant in which capacity utilization takes only negative values, and therefore we impose $\frac{\psi}{u} > -\frac{\beta_{uu}}{\beta_{u\psi}}$ to be satisfied everywhere.

so that we can analyze the profit-led case and the wage-led case by looking at the sign of the parameter $\beta_{w\psi}$.

6.1 Profit-Led Demand Regime

In the profit-led case, $\beta_{w\psi} < 0$. Thus, when the distributive curve has a positive slope, the determinant of our Jacobian matrix is positive. Given the negative trace, both eigenvalues are negative, and an equilibrium corresponding to this situation is locally asymptotically stable. For such reasons, we will refer to an equilibrium like E_1 in Figure 3 as a *stable depression* and to an equilibrium like E_3 as a *stable boom*, because of the low and high value of capacity utilization respectively. Conversely, when the slope of the locus $\dot{\psi} = 0$ is negative, the corresponding equilibrium will be a saddle point. Figure 3 displays the phase diagram corresponding to a profit-led economy when the slope of the locus $\dot{u} = 0$ is such that the two curves intersect three times. As shown in the figure, the dynamics around the steady states E_1 and E_3 feature the same counterclockwise behavior, although the negative trace of the Jacobian matrix ensures the convergence of these oscillations towards the steady states. As in Barbosa-Filho and Taylor (2006), this is due to the positive slope of the distributive curve in those regions, meaning that there is a stabilizing profit-squeeze effect. On the other hand, since at the intermediate equilibrium E_2 the distributive curve has a negative slope and intersects the demand regime ‘from above’, this steady state has the features of a saddle point.

[FIGURE 3 ABOUT HERE]

Thus, Figure 3 can be seen as somewhat combining the two cases discussed in Barbosa-Filho and Taylor (2006) with regard to the US Economy (1948-2001): they generally find a stabilizing profit squeeze effect, but in the period 1955-70 the forced saving switch in the distributive curve determines an unstable equilibrium. It must be noted that in their paper Barbosa-Filho and Taylor (2006) have explicit time series thresholds for the change in slope. The explanation we provide is instead in terms of quantitative values of the two variables of interest. Also, while in their paper they found a demand regime steeper than the distributive curve, here the opposite is true. Thus, our intermediate equilibrium E_2 is saddle path stable. Of course, the slope of the demand regime matters, in what determines how many equilibria we will find in our system.¹⁴

Given the negative slope of the demand regime in this case, we find a trade-off between short run growth and redistribution toward wages, which is a traditional feature of a profit-led economy (Bhaduri and Marglin, 1990b, Naastepad, 2006):

¹⁴Note however that the case in which the locus $\dot{u} = 0$ is so steep that there is only one intermediate equilibrium requires an intercept of the curve higher than 1, and this is a case we would like to rule out from the analysis. Clearly, a sound empirical analysis will be crucial on this respect, but we proceed here assuming that the $\dot{u} = 0$ -isocline is sufficiently flat.

in order to stimulate the economy toward a higher capital utilization rate over the business cycle, a reduction in the wage share is needed.¹⁵

Remark: If the economy is fluctuating around the extreme equilibria and comes closer to the intermediate one, a small shock may suffice to move it into one of the basins of attraction of the steady states E_1 or E_2 so that the business cycle will then change its course and converge either to a depressed or a boom situation. Convergence into the depressed basin may for example be the situation experienced in Germany, while the US economy seems to fluctuate outside the basins of attraction of the investigated dynamics, as Figure 1 shows.

6.2 Wage-Led Demand Regime

A wage-led demand regime leads to some complications in the phase plots. Since the locus $\dot{u} = 0$ has a negative intercept on the ψ -axis, an equilibrium with a very low rate of both capital utilization and wage share could disappear, as it is shown in Figure 4. According to the slope of the demand regime curve, there is even the possibility that the only surviving steady state is an intermediate one.

[FIGURE 4 ABOUT HERE]

As far as the stability properties of the equilibria in the wage-led case, again we look at the Jacobian matrix of the system. Given the positive value of β_u , when the slope of the distributive curve is negative, the determinant will be positive, and the negative trace will ensure stability. Then, the clockwise oscillations around an intermediate equilibrium like E'_2 will converge eventually to that steady state. Conversely, when the slope of the distributive curve is positive, the corresponding equilibrium will be a saddle. It is worth to observe that the wage-led case appears counterfactual to the empirical situation found to characterize the US economy after World War II: not only the estimated long-phase cycle, but also all business cycles have a counterclockwise orientation.

