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# Regime Dependence of the Fiscal Multiplier*


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# Regime Dependence of the Fiscal Multiplier*

Stefan Mittnik† and Willi Semmler‡

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**Abstract**

The reactions to the stimulus package of the Obama administration have been controversial. Some predict a multiplier effect in the order of 1.5, others argue that the multiplier will be less than 0.5. Such multiplier estimates typically stem from estimated linear vector autoregressions (VARs) or simulations generated generated by DSGE models. The latter, presuming intertemporally optimizing agents, largely negate any impact of the fiscal stimulus package. In this paper, we will argue that neither conventional VAR analysis nor DGSE simulations may be appropriate to evaluate demand effects arising from such a stimulus package. The reason is, as recent research suggests, that the timing of demand shocks matters. To assess the multiplier's variability, we adopt a regime–dependent VAR approach. As is shown in detail, our model specification is grounded on theoretical considerations. The empirical analysis presented here suggests that a regime–dependent VAR–specification is favored for U.S. output and employment data, and that the standard (one–regime) VAR methodology is inappropriate for analyzing multi–regime processes. Although we employ a nonlinear VAR framework, the chosen setup allows the use of largely familiar macroeconometric modeling tools. Estimating a two–regime VAR, we show that the fiscal multiplier varies with the state of the business cycle and the particular specifics of the measure taken. For the U.S. we find, for example, the fiscal expansion multiplier is much higher in a phase of a low economic activity than in periods of high activity.

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# 1 Introduction

The recent, close to $ 800 billion fiscal expenditure package by the Obama administration has led a number of economists to evaluate the multiplier effects of government–expenditure programs. This debate on the macroeconomic role of supply or demand factors is not new, and the earlier discussions by Kaldor, Solow, Tobin, and Okun[1] finds its echo in recent controversies. Today's discussion centers around the question of whether business cycles are driven by technology shocks or demand shocks, and what short– and long–run effects these shocks may generate. DSGE modelers stress the role of technology shocks in driving for business cycles and long–run growth, whereas the more Keynesian and New Keynesian literature emphasizes the role of demand shocks.[2]

From a more traditional Keynesian view Romer and Bernstein (2009) estimate a multiplier effect of roughly 1.5 to be effective by the year 2012. Studies using VAR methodology, frequently based on the work by Blanchard and Perotti (2002) where government spending is predetermined, obtain a considerably increase in output and employment as a result of government expenditure increase. Further studies along these lines are Perotti (2005), Gali et al. (2007) and Ramey (2009). Traditional Keynesian–oriented VAR studies typically estimate a fiscal policy multiplier of greater than one.[3]

An important response to the Keynesian–motivated macroeconomic VAR studies comes from economists relying on the DSGE methodology. Cogan et al. (2009) estimate a quickly rising multiplier reaching 1.03 in the first few quarters in 2009 which then gradually declines to 0.4. The decline, they argue, is due to (i) increasing interest rates, (ii) the anticipation of future tax increase by private agents, and, as a result of both, (iii) a crowding out of private consumption and investment spending.

The findings in Cogan et al. (2009), whose pessimistic view draws from a model by Smets and Wouters (2007), have received a number of responses in the literature. Among them are De Long (2009),[4] Ramey (2009), Christiano et al. (2009), Woodford (2010), and Uhlig (2009). One can also regard Gali et al. (2007) as an early position on this view. All of these responses argue that timing matters. If fiscal expenditure arrives at a time of low interest rates, liquidity–constrained households

[1]See Kaldor (1985), Solow (1997), Tobin (1993), and Okun (1962).

[2]Although we will focus on both aspects in this paper, we will mainly discuss the role of demand shocks. For a detailed discussion of the role of technology shocks see Appendices 1 and 2, where we evaluate the long– and short–run effects of technology shocks on unemployment.

[3]Other studies refer to war–time periods and defense spending rise to estimate the multiplier, see Barro (2009) and Ramey (2009). In another VAR study Mountfort and Uhlig (2005) report a multiplier of about 0.5.

[4]De Long argues that it is unreasonable to assume a rising interest rate and a fast reduction of liquidity by the Fed in the near future.

(and dominance of rule–of–thumb consumers) and no tax increase, then, the multiplier is considerable higher and usually seen to exceed one. The studies seem to imply that it is important to consider the particular state of the economy at the time fiscal expenditures become effective.

Most VAR–based studies consider a closed economy to assess the role of demand shocks, and associate demand shocks with shocks to general or specific spending positions, such as government consumption or investment demand (e.g., Blanchard and Perotti, 2002; Gali et al., 2007; and Ilzetzki et al., 2009). Ramey and Shapiro (1998) and Ramey (2009) take defense spending, arguing that the volatility of U.S. government spending is by and large driven by defense spending. In order to obtain real effects, most of this work follows the New Keynesian tradition and assumes, or demonstrates, stickiness of prices and wages.

The VAR studies follow the Sims tradition by estimating a linear VAR and conducting impulse–response analysis. In their seminal study Blanchard and Perotti (2002) estimate a VAR for three variables, a government–expenditure variable, GDP and net taxes (total tax revenues minus transfers). They demonstrate a positive effect of the government–expenditure multiplier and a negative one for tax increase. Further studies along these lines are Perotti (2002) 2005??????? and Mountford and Uhlig (2005). The latter see a favorable effect from tax cuts and less so from expenditure increases.

These earlier studies, indicating that output, employment, consumption, and real wages rise after a positive innovation in government spending, have recently been challenged. Eichenbaum et al. (2004) find that consumption and Cogan et al. (2009) that both consumption and investment fall after a government–spending shock.

Most research uses U.S. data. An international comparison of multiplier effects, which includes a number of developing countries, is provided in Ilzezki et al. (2009). They estimate bivariate VARs for GDP and government consumption and find a low or zero multiplier for very open economies and highly indebted countries. For the latter the multiplier levels off quickly.

In various studies derivations of the multiplier effect depend on particular restrictions on the behavior of households and on the timing when spending becomes effective. For example, Gali et al. (2004, 2007) consider Ricardian consumers (who can intertemporally smooth consumption) and "rule-of-thumb" consumers. They shift the emphasis toward variables used in New Keynesian models, namely, government purchases, GDP, employment, real interest rates, and, as an alternating fifth variable, private consumption, real wage or investment. They report a positive multiplier effect on GDP, employment and consumption, but less so on investment. The government–expenditure multiplier on output reaches 0.7 after one year and 1.3 after two years. Their study suggests that there is a considerable fraction of rule–of–thumb consumers, who are liquidity constrained, and that external spending shocks have positive effects.

However, they find no positive effects for investment. Investment is crowded out due to interest–rate effects.

Christiano et al. (2009) and Ramey (2009) emphasize that timing is an important factor for the effectiveness of government expenditure. Christiano et al. argue that government expenditure is normally effective with a delay, namely only when, at the time the expenditures come online, the interest rate is close to zero (zero–bound interest rate through the Taylor rule). Then, they argue, the multiplier can be large (reaching almost three) for the first six quarters and then declines. The decline will be less if, in the long–run, the Taylor rule is not enacted and the interest rate remains near zero. Yet, consumption is intertemporally smoothed and not, as in Gali et al. (2007), income and liquidity constrained. Other papers in the same vain where timing of the fiscal expenditure is important are Woodford (2010) and Uhlig (2009). In Woodford (2010), though only (!!!!!!!!!!!!) demonstrate in an intertemporal macro model, fiscal expenditure becomes effective when there is a persistence of the zero bound interest rate and there is a delay of price and wage adjustments. In this case the multiplier can be above one. In Uhlig (2009) the timing of the fiscal spending is important in the sense that the effect of fiscal spending should not be offset by an expected tax increase. Only in this case the multiplier is large.

Most of the work cited above, the quantification of the fiscal multiplier relies on—what we refer to as—a single–regime VAR with constant parameters which is assumed to hold over the whole sample and over all phases of the business cycle. Yet, in view of the arguments that the effectiveness of spending shocks depends crucially on their timing or, more specifically, on the particular state of the economy when expenditures come online, an assessment of the state dependency of multipliers is in order. We do so by estimating multi–regime VARs and, thus, adopt an empirical framework that is particularly suited for state–dependent multiplier analysis. As will be shown, the use of multi–regime models for empirical analysis is also well supported by theoretical arguments.

The remainder of the paper is organized as follows. Section 2 provides a more extensive review of recent macro studies and motivates the two–regime VAR adopted in our analysis. Section 3 introduces the multi–regime VAR approach and presents the empirical for the two–regime case. Section 4 evaluates the results and concludes the paper. The appendix sketches a theoretical model giving rise to two–regime decision–making process which motivates our empirical strategy.

# 2 Motivation of our Study

Before turning to our analysis, we start with a brief discussion of some of the theoretical and empirical backdrop of studies suggesting weak or zero multiplier estimates.

After that, we detail an alternative modeling approach that gives rise to a regime–change model and, thus, a state–dependent multiplier.