6.3 Policy Implications

Note also the difference between the two different demand scenarios in terms of policy implications. In a profit-led economy, starting from an equilibrium like E_1 in Figure 3, fiscal or monetary policy aimed at stimulating the economy needs to be very strong to be effective. If this is the case, it will lead to a situation of the type E_3 , but will pay a price in terms of distributive conflict. Conversely, suppose that policy makers worry about the distributional implications of an economy fluctuating around an equilibrium like E_3 in Figure 3, and that they would prefer a situation

¹⁵Note finally with respect to Figure 3 that there is of course a fourth steady state at the origin of the phase space, that however cannot be reached from the positive orthant.

like E_2 . Since there is only one stable saddle path leading to the desired equilibrium among the infinite possible ones, a restriction will need to be tailored very closely in order to achieve the desired goal. In fact, if the restriction is too weak, it will fail in moving the dynamics away from the basin of attraction of E_3 ; if the restriction is too strong, it will produce the undesired effect of leading the economy into a stable (i.e. hard to fight) recession.

On the other hand, in a wage-led scenario, if the economy is in (or around) equilibrium, a further stimulus to the economic activity can have a very hard time in achieving the desired effects, because of the uniqueness of the stable saddle path ensuring convergence to an equilibrium like E_3 . Conversely, if policy makers acting in a wage-led economy deem it overheated at an equilibrium like E_3 they will find easy to sort the desired effects adopting restrictive policy measures, but they will pay the price of a lower wage share.

The analysis of these two different kinds of asymmetry in the effectiveness of demand policy deserves further attention, and is left for future research here, together with deeper considerations about the mechanisms behind the agents' expectations.

7 A Simple Extension: Endogenous Productivity Growth

We now relax the assumption of exogenous Harrod-neutral technical change we made so far, in order to account for the dynamic effects that arise when labor productivity growth is allowed to respond to variations in capacity utilization and the wage share. The purpose of this section is to show that the behavior of the distributive curve doesn't change qualitatively when labor productivity varies endogenously with capacity and the distributive shares, so that dynamics of the model studied so far carry over to this more general scenario. First, it is quite standard to assume that the growth rate of labor productivity responds positively to the rate of capacity utilization. The justification for such assumption, known in the literature as the Kaldor-Verdoorn relation, can be found in the presence of increasing returns that make labor more productive as the utilization of installed capacity increases. On the other hand, the traditional (and very well known to economists working in Marxian or Post-Keynesian frameworks) induced technical change mechanism according to which capitalist firms innovate in order to reduce production (in particular labor) costs points toward imposing a positive relationship between the wage share and labor productivity growth (Kennedy, 1964, Drandakis and Phelps 1965 are cornerstone papers on induced technical change. Recent developments can be found in Tavani, 2009, 2010). To keep things as simple as possible, we impose the following linear relationship:

$$n_x = \bar{n} + n_{xu}(u - \bar{u}) + n_{x\psi}(\psi - \bar{\psi}) \quad (8)$$

with $n_{xu} > 0, \beta_{\psi\psi} > n_{x\psi} > 0$. In Appendix C, we show that this new specification for labor productivity growth implies exactly the same dynamics for the distribu-

tive curve.¹⁶ Thus, the dynamic analysis presented above survives the endogenous productivity growth scenario.

8 An Empirical Comparison

Consider Figure 1 again. Inspecting the measured long phase cycle for the US economy in more detail shows that the wage share can increase again during phases of significant unemployment of both labor and capital (top right panel). This further loop is in fact of the Goodwin (1957) type, and is not possible in the many conventional studies of Goodwin’s growth cycle model. However, it is perfectly in line with what we have derived in Figure 3, and shows that the area around the stable depression is indeed relevant for one episode in the evolution of the distributional conflict in the US economy.

We argue on the basis of this analysis that our simple model is already rich enough to allow for persistent periods of booms, and for considerably long depressions that may need economic policy interventions aimed at avoiding economic breakdowns along some of the trajectories of the dynamics.

We also control for the dramatic increase in wage inequality, in particular the increase in wage earnings at the top of wage distribution which is deemed to be largely responsible for the stability of the wage share and profit share (Piketty and Saez, 2003, 2006).¹⁷ Figure 5 plots annual data for employment rate and several measures of the labor share, obtained subtracting the share of top 10%, top 5%, and top 1% wage earners respectively from the standard measure of the wage share. The data on the labor share are constructed using the Hodrick-Prescott (HP) filter on the relevant series in the database by Piketty and Saez (2003), while the data on the employment rate is constructed HP-filtering the complement to one of the annual average of monthly unemployment rate from the Bureau of Labor Statistics.¹⁸ As apparent from the plots, the counterclockwise cycles are by and large robust to different specifications of the labor share in national income, at least as far as this

¹⁶Assumption 5 in Section 4 already incorporates induced technical change effects into the evolution of the wage share. In view of this argument, imposing $\beta_{\psi\psi} > n_{x\psi} > 0$ doesn’t seem a very stringent requirement. It must be said however that, if such an assumption is violated, the stability properties of the steady states in the model change (Rezai, 2010 analyzes theoretically all the possible cases). Nevertheless, the dynamics observed in the empirical plots in Figures 1 and 6 should be enough to convince the skeptical reader who likes to engage in Jacobian analyses.

¹⁷We thank an anonymous referee for suggesting we make such comparisons.

¹⁸The careful reader will have observed that the this plot displays data at the annual frequency, differently from Figure 1, which plots quarterly data. The reason is that Piketty and Saez (2003) have annual data in their dataset, which is the one we used to construct our series for these plots. Obviously, having to work with annual data like in the Netherlands makes it cumbersome to estimate ‘long-phase cycles’ using the methodology we adopted in Figure 1. In other words, there is a potential ‘apples and oranges’ problem that arises from different data frequencies for different countries. This is the reason why simple HP filtered data are used in Figure 6. The same considerations apply to the plots in Figure 5.

sample period is considered.

[FIGURE 5 ABOUT HERE]

Finally, for the sake of cross-country comparisons, it is interesting to look at the behavior of the employment rate against the wage share in a wage-led economy. Several influential studies in the Post-Keynesian tradition (Naastepad, 2006, Naastepad and Storm, 2008) have found evidence that the effective demand regime in the Netherlands is wage-led. Figure 6 displays the (annual) HP-trend of the employment rate plotted against the (annual) HP-trend of the wage share in the Netherlands. The series for the employment rate is taken from the World Bank website, whereas the wage share series is taken from the Extended Penn World Table (3.0) compiled by Duncan Foley and Adalmir Marquetti. Although World Bank data for the Netherlands don't go further back than 1983, and therefore it is hard to see at all the occurrence of cycles in the plot, the movement from top left to bottom right is clear. Such movement is compatible with the kind of wage-led dynamics around the stable equilibrium illustrated in Figure 4, but not with the profit-led case in Figure 3. Thus, this empirical plot can be seen as confirming the previous results on the wage-led character of the Dutch demand regime.

[FIGURE 6 ABOUT HERE]

9 Conclusions

In this paper, we studied the effects of an estimated non-linearity in the demand pressure term of a wage-Phillips curve into an otherwise standard Post-Keynesian macro model of the dynamic interaction between the distributive curve and the demand regime of an economy. To carry our analysis, we borrowed both from Flaschel and Krolzig (2006) and Barbosa-Filho and Taylor (2006).

We showed that, because of the non-linearity in the distributive curve, multiple equilibria are a likely outcome of this model. In a profit-led economy, which the empirical analysis substantiates to be the relevant case for post-war United States, there are three equilibria corresponding to different values of the wage share and of the rate of capacity utilization: i) an economic boom with relatively high capacity utilization and relatively low wage share, ii) a recession with relatively low capacity utilization and relatively high wage share, and iii) an intermediate equilibrium. Booms and recessions are locally stable in the profit-led scenario of our model, and generate counterclockwise dynamics in capital utilization rate and the wage share. These features are all qualitatively consistent with the available evidence on the so-called distributive-demand cycles in post-war United States (Barbosa-Filho and Taylor, 2006). Given the slope of the demand regime, the stability of booms and recessions in our model depends on the stabilizing profit squeeze effect in the distributive curve, consistently to previous findings in the related literature. We

also showed that the counterclockwise cycles don't seem to be affected, at least for the sample period considered in Figure 5, by the sharp increase in wage inequality occurred in the past two decades and documented in recent literature (Picketty and Saez, 2003, 2006).

The intermediate equilibrium we find is instead saddle-path stable, due to a downward-sloping distributive curve which crosses the profit-led demand regime from above in the relevant region of the phase space. We argued that the instability associated with a saddle-path intermediate equilibrium poses challenges to policy makers both willing to fight recessions and to cool down an overheated economy. In the first case, any stimulus to aggregate demand with the purpose of bringing back the economy to its virtuous circle needs to be very strong to be effective. If the policy makers think instead that the economy looping around its boom phase is overheated, they may end up either finding a small contraction ineffective in what it may not be enough to get away from the basin of attraction of the boom they want to fight, or stuck in a stable recession if the contraction is too strong. These implications are only sketched in the model, because we didn't include any explicit policy variables in it. However, they seem to be interesting enough to be explored in a framework with monetary and fiscal instruments available to policy makers.