Recent papers by Smets–Wouters (2007), Cogan et al. (2009) and Christiano (2009) suggest a weak multiplier. The results of these studies, which rely on a DSGE model with various extensions, may be questioned as the standard DSGE model by and large does, in our view, not provide a convincing basis for analyzing fiscal–policy effects. A DSGE model typically assumes intertemporally optimizing agents, market clearing, New Keynesian sticky prices, and mostly full utilization of capacity.[5] Consumption and labor effort are choice variables and the variation of employment is a reflection of the consumption–leisure choice. It is well known that, in general, the model does not perform well with respect to the empirically observed variability in employment. The problem of excessive smoothness in labor effort and its failure of explaining the actual variation of employment is a widely criticized feature of the DSGE model.[6]

Another puzzle of the baseline DSGE model is that it views the macroeconomy as being mainly driven by supply shocks. It predicts a high positive correlation between technology shocks and employment, though empirical research indicates, at least at business–cycle frequency, a negative or almost zero correlation—a phenomenon often referred to as the technology puzzle (see King and Rebelo, 1999; Francis and Ramey, 2003, 2006; and Basu et al., 2006). The stationarity of the labor effort is another issue, showing some effects in VARs on the relation of supply shocks and labor effort for the level variable, but different ones for first differences.[7] A detailed evaluation of the short– and long–run employment effects from productivity shocks is given in Chen et al. (2008).

The excessive smoothness of the variation in employment, the incorrect correlation of the macro variables and the correlation of the technology shocks with employment essentially arise from an unrestricted consumption–leisure (employment) choice model where economic agents can, in an intertemporal setting, freely and smoothly trade off consumption, leisure and employment whereby markets are cleared.[8] In the context of the smooth and unconstrained intertemporal choice of the DSGE model there are three marginal conditions that ensure three equilibrium conditions to be established:

[5]Or at least a choice of capacity utilization by households, as in Smets–Wouters, and capital adjustment frictions, as in Christiano et al. (2004, 2009).

[6]Critical evaluations of this issue include Mankiw et al. (1985), Summers (1986), Rotemberg and Woodford (1996), and Schmidt–Grohe (2001). Recent models include search and matching on the labor market to improve the volatility of employment, see Blanchard and Gali (2008).

[7]See Christiano et al. (2004) versus Rabanal and Gali (2005) and Basu et al. (2006).

[8]An earlier test of this assumption was undertaken by Mankiw et al. (1986), 1985!!!!!!???? who state that their empirical analysis "casts serious doubts on the premise of most classical macroeconomic models that observe a labor supply that represents unconstrained choices given opportunities"(p. 241).

(i) the Euler equation that ensures an equality in the intertemporal trade off of consumption in consecutive periods,

(ii) the marginal rate of substitution equal to the real wage (the cost of trading off leisure against consumption is equal to the real wage), and

(iii) the optimal decision making of the firm ensures that the marginal product of labor is equal to the real wage.

The establishment of the above equalities, arising from the first–order conditions, often presumes[9] frictionless labor markets.[10] Moreover, the first–order conditions derived, are usually solved only after crude simplification, such as log–linearization or use of first– or second–order approximations. Yet, regarding the accuracy of the solutions, studies have shown that the size of the shocks matter significantly.[11]

Despite of the potential inaccuracies, linear VAR analysis is commonly conducted to show that technology shocks drive business cycles and employment. As mentioned above, there is, empirical evidence that technology shocks are not correctly measured, and purified (filtered ???????) technology shocks may actually be negatively correlated with hours worked (e.g., Francis and Ramey, 2003; Gali et al., 2005; and Basu et al., 2006).

Typical extensions of the DSGE model can be found in Smets and Wouters (2007) and Christiano et al. (2009). They employ a New Keynesian sticky price model with Calvo price and wage setting[12] and habit formation in consumption. The Smets–Wouters model, for example, has five decision variables for households (consumption, hours worked, bond issuing, investment, and capital utilization). Log–linearization of their model produces a linearized equation, motivating a standard VAR analysis with seven shocks.[13]

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[9]Recently, Gali, Gertler and Lopez–Salido (2003) have considered the welfare cost for the case when conditions (ii) does not hold, i.e., when the marginal rate of substitution differs from the real wage and, thus, from the marginal product of labor given by (iii).

[10]The absence of frictions is alse presumed for product and capital markets.

[11]The question to what extent solutions for the decision variables can be accurately obtained through linearizations is a crucial one. A study of the accuracy of solutions of first– or second–order approximation methods used to solve these nonlinear models is undertaken in Becker et al. (2008). Their main result is that the extent of the shocks matter: decision variables are only accurate close to the steady state and not so further away. The welfare function, however, has large errors even close to the steady state and they increase with the size of the shocks. For a recent criticism of the linearization technique, see also Brunnermeier and Sannikov (2009).

[12]For DSGE models with Keynesian features see Rotemberg and Woodford (1995, 1999), King and Wollman (1999), Gali (1999), Erceg, Henderson and Levin (2000), Woodford (2003), and Smets and Wouters (2007), who present a variety of models with monopolistic competition and sticky prices.

[13]These are: risk premium shocks, investment specific shocks, wage and price mark–up shocks, and two policy shocks, namely, exogenous–spending and monetary–policy shocks.

We will be mainly interested in the Smets–Wouters results on exogenous spending shocks, giving rise to the aforementioned weak multiplier effects. In order to calibrate their model, they apply a Bayesian estimation strategy with some parameters being set exogenously and some being estimated from U.S. time series data. From their model, they infer that spending shocks result in a short–term rise (though below 1) in output and employment, with both (in particular employment) dissipating quickly within a few quarters.

Some of the DSGE literature has moved away from assuming frictionless labor markets.[14] With the labor market sluggishly adjusting, output and employment effects of shocks remain small. In Smets and Wouters (2007) there is also a wedge driven between the marginal product of labor and the real wage, so that Condition (iii) would not immediately hold.

It seems to be called for to go beyond these approaches in order to study output, employment and consumption effects of demand shocks. In spite of recent attempts to include real frictions into the model and employing search and matching technology to explain the actual variation in employment and, thus, unemployment (see Blanchard and Gali, 2008), DSGE models suffer from the fact that they often ignore production and capital and that the relative variation in vacancies and labor effort does not fit the data well.[15]

One alternative could be to permit non-clearing labor market. There is a long tradition of studies on non-clearing labor markets of the Malinvaud–Benassy type,[16] and it seems worthwhile to extend this earlier work by including intertemporal decisions.[17] Here, we contrast the above DSGE market–clearing model with the tradition of a dynamic decision model of Keynesian type along the line of a model with non-

[14]Further frictions in the labor market are discussed in Hall (2005), Shimer (2005), Gali and Blanchard (2005), and Blanchard and Gali (2008).

[15]The search and matching technology has recently been included in DSGE models. These models can generate unemployment. Recent studies on modeling unemployment in a DSGE framework employing search and matching theory are Merz (1999), Ernst and Semmler (2009) and Walsh (2002) among others. However, search and matching models have difficulties in capturing the observed volatility of the ratio of vacancies and unemployment (see Shimer, 2005).

[16]See Malinvaud (1978, 1994) and Benassy (1984). A recently developed model of non-clearing markets of the French disequilibrium tradition, which resembles ours, can be found in Portier and Puch (2004). Uhlig (2004) also presumes that models with exogenous wage sequence at non-clearing market level are better suited to match actual labor market movements.

[17]In these studies with non–clearing labor markets, an explicit labor demand function is introduced from the firm's perspective of the decision problem. However, as in the early rationing models of the Malinvaud–Benassy type, the decision rule with regard to labor supply is often dropped in these models, because labor supply no longer appears in the welfare function of the household. Consequently, the moments of labor effort become purely demand-determined. Implicitly, labor supply is typically assumed to be exogenous and not determined within the model.

clearing markets.[18] This framework gives rise to the empirical modeling strategy adopted below, namely, the use multi–regime VARs.

We can summarize the proposed model by referring to two types of decision sequences. Details of this model are given in the appendix.[19] The basic dynamic mechanism works as follows. In the first stage of the decision sequence, there are intertemporal decisions of households creating a notional labor supply without constraints on the consumption–labor choice. However, the resulting labor supply does not necessarily become effective. We also presume a Calvo–type updating scheme for the partial adjustment of actual wages to the optimal wage level causing sticky wages. Given the wage sequence the firms adjust their notional demand for labor.

In the second stage, given the imbalance of supply and demand for labor, a decision rule for determining the actual employment level is required. When households face a constraints on the labor market, they need to re-optimize, to adjust their optimal consumption sequence to the labor market constraint they are facing. This second stage has also effects on the demand for goods of firms, since the re-adjusted household decisions are likely to feed back to the product market, where firms may now be faced with output constraints.

It is only the first stage where firms and households can make decisions without market constraints. This is the stage of decision making that DSGE models typically generalize, assuming a smooth and unconstrained consumption–leisure choice. "Unconstrained" means that decisions on consumption can be made without considering employment, as noted in Mankiw et al. (1986). In the second stage the consumption–leisure (labor effort) choice is constrained through the fact that there is excess supply of labor in the market and that income is reduced through unemployment.[20]

Formally, one can show that the typical DSGE model disregards the possibility of a two–stages decision sequence. Agents can simply intertemporally smooth consumption; and consumption and employment are only constrained by the capital stock and the technology shock.[21] The solution is obtained by linearization procedures, leading to a single–regime VAR analysis and, thus, a regime–independent fiscal multiplier.

In our case (see the appendix for an explicit derivation), current consumption is not constrained in the first stage, where there are no employment constraints to be

[18]Details are worked out in Gong and Semmler (2006) and Semmler and Gong (2009).

[19]However, unlike the other recent models of non-clearing labor markets, we view the decision rule of the labor effort derived from a dynamic decision problem as being a natural way to reflect the desired labor supply.

[20]As is the case in the previous models of non-market clearing of Malinvaud type.

[21]See Uhlig (1999) for a derivation of the linear consumption equation using log–linearization of the baseline DSGE model. Smets and Wouters (2004) have current employment impacting consumption positively, but the expected employment for the next time period will negatively impact consumption—which is quite a counterintuitive result. Yet, accordingly a one–regime VAR analysis is pursued. See also Uhlig (2009).

considered. For the constrained stage, however, consumption will not only depend on capital stock and technology but also on the actual employment. In terms of the Gali et al. (2007) model, with Ricardian and "rule-of-thumb" consumers, this just means that the fraction of the rule-of-thumb consumers becomes dominant.[22] The decision framework we propose appears to fit the data better than market–clearing models.[23]

There are mechanisms that may enforce the two stages described above. First, if there are actual income constraints due to employment, there is likely to be little intertemporal consumption smoothing and, thus, there are credit constraints.[24] Yet, credit constraints by some households will create income and credit constraints for others, characterisic for a regime of low employment and income. Given those externalities of spending, the second stage is, thus, characterized by liquidity and credit constraints.[25] Any relaxation of liquidity and credit constraints,[26] such as arising from additional government spending, is likely to have an amplifying effect on output and employment.

Second, regarding monetary policy, as Christiano et al. (2009) argue, if the interest rate will be low—and a zero bound interest rate is prevailing—this may make exogenous spending shocks, like government spending, more effective, and this the more so the lower the interest rate is kept. If interest rates are low, there is little crowding out of consumption and investment by government spending and the expenditure multiplier will be higher.[27] This could, however, not be so in the first stage of the decision sequence, where monetary institutions are free to respond to the level of macro activity, and endogenous monetary policy causing a rise of interest rates.[28]

In order to evaluate responses to demand shocks, the issue of timing and, thus, the decision sequence the economic agents find themselves in is important. It is with respect to th different stages that impulse responses are expected to be different. These arguments motivate our empirical two–regime analysis. It allows for two phases

[22] Our model does, indeed, resemble resemble that of Gali et al. (2007), who allow only for a fraction of the consumers adaptive re-optimizing and the other fraction following some rule of thumb. We can think of these two rules of being prevalent in the two stages of the business cycle, as discussed above. It also resembles a recent model by Christiano et al. (2009), where there are two stages: one with zero bound and one with high interest rates.

[23] See Semmler and Gong (2009) for details.

[24] Or, in terms of Woodford's (2010) analysis, significant credit spreads because of default premia.

[25] This is the argument of the financial accelerator, referring to the ease of collateralized borrowing in booms and the tightness in recessions (see also Ernst et al., 2009).

[26] This is usually also accompanied by reducing default premia and credit spread. See also Woodford (2010).

[27] This is the period for which Christiano et al. (2009) estimate a government–expenditure multiplier that is considerably greater than one. Ramey (2009) also refers to the issue of timing when government expenditure comes online and sees the multiplier also varying strongly in different stages.

[28] For details of two such stages with respect to interest rates, see Christiano et al. (2009), and with respect to credit spreads, see Woodford (2010) and Ernst et al. (2009).

in a business cycle: one with less external constraints and one with more severe constraints. Both stages result in different spending effects. Given the possibility of such different stages, it is clear that a linearization of the non–linear model will lead to distortions and potentially render regime–invariant response analysis meaningless. We, therefore, propose the use of multi–regime VAR analysis, as it allows us to learn about regime–specific responses dynamics.

# 3 Empirical Analysis

Having motivated the need for a multi–regime VAR approach we now turn to the empirical analysis. We begin with a brief discussion of two candidate models, namely, Markov–switching and multi–regime VARs. We will use the latter to study the regime dependence of the effects of demand shocks for the U.S. economy. We present estimation results and, in the last subsection, describe the findings obtained from a response analysis based on a two–regime VAR.

## 3.1 Methodolgy

Conventional VAR models are not capable of capturing regime dependencies. They approximate time series in terms of linear dynamic models, which have the property that (impulse and cumulative) responses to shocks are independent of an economy's state at the time a shock. Moreover, VAR response profiles are invariant with respect to the sign and size of a shock; that is, responses to positive and negative shocks are mirror images of each other, and the response to shocks of different sizes are simply scaled versions of the unit–shock response.

To capture state dependencies and asymmetries of shock responses, a nonlinear time series model needs to be specified. To do so, some nonlinear functional form of the type $y_t = f(\varepsilon_t, y_{t-1}, y_{t-2}, \ldots, y_{t-p}; \theta)$ or a linear relationship with state–dependent parameters, such as $y_t = c(s_t) + \sum_{i=1}^{p} A_i(s_t)y_{t-i} + \varepsilon_t$, where $s_t$ represents the state at time $t$, could be specified. In the analysis below, we essentially follow the latter and entertain the mildest form of generalizing a linear, constant–parameter VAR by adopting a piecewise linear VAR. Two model classes have been proposed for this strategy: (i) Markov switching autoregressions, put forth by Hamilton (1989); and (ii) multi–regime (or threshold) autoregressions, proposed by Tong (1978, 1983).

A multivariate Markov switching autoregression (MSVAR) with $M$ regimes is given by

$$y_t = c(s_t) + \sum_{i=1}^{p} A_i(s_t)y_{t-i} + \varepsilon_t, \quad \varepsilon_t \mid s_t \sim NID\,(0, \Sigma(s_t)) \tag{3.1}$$

where the model parameters shift according to the state at time $t, s_t$; i.e.

$$\theta(s_t) = \begin{cases} \theta_1, & \text{if} \quad s_t = 1 \\ \theta_2, & \text{if} \quad s_t = 2 \\ \vdots & \\ \theta_s, & \text{if} \quad s_t = M \end{cases} \tag{3.2}$$

with vector $\theta$ capturing the parameters in $c, A_i$ and $\Sigma$ in (3.1). It is assumed that the states reflect some unobservable regime and that the regime–generating process is governed by a finite–dimensional Markov chain with transition probabilities

$$p_{ij} = Pr\left(s_{t+1} = j \mid s_t = i\right), \quad \sum_{j=1}^{M} p_{ij} = 1, \quad i, j \in \{1, \ldots, M\}. \tag{3.3}$$

The conditional transition probabilities, $p_{ij}$, give rise to the state transition matrix

$$P = \begin{bmatrix} p_{11} & \cdots & p_{1M} \\ \vdots & & \\ p_{M1} & \cdots & p_{MM} \end{bmatrix}.$$

For a detailed discussion of MSVAR models see Krolzig (1997).

A crucial characteristic of MSVAR models is that the states are unobservable and, hence, do not necessarily have an obvious interpretation. Also, a given observation cannot directly be associated with any particular regime. Only conditional probabilistic assignments are possible via statistical inference based on past information.

For our interests, i.e., conducting business–cycle dependent response analysis, states can be straightforwardly defined in terms of output growth and can be observed. Therefore, regime–specific VAR analysis can be conducted. We refer to these models as multi–regime vector autoregression (MRVAR) models. They correspond to the class of threshold autoregression models of Tong (1978, 1983) or, in a vector setting, to multivariate threshold autoregressions (Tsay, 1998).[29] In contrast to MSVARs or standard multivariate threshold autoregressions, in our approach we assume that we can, based on some observable variable, define upfront a meaningful set of regimes and that they are not the implication of some estimation procedure. This is preferable in situations where we are, for example, interested policy analysis designed for a particular state (regime) of the economy.

[29] We prefer the term multi–regime VAR, because the regime–specific analysis of the process is our focus.

A general MRVAR specification is given by (cf. Tsay, 1998)

$$y_t = c_i + \sum_{j=1}^{p_i} A_{ij} y_{t-j} + \varepsilon_{it}, \text{ if } \tau_{i-1} < r_{t-d} \le \tau_i, \ \varepsilon_{it} \sim NID(0, \Sigma_i), \ i = 1, \dots, M, \quad (3.4)$$

where $r_{t-d}$ is the value of the threshold variable observed at time $t - d$. The regimes are defined by the (prespecified) threshold levels $-\infty = \tau_0 < \tau_1 < \cdots < \tau_M = \infty$. In a business–cycle context, we could think of a two–regime VAR with the threshold variable being the output–growth rate with the threshold level being, for example, zero or, as done below, an average growth rate.

Apart from the more straightforward regime interpretation, MRVAR models are also more appealing than MSVARs as far as estimation is concerned. Rather than EM–estimation, as is practice with MSVAR models, MRVARs with given threshold levels resemble conventional VARs and can be estimated regime by regime, using standard least–squares estimation. Extensions to cointegrated MRVAR processes have been proposed by Balke and Fomby (1997), where the error–correction term defines the threshold variable. This permits asymmetric adjustments to an equilibrium.[30]

Response analysis for linear VAR models is straightforward. Point estimates and asymptotic distributions of shock response can be derived analytically from the estimated VAR parameters (cf. Mittnik and Zadrozny, 1993). In nonlinear settings, this is, in general, not possible and one has to resort to Monte Carlo simulations. Following Koop et al. (1996), so–called generalized impuls responses (GIRs), which depend on the overall state, $z_t$, type of shock, $v_t$, and the response horizon, $h$, are defined by

$$GIR_h(z_t, v_t) = \mathrm{E}\left(y_{t+h} \mid z_t, u_t + v_t\right) - \mathrm{E}\left(y_{t+h} \mid z_t, u_t\right). \quad (3.5)$$

Here, the overall state, $z_t$, reflects all relevant information for $y_t$. For an MSVAR process, $z_t$ comprises information about the past realizations of $y_t$ and the states; for an MRVAR process with known threshold levels, only information about past realizations $y_{t-1}, \cdots, y_{t-p_{\max}}$, with $p_{max} = \max(p_1, \dots, p_M)$, is required.