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A Derivation of Equation 2

Using the newly defined variables e, u , the two equations in (1) are then modified as follows:

$$\begin{aligned}\hat{w} - n_x &= \beta_w((1 - \bar{e}) - (1 - e)) + (1 - \kappa_w)\pi + \kappa_w\hat{p} = \beta_w(e - \bar{e}) + (1 - \kappa_w)\pi + \kappa_w\hat{p} \\ \hat{p} &= \beta_p((1 - \bar{u}) - (1 - u)) + \kappa_p(\hat{w} - n_x) + (1 - \kappa_p)\pi = \beta_p(u - \bar{u}) + (1 - \kappa_p)\pi + \kappa_p(\hat{w} - n_x)\end{aligned}$$

These equations capture the dynamics of (nominal) wage share growth and of the inflation rate. Subtracting π on both sides in both of the last equations, we obtain a two equation system in the variables $\hat{w} - n_x - \pi, \hat{p} - \pi$ which, given that the term π captures the agents' expectations about the evolution of prices in the economy, captures the expected evolution of the (real) wage share and the forecasting error in the inflation rate. This system can be rewritten in matrix form as:

$$\begin{pmatrix} 1 & -\kappa_w \\ -\kappa_p & 1 \end{pmatrix} \begin{pmatrix} \hat{w} - n_x - \pi \\ \hat{p} - \pi \end{pmatrix} = \begin{pmatrix} \beta_w(e - \bar{e}) \\ \beta_p(u - \bar{u}) \end{pmatrix} \quad (9)$$

Defining $\kappa \equiv (1 - \kappa_p\kappa_w)^{-1}$, the solution of (2) yields:

$$\begin{aligned}\hat{w} - n_x - \pi &= \kappa(\beta_w(e - \bar{e}) + \kappa_w\beta_p(u - \bar{u})) \\ \hat{p} - \pi &= \kappa(\kappa_p\beta_w(e - \bar{e}) + \beta_p(u - \bar{u}))\end{aligned}$$

Using the fact that $\hat{\psi} \equiv \hat{w} - \hat{p} - n_x$ in subtracting the second equation from the first one, we immediately have (2).

B Penalized Spline Estimates

For the estimation of the wage-Philips Curve a Penalized Spline approach has been used, see for instance Ruppert *et al.* (2003), such that the (penalized) log likelihood for normal errors can be written as

$$l[\theta] = -(y - C\theta)^2 / \sigma^2 - \lambda_1\theta^T D_1\theta - \dots - \lambda_m\theta^T D_m\theta \quad (10)$$

with the combined design matrix $C = (X Z)$ containing the fixed effect design matrix $X = (1 x_{1i} \dots x_{1i}^{q_1} x_{2i} \dots x_{2i}^{q_2} \dots x_{mi} \dots x_{mi}^{q_m})_{i=1, \dots, n}$ and the truncated spline basis $Z = (Z_1 \dots Z_m)$ with the j -th truncated spline basis defined by $Z_j = ((x_{ji} - \tau_{1,j})_+^{q_j} + \dots + (x_{ji} - \tau_{K,j})_+^{q_j})_{i=1, \dots, n}$ which are constructed with the truncation function $(x)_+^q := \max\{0, x\}^q$ and the K knots $\tau_{1,j}, \dots, \tau_{K,j}$ for the j -th dependent variable x_{j1}, \dots, x_{jn} . We have chosen different orders of the truncated polynomial, i.e. q_1, \dots, q_n , just to ensure that the structure for the unknown functions for some dependant variables have not been chosen to be too complex and for some variables we need to choose a higher order to visualize the first derivative of the estimated function in a smooth way. In the same way we could have choose different numbers of knots for each variable but to keep things simple we have used the same

number of knots for all variables. The main diagonal of the penalty matrix D_l contain a one if the index belong to the truncated spline basis Z_l and otherwise the element contain a zero, i.e. $D_l = (d_{ij})_{i=1, \dots, mK+q+1}^{j=1, \dots, mK+q+1}$ with $q = \sum_{i=1}^m q_i$ and $d_{ij} = 1_{\{i=j\}} 1_{\{i \in \{q+2+(l-1)K, \dots, q+1+lK\}\}}$. The smoothing parameters $\lambda_1, \dots, \lambda_m$ control the complexity of the structure for the unknown functions and should be chosen carefully. No penalization $\lambda_j = 0$ result in a too complex function with $q_j + K$ degrees of freedom and a highly penalized function ($\lambda_j \rightarrow \infty$) result in a function of order q_j . We are following the suggestion of Krivobokova and Kauermann (2004) to use the REML estimator for smoothing parameters to avoid misleading parameters because of misspecified autocorrelated errors.

For the Wage Philips Curve we are describing the wage inflation (y) by the variables price inflation ($x_1.$), the log of the wage share ($x_2.$),¹⁹ the employment rate ($x_3.$) and the price inflation climate ($x_4.$). In a first step we have set the order of the truncated splines to one, i.e. $q_1 = \dots = q_4 = 1$, to avoid misleading estimations because of too complex functional relationships. The resulting estimating show, that the price inflation and the wage share are linear related with the wage inflation. The employment rate and the price climate are in a non-linear way related with the wage inflation, and even more the functional form for employment rate uses more than three degrees of freedom, such that a higher polynomial order could be used. In our second step, setting $q_3 = 2$, the resulting estimation is nearly similar to our first one such that the same smoothing parameters λ_1 and λ_2 and the same shape of functions for the employment rate and the price climate are estimated.