To understand the differences in the dynamic characteristics between the different regimes, regime–specific response analysis as in Ehrmann et al. (2003) is helpful. Regime–specific responses of MRVAR models assume that the process remains within a given regime during the next $h$ periods. This is particularly reasonable when regimes tend to persist or when we are interested in short–term analysis.

[30]Cointegrated MSVAR models are discussed in Krolzig et al. (2002).

## 3.2 Estimation

For our bivariate analysis, we use quarterly data on U.S. output and employment over the period 1954:1 to 2008:4.[31] The logarithms of both series exhibit non–stationarity over the period considered and can be classified as I(1). Johansen cointegration tests on the bivariate series $y_t = 400(\Delta \log GDP_t, \Delta \log EMP_t)'$ suggest the absence of (linear) cointegration for the period under investigation.

We estimate a standard VAR and an MRVAR model and use the AIC for model selection. For MRVAR model (3.4), the AIC is given by

$$AIC\,(M, p_1, \ldots, P_M) = \sum_{j=1}^{M} \left[ T_j \ln |\hat{\Sigma}_j| + 2n \left( np_j + \frac{n+3}{2} \right) \right], \tag{3.6}$$

where $M$ is the number of regimes; $p_j$ is the autoregressive order of Regime $j$; $T_j$ reflects the number of observations associated with Regime $j$; $\hat{\Sigma}_j$ is the estimated residual covariance matrix for Regime $j$; and $n$ denotes the number of variables in vector $y_t$. Formulation (3.6) differs from that in Chan et al. (2004) in that we account for possible heterogeneity in the constant terms, $c_j$, and residual covariance, $\Sigma_j$, across regimes.[32]

Based on the AIC, a VAR of order $p = 5$ is suggested. Specifying a two–regime MRVAR with the threshold, $\tau$, set to the sample mean of the output–growth rate, given by 3.18, we assign observations associated with below–mean (above–mean) growth rates to Regime 1 (Regime 2). Then, the AIC suggests an autoregressive order of three for Regime 1 and order two for Regime 2. The AIC favors the two–regime MRVAR with AIC $(M = 2, p_1 = 3, p_2 = 2) = 483.5$ (and regime sample sizes $T_1 = 113$ and $T_2 = 104$) over a standard VAR with AIC$(M = 1, p = 5) = 617.6$.

The clear selection in favor of the MRVAR over the VAR results from the fact that it has only a few additional free parameters (30 vs. 25), but considerably smaller estimates for the residual covariance matrices. The residual variance for output growth (see Table 1) is 8.88 in the fifth–order VAR case. For the MRVAR model, this reduces to 5.76 in the third–order specification for Regime 1 and to 4.00 for the second–order specification for Regime 2. The residual variance for unemployment growth is 2.57 for the VAR, and reduces to 1.57 in Regime 2, whereas it slightly increases to 2.74 in Regime 1.

[31] For employment we use the seasonally–adjusted (end–of–quarter) monthly data on total nonform employment (Series Id. CES0000000001) from the Bureau of Lahor Statistics; and for output we use seasonally–adjusted real GDP (Series Id. GDPC96) from the U.S. Department of Commerce.

[32] When employing (3.6) to discriminate between an MRVAR and a standard VAR specification (i.e., a one–regime MRVAR), we need to include the $n$ parameters in the intercept vector, $c$, and the $n(n+1)/2$ parameters in the residual covariance matrix for an equivalent parameter count.

Table 1: VAR and MRVAR Estimation Results

| | VAR | | MRVAR | | | |
|---|---|---|---|---|---|---|
| | | | Regime 1 | | Regime 2 | |
| $c'$ | 2.881 | 0.297 | 0.509 | -0.675 | 5.220 | 1.610 |
| $A_1$ | -0.164 | 1.225 | -0.083 | 0.577 | -0.099 | 0.804 |
| | 0.031 | 0.738 | 0.049 | 0.473 | 0.070 | 0.487 |
| $A_2$ | 0.028 | -0.408 | 0.125 | -0.308 | 0.040 | -0.533 |
| | 0.082 | -0.173 | 0.149 | -0.107 | 0.017 | -0.101 |
| $A_3$ | 0.009 | -0.156 | 0.061 | -0.202 | | |
| | 0.055 | 0.058 | 0.052 | 0.203 | | |
| $A_4$ | 0.096 | -0.432 | | | | |
| | 0.056 | -0.337 | | | | |
| $A_5$ | -0.041 | 0.043 | | | | |
| | -0.022 | 0.204 | | | | |
| $\Sigma$ | 8.881 | 3.073 | 5.762 | 2.573 | 4.000 | 1.300 |
| | 3.073 | 2.575 | 2.573 | 2.741 | 1.300 | 1.565 |
| Steady State | 3.152 | 1.828 | 0.470 | -1.294 | 5.815 | 3.448 |
| $T$ | 217 | | 113 | | 104 | |
| # Parameters | 25 | | 30 | | | |
| AIC | 617.7 | | 476.5 | | | |

The estimation results are shown in Table 1. The intercept estimates for the two MRVAR regimes reflect the definition of the regimes; they are low for the below–average–growth regime (Regime 1) and high for the other. Concerning the autoregressive parameter estimates, it turns out that the autoregressive coefficient estimates for the VAR and MRVAR–regimes have the same sign, whenever there have lag orders in common. The differences are only in their magnitudes. The steady state implied by the VAR parameters is 3.152 for output growth and 1.828 for employment growth. The MRVAR estimates imply a regime–specific steady states of 0.470 (5.815) for output growth in the low–growth (high–growth) regime and -1.294 (3.448) for employment growth. The eigenvalues of the companion matrices associated with the autoregressive parameters, shown in Table 2, indicate that the dynamics in the below–average regime are more persistent than in the above–average regime. The fact that the VAR dynamics display an even higher persistence could be the result of model misspecification and is in line with Perron (1989), who suggests that ignoring the presence of structural break tends to increase persistence.

Table 2: Eigenvalues of Estimated VAR and MRVAR Models

| VAR | | MRVAR | | | |
|---|---|---|---|---|---|
| | | Regime 1 | | Regime 2 | |
| Eigenvalues | Size | Eigenvalues | Size | Eigenvalues | Size |
| 0.6959±0.2817$i$ | 0.7507 | 0.6747±0.1292$i$ | 0.6870 | 0.3734±0.2452$i$ | 0.4467 |
| -0.4994±0.5417$i$ | 0.7367 | -0.1890±0.5993$i$ | 0.6283 | -0.2701 | 0.2701 |
| -0.6156 | 0.6156 | -0.2903±0.1949$i$ | 0.3497 | -0.0888 | 0.0888 |
| 0.3279±0.5092$i$ | 0.6056 | | | | |
| -0.1066±0.5412$i$ | 0.5516 | | | | |
| 0.3534 | 0.3534 | | | | |

## 3.3 Response Analysis

To assess the effects of linear versus nonlinear model specification, we first look at the estimates of the cumulative responses of the VAR model and the regime–specific responses of the MRVAR model. Subsequently, we analyze the systems overall rather than regime–specific responses. To derive structural responses, we assume that shocks to output simultaneously affect output and employment, whereas output reacts with one period delay to employment shocks.

The results for the VAR model (Figure 1) suggests that a one–percent shock to output growth (left panel) has a positive cumulative growth effect of about 1.6% after one year and settles after about three years at 1.2%. Employment growth responds to the same shock in a similar fashion, peaking at 1.3% after six quarters and settling at about 1.2% after three years. The regime–specific MRVAR responses differ from the VAR responses. As long as output growths at a below–average rate (Figure 2), a one–percent output shock implies long–term effects of 1.3% on output growth and 1.8% on employment growth. The same shock applied in a state of high–growth (Figure 3) has a cumulative effect of only about 1.1% on output and merely 0.7% on employment growth.

The cumulative VAR response of output to a labor–supply shock peaks at 1.6% after three quarters and decreases to about 0.5% after 20 quarters, and the response to employment itself has its maximum at 2.5% in quarter three and settles at about 2.2% (right panel of Figure 1). The corresponding regime–specific MRVAR are quite different. In the below–average regime cumulative output growth jumps to 0.6% in the first quarter and reduces gradually to about 0.2% thereafter, and the cumulative response of employment increases for about two years and settles there at 2.4%. Finally, in the above–average regime cumulative output jumps to 0.8% and settles at 0.4%, whereas employment growth stabilizes at 1.7% after three quarters.