Similarly, for the Price Philips Curve we are describing the price inflation (y) by the variables wage inflation ($x_1.$), the log of the wage share ($x_2.$), the utilization rate ($x_3.$) and the price inflation climate ($x_4.$). But in contrast to the Wage Philips Curve the functional shape of the Price Phillips Curve with respect to the utilization rate is not distinctively different from a linear curve (as was the functional shape of the WPC with respect to the employment rate), which gives the reason why we have omitted the visualization of the PPC estimation.

For the joint estimation of the employment rate ($y_1.$) and the log of the wage share ($y_2.$) we are distinguishing between long term and short term trends which is usually done when estimating business cycles. But instead of treating the deviations from the long term trend as errors we assume that the business cycles can be described by a functional form. Following Kauermann *et al.* (2008) we are assuming that the observations $y_t := (y_{1t}, y_{2t})^T$ can be described by a long term trend $c[t] := (c_1[t], c_2[t])^T$ and a short term trend $g[t] := (g_1[t], g_2[t])^T$, i.e. $y_t = c[t] + g[t] + \epsilon_t$ with normal residuals $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t})^T \sim \mathcal{N}(0, \sigma)$. The structure of the short term trend is even more specified by setting $g[t] := (\rho[t] \cos \phi[t], \rho[t] \sin \phi[t])^T$ with $\rho[t]$ representing the radius and $\phi[t]$ the angle around the center $c[t]$. For the estimation of the short term trend $g[t]$ polar coordinates are preferred because we assume that the speed and the direction of the trajectory for the detrended time series $y_t - c[t]$ are

¹⁹as in Blanchard and Katz (1999).

smooth functions over the time. The unknown functional forms of the radius $\rho[t]$, the angle $\phi[t]$ and the long term trends $c_1[t]$ and $c_2[t]$ are captured by a Penalized Spline approach such that the structure and the degree of complexity has to be estimated with the data at hand. But instead of estimating the short and long term functions simultaneously a hybrid version has been used because of numerical reasons. At the first stage, the long term trend is fitted by a given pair of long term penalty parameters. At the second stage, the resulting detrended observations $y_t - \hat{c}[t]$ are used to get the estimations for the short term functions using the REML estimation for choosing the optimal amount of smoothing for the radius and the angle. The optimal pair of long term smoothing parameters has been chosen by the Akaike Information Criterion, see for justification Kauermann et al. (2008).

C The Distributive Curve with Endogenous Productivity Growth

Making use of Equation (8) into (4) yields, after some algebra, the following equation in the growth rate of the real wage:

$$\hat{w} - \hat{p} = \bar{n} + \kappa((1 - \kappa_p)\beta_w[\sigma(u - \bar{u})] - (1 - \kappa_w - \kappa_n)\beta_p(u - \bar{u})) - (\beta_{\psi\psi} - n_{x\psi})(\psi - \bar{\psi}) \quad (11)$$

where $\kappa_n \equiv n_{xu}/\kappa$. This equation constitutes the distributive curve in the model with endogenous productivity growth letting $\hat{\psi} \equiv \hat{w} - \hat{p} - \bar{n}$.²⁰

Under $\beta_{\psi\psi} > n_{x\psi}$, that is under the assumption that the effect of the wage share on its own growth rate (which as discussed in Assumption 5 in Section 4 already incorporates induced technical change considerations) dominates the (pure) induced technical change effect, it is easy to see that the presence of endogenous labor productivity growth makes very little difference as far as the behavior of the distributive curve is concerned. Indeed, it is actually pretty easy to check that the presence of κ_n on the linear term appearing in (11) actually makes the inequalities that must be fulfilled for the distributive curve to have the shape under investigation less stringent.

D Tables and Figures

The data for the plots used in Figure 1 are taken from the Federal Reserve Bank of St. Louis (see <http://www.stls.frb.org/fred>). The data are quarterly, seasonally adjusted and are all from 1956:1 to 2004:4. Except for the unemployment rate U the log of the series are used.

²⁰Of course, imposing $n_{xu} = n_{x\psi} = 0$ gives back (5) as a special case.