The regime–specific MRVAR response dynamics indicate that the short– and long–

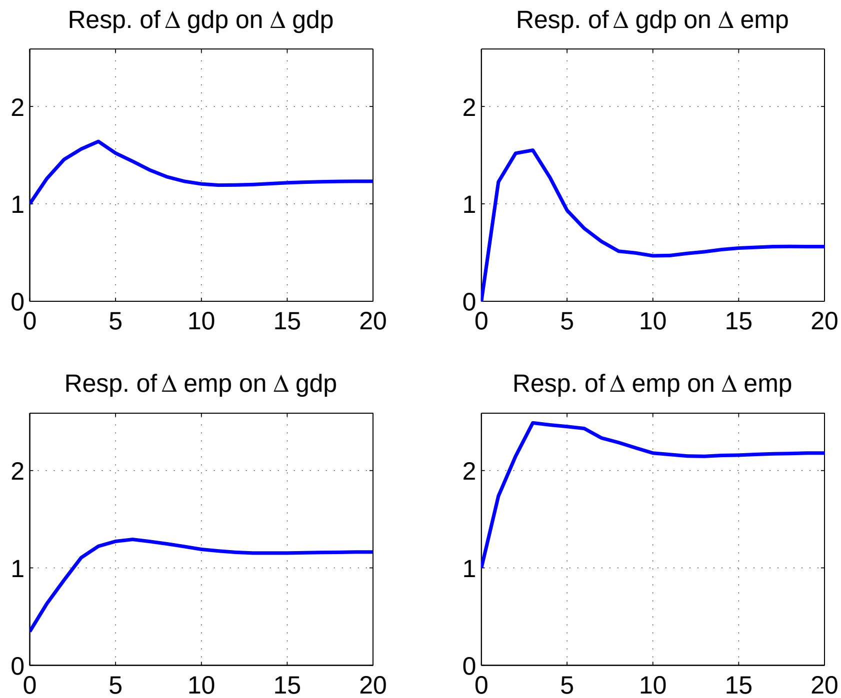


Figure 1: Cumulative Responses from a Linear VAR Model

term impacts of a shock may vary substantially according to the regime that governs the economy. This holds especially for employment responses to either shock, suggesting that the effectiveness of employment policies varies over the business cycle. On the other hand, the regime–specific responses of output display less variation across regimes.

The regime–specific response estimates help to understand the dynamic properties of the regimes. For two reason, they are, however, of limited use when trying to assess the overall impact of a shock. First, the process is not expected to stay within a given regime for extended period of time; it will rather switch between regimes. Secondly, by looking at the within–regime dynamics, we solely focus on the regime–specific autoregressive parameters and ignore the level effects induced by the differences in the regime intercepts. They will induce additional variation in the dynamics as the process switches between regimes.

To investigate the system's overall reaction to shocks, we simulate generalized cumulative response functions to unit–impulse shocks. This requires us to also specify the state at which a shock applies. Rather than defining some artificial state, we select two states observed in the sample. One is given by the very last observations in our sample, i.e., $y_{2008:2}, y_{2008:3}, y_{2008:4}$, where the economy was in a rather depressed state

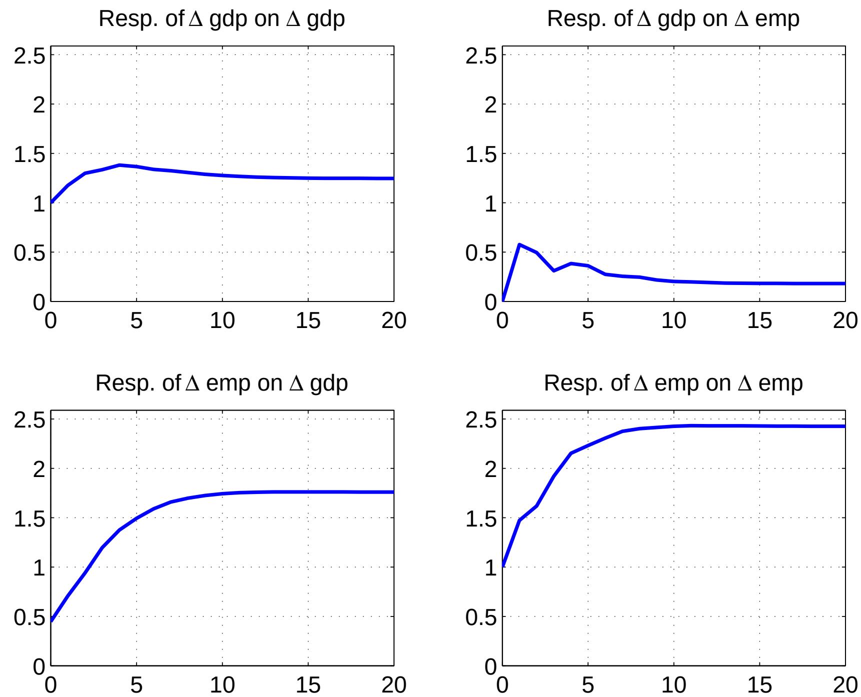


Figure 2: Cumulative Responses in the Low–growth MRVAR–Regime

with

$$y'_{2008:2} = [2.79, -1.33],\ \ y'_{2008:3} = [-0.51, -1.82],\ \ y'_{2008:4} = [-3.88, -4.88].$$

The responses and one–standard deviation confidence bands are shown in Figure 4.[33] A second set of cumulative responses (shown in Figure 4) was simulated for a strongly growing economy by specifying observations $y_{1982:4}, y_{1983:1}, y_{1983:2}$ as initial state, with

$$y'_{1982:4} = [7.82, 5.40],\ \ y'_{1983:1} = [8.10, 4.27],\ \ y'_{1983:2} = [7.75, 5.18].$$

The point estimates of the cumulative responses strongly suggest that the impact of a shock depends on the state of the economy—especially with respect to employment. A one–percent output shock in the contraction period 2008:4 causes employment to increase by about 1.8% in two years. The same shock applied in the expansionary period 1983:2 results in an increase of only about 0.8%. After two years, employment growth remains in both cases at these respective levels. Output itself

[33] The generalized cumulative responses were simulated based on 2,500 replications, which were repeated 100 times to approximate the standard errors of the responses.

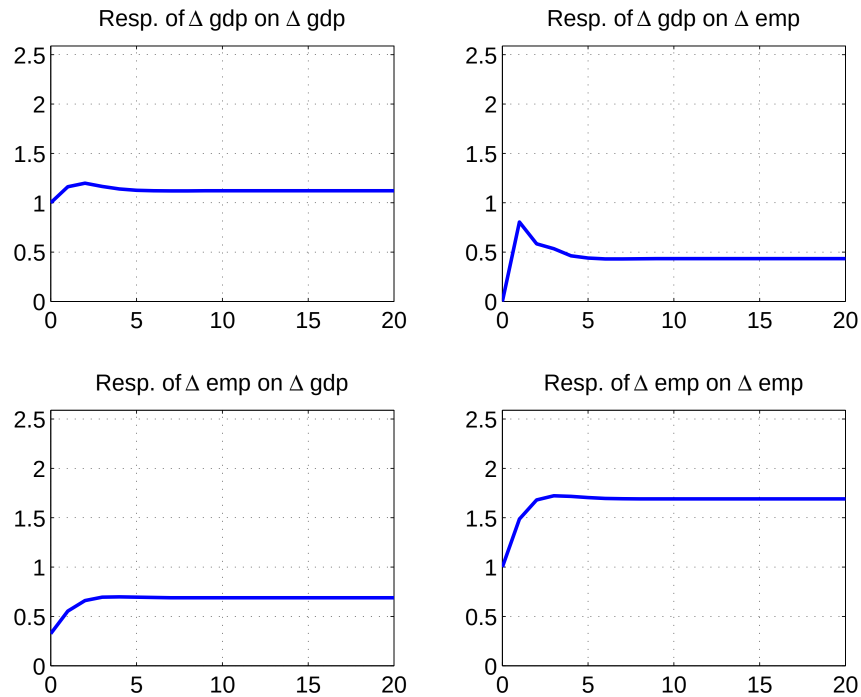


Figure 3: Cumulative Responses in the High–growth MRVAR–Regime

reacts initially more strongly in the recession than in the boom period (1.65% vs. 1.25% after one year). Both responses settle at about 1.6% and 1.4%, respectively, after five years.

A positive employment shock in 2008:4 causes an initial output response of 0.6% and settles at 0.4% thereafter. Applying the shock in 1983:2 produces a slightly stronger initial response, but it vanishes within five years. Employment itself exhibits a stronger state–dependence than output. In the recession state 2008:4, it responds by an increases of 2.4% vs. 1.5% for the expansion state in 1983:2.

The generalized cumulative response functions are pretty much compatible with the within–regime responses. Given their dependence on the initial state, the former are not necessarily convex combinations of the latter.

The response analysis based on nonlinear MRVARs indicates that the consequences of shocks may vary considerably depending on the state of the economy. The state dependence is especially strong for employment. But also the output multiplier shows a different profiles. It reacts more quickly in a recession state with being about one third larger after one year than during an expansion. Though there seems no difference in the longer run. For policy making, this implies that the timing of policy actions may be a crucial aspect in the design of an optimal policy.