Table 1: Data Used for the Plots in Figure 1

Variable	Transformation	Mnemonic	Description of the untransformed series
U	UNRATE/100	UNRATE	Unemployment Rate
w	$\log(\text{COMPNFB})$	COMPNFB	Nonfarm Business Sector: Compensation Per Hour, 1992=100
p	$\log(\text{GDPDEF})$	GDPDEF	Gross National Product: Implicit Price Deflator, 1992=100
$y - l$	$\log(\text{OPHNFB})$	OPHNFB	Nonfarm Business Sector: Output Per Hour of All Persons, 1992=100

The employment data for the Netherlands are taken from the WDI website: <http://data.worldbank.org/data-catalog>. Data for the wage share in the Netherlands are taken from EPWT 3.0, available at <http://homepage.newschool.edu/~foleyd/epwt/>

Table 2: Four Baseline Real Wage Adjustment Scenarios

	wage-led goods demand	profit-led goods demand
labor market-led real wage adjustment	adverse (divergent)	normal (convergent)
goods market-led real wage adjustment	normal (convergent)	adverse (divergent)

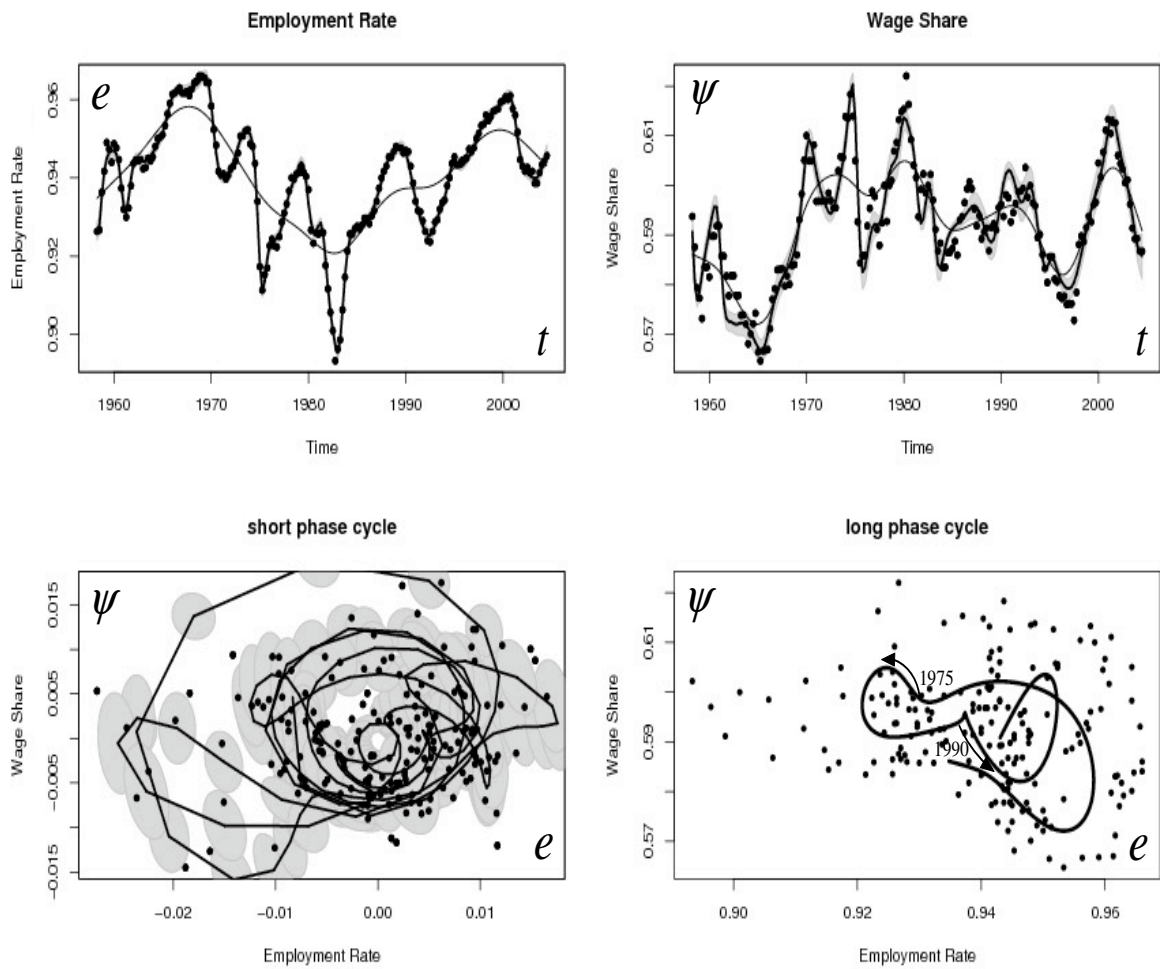


Figure 1: Long phase and business cycles in the U.S. economy after World War II (see the appendix for the details of the applied econometric technique)

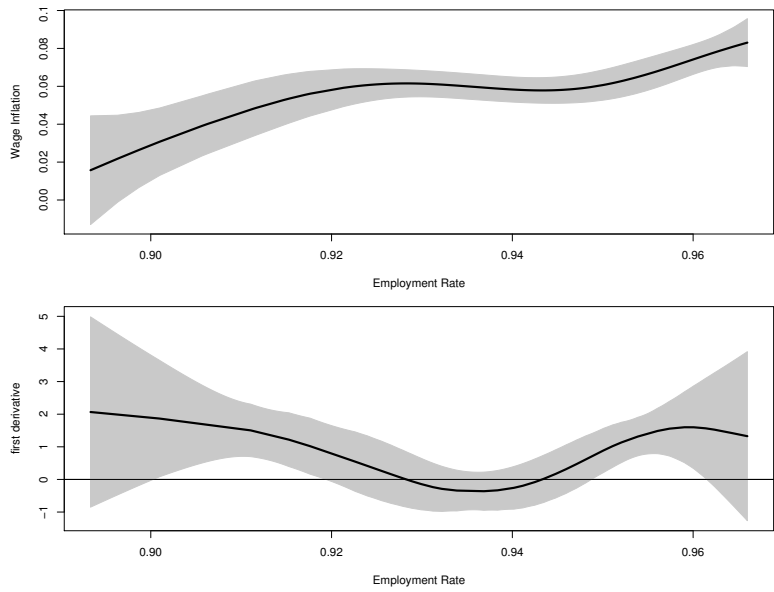


Figure 2: P-spline estimation of the wage-inflation/employment-rate schedule and its first derivative (with confidence intervals shown as grey areas)