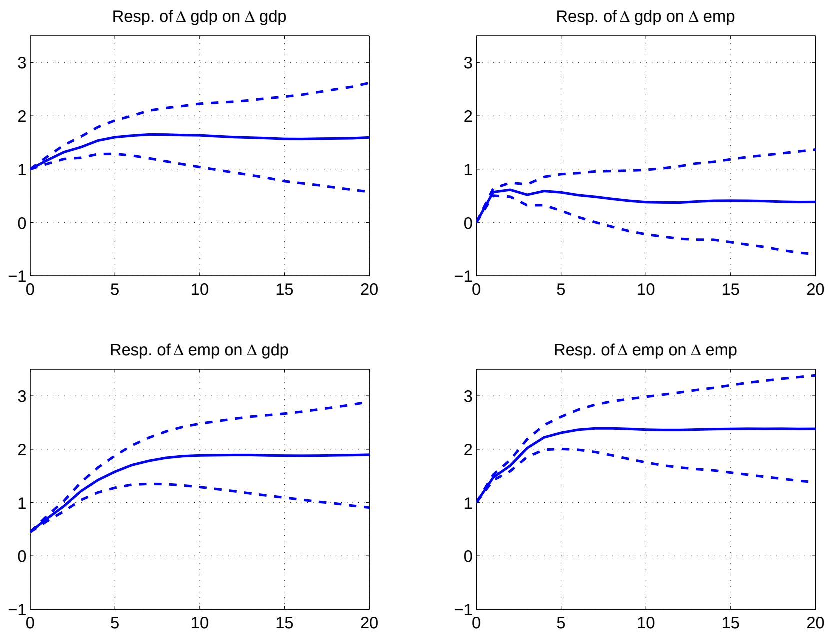


Figure 4: Cumulative MRVAR Responses Originating 2008:4

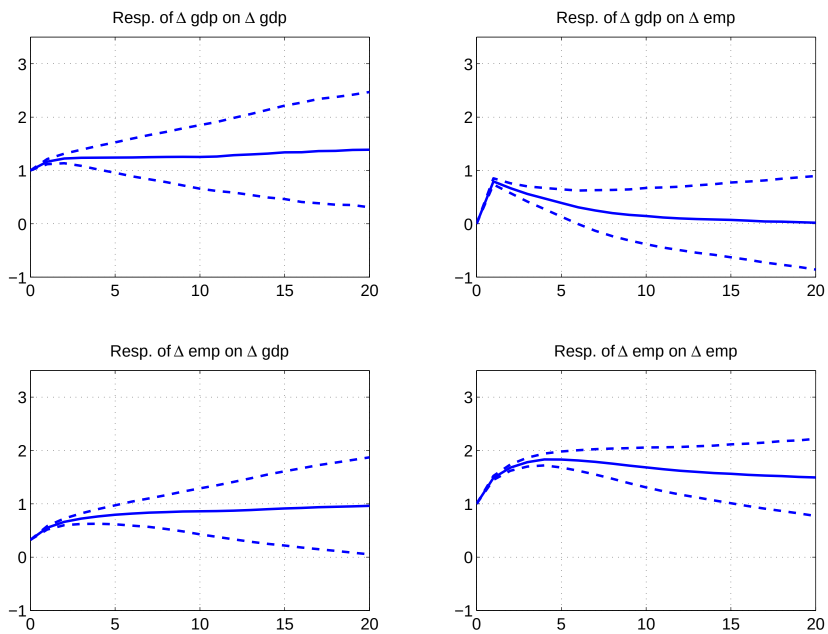


Figure 5: Cumulative MRVAR Responses Originating 1983:2

# 4 Conclusions

The results of our empirical study point to a similar conclusions as some other recent papers:[34] The timing of (policy) shocks matters. We have argued that response analysis based on standard, linear VAR methodology can easily mislead as it ignores the fact that process dynamics are likely to be state dependent. Although our MRVAR specification is nonlinear, piecewise or regime–specific linearity allows us to make use of tools that are well–established in linear time series econometrics. Employing multi–regime VARs one can investigate whether or not impulse responses are affected by the state of the business cycle. If this is the case, proper timing of government expenditure programs couldincrease their effectiveness.

In our analysis, we estimate a two–regime model and define the regimes in terms of the level of output growth, namely a below–average and an above–average regime. The regime or, for that matter, the stage of the business cycle affects the response to output shocks, which we interpret here as demand or government–spending shocks. As compared to the standard, one–regime VAR framework, which, by design, restricts responses to be independent of the prevailing growth rate, our MRVAR analysis indicates that the impact of shocks does, in some cases, vary substantially across regimes. A positive demand shock in a below–average growth regime produces a multiplier effect on output that is initially about one third higher than in a high–growth regime. However, the multipliers settle more or less at the same level after five years. During a low–growth regime, a unit–shock to output produces employment growth that is about two and a half times larger than in a period of high growth. These findings are reconfirmed when specifying particular historically observed states when deriving the MRVAR responses. A regime of very low and even negative growth rates as observed in 2008:4 lets output and employment responses to a demand shock rise faster and higher compared to the high–growth regime in 1983:2. This points to a regime dependence of demand effects.

Our MRVAR approach is motivated on theoretical grounds. As compared to DSGE models, where the agents find themselves always in the same regime, we sketched a model with regime changes, where agents can find themselves in two different regimes and follow different decision sequences. The theoretical framework establishes regimes that are compatible with those in our econometric MRVAR specification. In a regime with no severe labor market, credit and liquidity constraints agents can intertemporally choose consumption and employment. In a second regime, agents are constrained in the labor market and face credit and liquidity constraints. In addition, in the latter regime firms may face both output and credit constraints. Given positive demand shocks in the low–growth state and provided that interest

[34] Such as Ramey (2009) and Christiano et al. (2009)

rates will stay low, constraints for households as well firms will be reduced and the multiplier is predicted to be larger. This is also what we find in our MRVAR analysis.

As to the Obama administration's spending plans, some of the criticism raised is based on empirical multiplier estimates from standard, linear VAR models. Others use a (single–regime) DSGE model with unconstrained optimizing behavior of agents and suggest—resorting again to standard VAR analysis—a very weak multiplier. To directly investigate the effects of government spending, a government expenditure variable could be added as in Blanchard and Perotti 2002) and Gali et al. (2007).

Further qualifying remarks are in order. It has been stated that the government spending multiplier has recently become smaller.[35] Various studies use data from different subperiods, ranging from the early 1950s up to now. For example, Blanchard and Perotti (2002) use the subperiod 1960:1–1997:4; Gali et al. (2007) 1954:1–1998:4; Smets and Wouters (2007), on which the study of Cogan et al. (2009) is based, use 1966:1–2004:4; and Erceg et al. (2008) look at the period 1983:1–2003:4.

With a period covering 1954:1–2008:4, we take a rather long post World War II sample. We do this for two reasons. First, we are interested in an overall assessment rather than the characteristics of specific subperiods. Second, the need for estimating different models for different subperiods may just be the consequence of working with inadequate linear specifications.[36]

Although the analysis with the MRVAR approach adopted here is at an early stage and extended studies—which specify higher–dimensional MRVAR models, to explicitly consider different types of government expenditure, and which look at different subperiods—could be undertaken, we think the empirical results from the methodology presented here promises new and deeper insights in business–cycle and policy modeling.

[35] Ilzetzki et al. (2009), for example, find that pre-1980 periods show a multiplier of roughly 1.5 and the later period roughly 0.5.

[36] Various factors may affect the demand shocks across countries. Ilzetzki et al. (2009), for example, demonstrate, yet using only government consumption as expenditure shocks, for small subperiods that the expenditure multiplier may depend on the exchange rate regime (floating rates show lower multipliers), the degree of openness and the degree of financial fragility (foreign debt). These are, so the authors, important factors reducing the multiplier.

# A A Model of Non-Cleared Markets: Regimes or Linearization?

There are now many models where a non-clearing labor market could occur, see Malinvaud (1978, 1994) and Benassy (1984). Yet, the latter models of non-clearing markets of the French disequilibrium tradition are mostly static and not embedded in a dynamic decision framework. We will derive a dynamic regime change model and discuss the issue of linearization.

## A.1 Decision Sequence Under Unconstrained Choice

### A.1.1 Production and Household Behavior

The model we present here[37] starts with an unconstrained choice in a intertemporal macromodel. The state equation for the capital stock takes the form:

$$K_{t+1} = (1-\delta)K_t + I_t - Q_t \tag{A.1}$$

where $K_t$, $I_t$ and $Q_t$ are respectively the capital stock, investment and adjustment cost, all in real terms; $\delta$ is the depreciation rate. Here we allow

$$I_t = A_t K_t^{1-\alpha}(N_t X_t)^{\alpha} - C_t$$

with $C_t$ to be consumption; $N_t$ per capita working hours; $A_t$ the temporary shock in technology; and $X_t$ is the permanent shock (including both population and productivity growth) growth rate that follows a growth rate $\gamma$. The model is non-stationary due to $X_t$. To transform the model into a stationary version we need to detrend the variables. For this, we divide both sides of equation (A.1) by $X_t$:

$$k_{t+1} = \frac{1}{1+\gamma}\left[(1-\delta)k_t + i_t\right]$$

Above, we have defined $k_t$, $i_t$ to be the detrended variables for $K_t$, $C_t$ and $Q_t$: $k_t \equiv \frac{K_t}{X_t}$, $c_t \equiv \frac{C_t}{X_t}$ . In particular,

$$i_t = A_t k_t^{1-\alpha}(n_t \bar{N}/0.3)^{\alpha} - c_t$$

where $c_t \equiv \frac{C_t}{X_t}$ and $n_t \equiv \frac{0.3N_t}{\overline{N}}$ with $\overline{N}$ denoting the sample mean of $N_t$.

Let us assume a simple households‘ welfare function such as

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \theta \log(1-n_t)\right].$$

[37]Details of the subsequent model are given in Gong and Semmler (2006, chapter 8).

### A.1.2 Labor Market Features

We shall follow the standard assumptions on households and firms. There are three types of commodities in our model and therefore we have three types of prices, the output price $p_t$, the wage rate $w_t$ and the rental rate of capital stock $r_t$. One of them should serve as a numeraire, which we assume to be the output. This implies that the output price $p_t$ always equals 1 and thus the wage $w_t$ and the rental rate of capital stock $r_t$ are all measured in terms of the physical units of output.

### A.1.3 Wage Setting

At the beginning of period, $t$, the household should first choose the optimal wage $w_t^*$ by building on the following dynamic decision problem:

$$\max_{w_t^*, \{c_{t+i}\}_{i=0}^{\infty}} E_t \left[ \sum_{i=0}^{\infty} (\xi\beta)^i U(c_{t+i}, n_{t+i}) \right] \tag{A.2}$$

subject to

$$k_{t+i+1} = \frac{1}{1+\gamma} \left[ (1-\delta)k_{t+i} + f(k_{t+i}, n_{t+i}, A_{t+i}) - c_{t+i} \right]; \tag{A.3}$$

$$w_t^* = f_n(k_{t+i}, n_{t+i,t+i}). \tag{A.4}$$

Above, $U(\cdot)$ is the welfare function which depends on consumption $c_{t+i}$ and employment $n_{t+i}$; $f(\cdot) \equiv A_{t+i}k_{t+i}^{1-\alpha}(n_{t+i}\bar{N}/0.3)^{\alpha}$ is the production function in a stationary form, which is implied by (4 ); $f_n(\cdot)$ in (5) is the marginal product of labor derived from $f(\cdot)$; $\beta$ is the discount factor; $\xi$ is the probability that the wage rate $w_t^*$ will remain in period $t+1$;[38] and finally, $E_t$ is the expectation operator. Note that here we have assumed that the households know the production function $f(\cdot)$ and therefore know the firm's demand curve for labor as expressed in (A.4).

Solving this dynamic decision problem as expressed in (A.2) - (A.4) will allow us to obtain $w_t^*$ which depends on the expectation on the technology sequence $\{A_{t+i}\}_{i=0}^{\infty}$.[39] Next in the spirit of Calvo (1983) we presume that the existence of adjustment costs entailed by the economy as a whole, a probability $\xi$, that a fraction of wages will be sticky and the other fraction $(1-\xi)$ will be adjusted. This implies a partial adjustment process, such as

$$w_t = \xi w_{t-1} + (1-\xi)w_t^*, \tag{A.5}$$

where $w_t$ is the actual wage rate at period $t$.

[38] and therefore, $\xi^i$ is the probability that $w_t^*$ will remain in period $t+i$.

[39] For more details of this solution, see Semmler and Gong (2009).

### A.1.4 The Decision of Households

Given the wage rate as expressed in (A.5), the household will decide about its preferences for output demand and factor supply $\left\{c^d_{t+i}, i^d_{t+i}, n^s_{t+i}, k^s_{t+i+1}\right\}_{i=0}^{\infty}$. Note that here we have used the superscripts $d$ and $s$ to refer to the agent's desired, or notional, demand and supply. The decision problem for the household to derive its demand and supply can be formulated as

$$\max_{\left\{c^d_{t+i}, n^s_{t+i}\right\}_{i=0}^{\infty}} E_t \left[\sum_{i=0}^{\infty} \beta^i U(c^d_{t+i}, n^s_{t+i})\right] \tag{A.6}$$

subject to

$$k^s_{t+i+1} = (1-\delta)k^s_{t+i} + f(k^s_{t+i}, n^s_{t+i}, A_{t+i}) - c^d_{t+i}. \tag{A.7}$$

For the given technology sequence $\{A_{t+i}\}_{i=0}^{\infty}$, equation (A.6) and (A.7) form a standard intertemporal decision problem. The solution to this problem can be written as:

$$c^d_{t+i} = G_c(k^s_{t+i}, A_{t+i}); \tag{A.8}$$

$$n^s_{t+i} = G_n(k^s_{t+i}, A_{t+i}). \tag{A.9}$$

Note that consumption demand in linearized form can be written as:[40]

$$c^d_t = G_{11}A_t + G_{12}k_t + g_1 \tag{A.10}$$

with $G_{ij}$ and $g_1$ coefficients.

We shall remark that although the solution appears to be a sequence $\left\{c^d_{t+i}, n^s_{t+i}\right\}_{i=0}^{\infty}$ only $(c^d_t, n^s_t)$ along with $(i^d_t, k^s_t)$, where $i^d_t = f(k^s_t, n^s_t, A_t) - c^d_t$ and $k^s_t = k_t$, are actually carried into the market by the household for exchange due to the switch to a new decision sequence, see below.

### A.1.5 The Decision of Firms

Since the firms rent capital and hire labor on a period-by-period basis, the problem faced by firms at period $t$ is to choose the current input demands and output supplies $(n^d_t, k^d_t, y^s_t)$ that maximizes the current profit:

$$\max \; y^s_t - r_t k^d_t - w_t n^d_t$$

subject to

$$y^s_t = f(A_t, k^d_t, n^d_t) \tag{A.11}$$

[40] For details of the derivation, see Gong and Semmler (2006).

The solution to the above problem will allow us to obtain the demand for inputs:

$$k_t^d = K(w_t, r_t, A_t) \tag{A.12}$$
$$n_t^d = N(w_t, r_t, A_t) \tag{A.13}$$

while the supply of output is given by (A.11).

### A.1.6 Transactions in Factor Market

Next we shall consider the transactions in our three markets: the capital, labor and product markets. Let us first consider the two factor markets. Given the wage rate $w_t$ as expressed in (A.5), the rental rate of capital $r_t$ is adjustable to clear the capital market so that we have

$$k_t = k_t^s = k_t^d \tag{A.14}$$

This equilibrium condition allows us to obtain $r_t$.[41]

## A.2 Decision Sequence Under Constrained Choice

Given that markets are not cleared, as shown above, a new decision sequence needs to take place.

### A.2.1 Labor Market constraints

In particular, given $r_t$ as determined by the equilibrium condition (A.14) and $w_t$ as expressed in (A.5), there is no reason to believe that the labor market can be cleared. In this case, we shall have to specify what rule applies regarding the realization of actual employment.

> Employment Rules: When a nonclearing of the labor market occurs either of the following rules might be applied:
>
> $$n_t = \min(n_t^d, n_t^s), \tag{A.15}$$
> $$n_t = \omega n_t^d + (1 - \omega) n_t^s. \tag{A.16}$$
>
> where $\omega \in (0, 1)$.

[41] Note that here the capital market is cleared. A model with non-clearing capital market is presented in Ernst and Semmler (2009).

The first rule is the famous short-side rule when non-clearing of the market occurs. It has been widely used in the literature on disequilibrium analysis (see, for instance, Benassy, 1984, among others). Yet we want to suggest a second rule, which we find more convincing.

This second rule might be called the compromise rule. This rule indicates that when non-clearing of the labor market occurs both firms and workers have to compromise. If there is excess supply, firms will employ more labor than what they wish to employ.[42] On the other hand, when there is excess demand, workers will have to offer more effort than they wish to offer.[43] Such a mutual compromises may be due to institutional structures and moral standards of the society. Such a rule that seems to hold for many other countries was already discussed early in the economic literature, see Meyers (1968) and Solow (1979). [44]

### A.2.2 Product Market Constraints

After the transactions in these two factor markets have been carried out, the firm will engage in its production activity. The result is the output supply, which, instead of (A.11), is now given by

$$y_t^s = f(k_t, n_t, A_t).$$

Then the transaction needs to be carried out with respect to $y_t^s$.

### A.2.3 Constraints for Households' Choice

It is important to note that when the labor market is not cleared, the previous consumption plan as expressed by (A.8) becomes invalid due to the improper budget constraint, which further points the improper transition law of capital (A.7), for deriving the plan. Therefore, households will be required to design a new consumption plan, which should be derived from the following dynamic decision problem:

$$\max_{(c_t^d)} U(c_t^d, n_t) + E_t \left[ \sum_{i=1}^{\infty} \beta^i U(c_{t+i}^d, n_{t+i}^s) \right]$$

---

[42] This could also be realized by firms by demanding the same (or less) hours per worker but employing more workers than being optimal. This case corresponds to what is discussed in the literature as labor hoarding where firms hesitate to fire workers during a recession because it may be hard to find new workers in the next upswing, see Burnside et al. (1993).

[43] This could be achieved by employing the same number of workers but each worker supplying more hours (varying shift length and overtime work); for a more formal treatment of this point, see Burnside et al. (1993).

[44] See also Ernst et al. (2006) where a test of this rule is performed for many European countries.

subject to

$$\begin{aligned} k^s_{t+1} &= (1-\delta)k_t + f(k_t, n_t, A_t) - c^d_t \\ k^s_{t+i+1} &= (1-\delta)k^s_{t+i} + f(k^s_{t+i}, n^s_{t+i}, A_{t+i}) - c^d_{t+i} \\ i &= 1, 2, ... \end{aligned}$$

Note that in this program the only decision variable is $c^d_t$ and the data includes not only $A_t$ and $k_t$ but also $n_t$, which is given by either (A.15) or (A.16). We can write the solution in terms of the following equation:[45]

$$c^d_t = G_{c2}(k_t, A_t, n_t) \tag{A.17}$$

Given this adjusted consumption plan, the product market is cleared if the household demand $f(k_t, n_t, A_t) - c^d_t$ for investment. Therefore, $c^d_t$ in (A.17) should also be the realized consumption.[46] Yet, overall, the consumption does not only depend on the capital stock and technology, as in the DSGE model, but also on actual employment. Moreover, if there are actual income constraints due to employment, there are likely to be credit constraints and little intertemporal consumption smoothing. Yet, credit constraints by some households will create income and credit constraints for others, generating a regime of low employment and income.[47] Here then the linearized form of the consumption demand is:

$$c^d_t = G_{21}A_t + G_{22}k_t + G_{23}n_t + g_2 \tag{A.18}$$

where $G_{ij}$ and $g_2$ are coefficients.