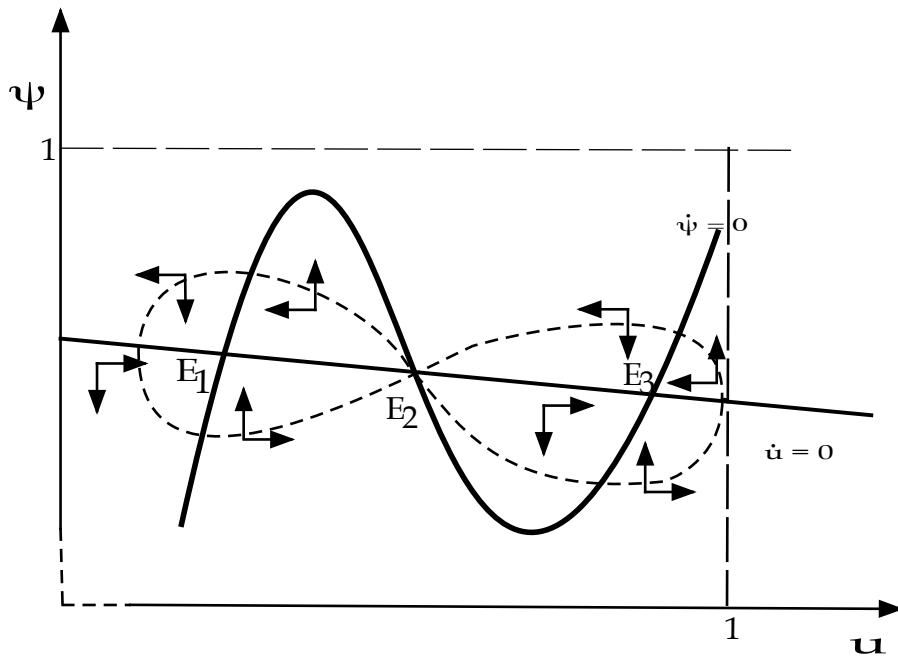


Figure 3: Phase Diagram for the Profit-Led Demand Regime

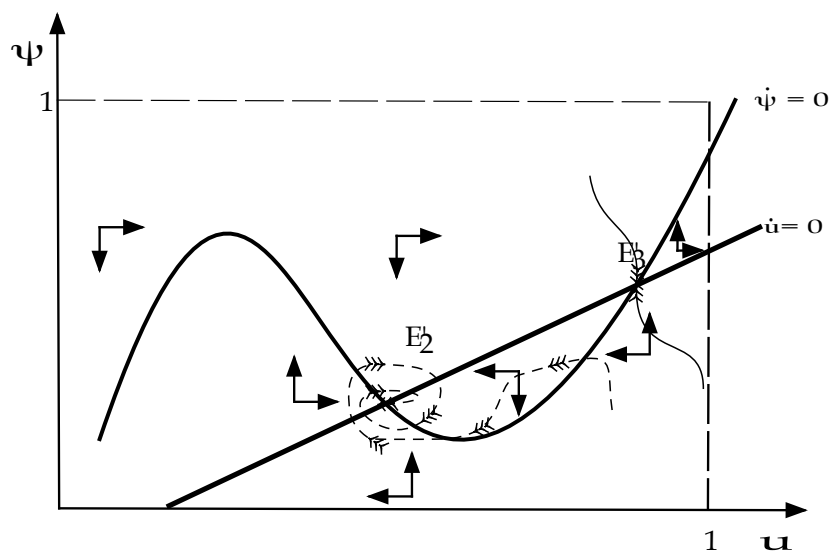


Figure 4: Phase Diagram for the Wage-Led Demand Regime

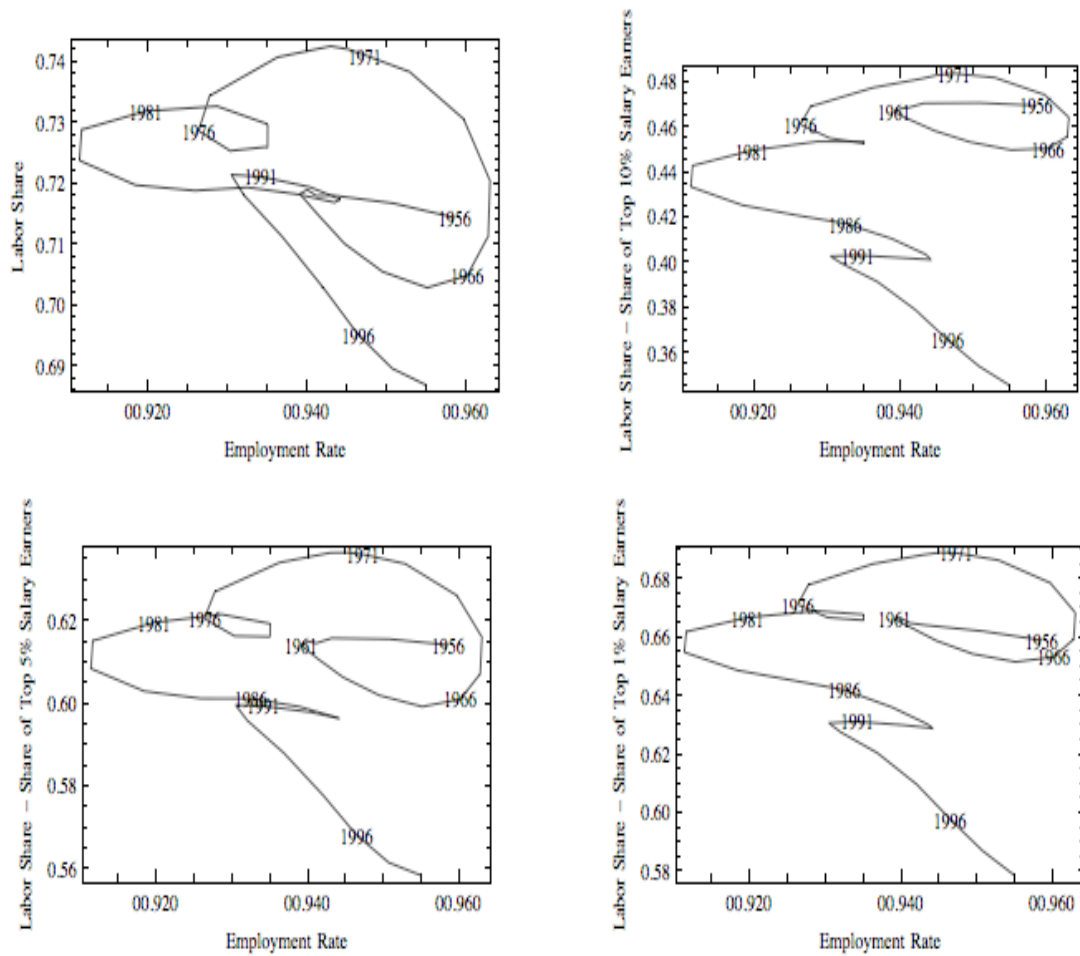


Figure 5: Cycles in employment rate and the wage share in the US (1956-1998). Sources: Piketty and Saez (2003) (Labor Share), annual average of BLS monthly data (Employment Rate).

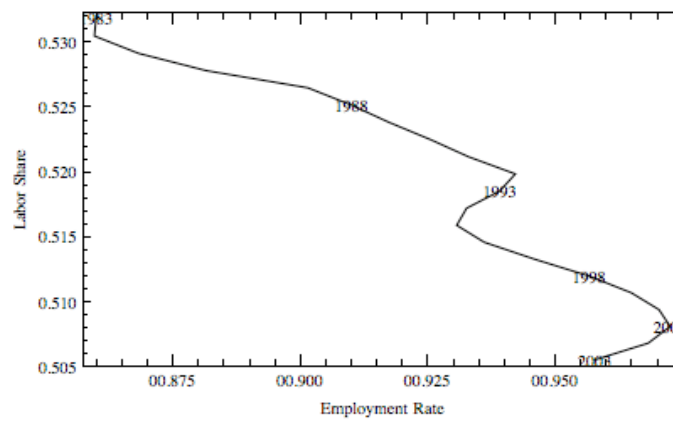


Figure 6: Employment Rate and Labor Share in the Netherlands (1983-2003). Sources: WDI (employment rate), Extended Penn World Table 3.0 (wage share).