### A.2.4 Non-cleared Product Market and Demand for Labor

The demand for labor will depend on what regime in the product market is realized:

$$n^d_t = \begin{cases} (0.3/\overline{N})(Ey_t/A_t)^{1/\alpha} k_t^{(\alpha-1)/\alpha}, & \text{if } Ey_t < (\alpha A_t/w_t)^{\alpha/(1-\alpha)} k_t A_t \\ (\alpha A_t/w_t)^{1/(1-\alpha)} k_t (0.3/\overline{N}), & \text{if } Ey_t \geq (\alpha A_t/w_t)^{\alpha/(1-\alpha)} k_t A_t \end{cases}$$

So there is potentially also a constraint of the demand for labor (from the side of firms), when firms are constrained on the product market. It is then the interaction of

[45]See Gong and Semmler (2006, chapter 8) for details.

[46]Note that this comes close to the scenario used by Gali et al. (2007) where the "rule-of-thumb" consumers dominate.

[47]As shown in Gong and Semmler (2006) this non-market clearing model generate data series that is much more close to the variation of observed time series in comparison with the standard intertemporal model that presumes market clearing.

the households' constrained choice of consumption goods and the non-cleared product market that is likely to exacerbate the downward spiral. The result is similar to the one from the model by Gali et al (2007), with Ricardian and rule of thumb consumers. We can interpret our second period as one where the fraction of "rule-of-thumb" consumers dominate.

There are two mechanisms that complement our points made above. First, we have not included financial or credit market conditions that may affect private demand in the second stage of the decision sequence. This is usually a regime where credit is constrained or obtained only at a risk premium.[48] Here then, in this second stage, the agents are also more liquidity constrained and any additional government expenditure will relax credit and liquidity constraints.

Second, as concerning monetary policy, we have not included additional liquidity provision and low interest rate changes. Our second stage of decision sequence coincides with what Christiano et al. (2009) describe as regime of a zero bound interest rates, whereas our first stage is more akin to their stage of endogenous rise of interest rates. If interest rates rise endogenously, as in the DSGE models, the fiscal expenditure effects would be mitigated. Thus, overall, because of these two reason a stronger fiscal multiplier could be expected in the second stage of the decision sequence.

## A.3 The DSGE Model and Linearization

Note that from a typical DSGE model, with unconstraint consumption-leisure choice (and no constraints on the product market) one would typically get a consumption demand function that is independent of employment. A typical log-linearized version would look like[49]

$$-c_t = \lambda_t \tag{A.19}$$

$$\lambda_t = \eta_{\lambda k} k_{t-1} + \eta_{\lambda z} z_t \tag{A.20}$$

$$k_t = \eta_{kk} k_{t-1} + \eta_{kkz} z_t \tag{A.21}$$

$$n_t = y_t + \lambda_t \tag{A.22}$$

hereby $\eta_{ij}$ are the elasticities, and $c_t$, $\lambda_t$, $k_t$ $z_t$, $y_t$ are log-linear deviations from consumption, Lagrangian multiplier, capital stock, technology shocks and output. Thus consumption, and labor demand (equal to labor supply) is in the unconstrained DSGE model only driven by capital stock and technology shocks.[50]

[48] See Ernst and Semmler (2009) on the role of credit market constraints.

[49] For the derivation of the subsequent results, see Uhlig (1999).

[50] A further slightly more complicated model in terms of preferences is log-linearized in Uhlig (2004), where then log-linear deviations of consumptin on and leisure are linearily depending on $\lambda_t$.

Though in the Smets and Wouter (2007) model the log-linearized equation for consumption demand contains employment, but it results from an unconstrained choice of employment:

$$c_t = c_1 c_{t-1} + (1 - c_1) E_1 c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (r_t - E_t \pi_{t+1} + \varepsilon_t)$$

with $c_i$, $c_t, E_1 c_{t+1}$ log-linearized consumption and expected consumption respectively, $l_t, E_t l_{t+1}$, employment and expected employment, and $r_t - E_t \pi_{t+1}$, the real interest rate. Note this represents a hybrid consumption equation (not only a purely forward looking consumption equation) which is however, not empirically established. Whenever those forward looking equations have been empirically estimated, the results are very mixed.[51] Other linearized equations are derived in Smets and Wouters (2007) in Section 2 of their paper. In the DSGE as well in the Smets and Wouters models there is only a one regime decision making process. Although the issue of the accuracy of the solutions of decision variables (and the value function) has not satisfactorily been resolved,[52] VAR exercises are then undertaken.

# References


[1] Balke N.S. and T.B. Fomby (1997): "Threshold Cointegration," International Economic Review, 38, 627-645.

[2] Barro, R. (2009): "Government Spending is no Free Lunch," http://online.wsj.com/article/SB123258618204604599.html

[3] Basu, S., J. G. Fernald and M.S. Kimball (2006): "Are Technology Improvements Contractionary?" American Economic Review, 9, 1384-1418.

[4] Becker S., L. Grüne and W. Semmler (2007): "Comparing Accuracy of Second–Order Approximation and Dynamic Programming," Computational Economics, 30, 65-91.

[5] Benassy, J.-P. (1984): The Economics of Market Disequilibrium, New York: Academic Press.

[6] Blanchard, O. and J. Gali (2006): "Real Wage Rigidities and the New Keynesian Model," Journal of Money, Credit and Banking, 39, 35-65.

[7] Blanchard, O. and J. Gali (2008): "Labor Markets and Monetary Policy: A New-Keynesian Model with Unemployment," unpublished manuscript, MIT.


[51] See for example the evaluation of forward looking Phillips Curve by Gorden et al (2003).

[52] For a detailed study of those issues, see Becker et al. (2007).

[8] Blanchard, O. and R. Perotti (2002): "An Empirical Characterization of the Dynamic Effects Of Changes in Government Spending and Taxes on Output," Quaterly Journal of Economics, 117, 1329-1368.

[9] Burnside, A. C., M. S. Eichenbaum and S. T. Rebelo (1996): "Labor Hoarding and the Business Cycle,"Journal of Political Economy, 101, 245-273.

[10] Brunnermeier, M. and Sannikov, Y. (2009): "A Macroeconomic Model with a Financial Sector," Department of Economics, Princeton University.

[11] Calvo, G. A. (1983): "Staggered Contracts in a Utility Maximization Framework," Journal of Monetary Economics, 12, 383-398.

[12] Chan, W.-S., A. C. S. Wong and H. Tong (2004): "Some Nonlinear Threshold Autoregressive Time Series Models for Actuarial Use," North American Actuarial Journal, 8, 37-61.

[13] Chen, P., A. Rezai and W. Semmler (2008): "Productivity Shocks in the Short and Long Run: An Intertemporal Model and Estimations," Investigacion Economica, Vol LXVII, No. 264, 39-63.

[14] Christian, L. J., M. Eichenbaum, and R. Vigfusson (2004): "The Response of Hours to a Technology Shock," NBER Working Paper No. 10254.

[15] Christiano, L. J., M. Eichenbaum and C. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," Journal of Political Economy, 113, 1-45.

[16] Christiano, L. J., M. Eichenbaum, and S. Rebelo (2009): "When is the Government Spending Multiplier Large?" NBER Working Paper No. 15394.

[17] Cogan, J. F., T. Cwik, J. Taylor, and V. Wieland (2009): "New Keynesian versus Old Keynesian Government Spending Multiplier," www.volkerwieland.com/docs/CCTW%20Mar%202.pdf

[18] Danthine, J. P. and J. B. Donaldson (1995): "Non-Walrasian Economies," in T. F. Cooly (ed.), Frontiers of Business Cycle Research, Princeton: Princeton University Press.

[19] De Long, B. (2009): "Comments on the Multiplier," http://siepr.stanford.edu/policyforum_jobs_agenda.

[20] Ehrmann, M., M. Ellison and N. Valla (2003): "Regime-dependent Impulse Response Functions in a Markov-switching Vector Autoregressive Model," Economic Letters, 78, 295-299.

[21] Erceg, C. J., D. W. Henderson and A. T. Levin (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," Journal of Monetary Economics, 46, 281-313.

[22] Erceg, C. J., J. Guerrieri, and C. Gust (2008:" Trade Adjustment and the Composition of Trade," Journal of Economic Dynamics and Control, 32: 2622-50.

[23] Ernst, E., G. Gong, W. Semmler and L. Bukuviciute (2006), "Quantifying the Impact of Structural Reforms," European Central Bank, Working Paper, No. 666.

[24] Ernst, E., S. Mittnik and W. Semmler (2009), "Interaction of Labor and Capital Market Frictions: A Theoretical and Empirical Analysis," paper presented at an Econometric Society session at the ASSA Meeting in Atlanta, Jan. 3-5, 2010, www.newschool.edu/nssr/cem

[25] Ernst, E. and W. Semmler (2009), "Global Dynamics in a Model with Search and Matching in Labor and Capital Markets," forthcoming, Journal of Economic Dynamics and Control, www.newschool.edu/nssr/cem

[26] Francis, N. and V.A. Ramey (2003): "The Source of Historical Economic Fluctuations: An Analysis Using Long-run Restrictions,"University of California, San Diego.

[27] Francis, N. and V. A. Ramey (2006): "'A Century of Work and Leisure," NBER Working Paper No. W12264.

[28] Gali, J. (1999): "Technology, Employment, and the Business Cycles: Do Technology Shocks Explain Aggregate Fluctuations?" American Economic Review, 89, 249-271.

[29] Gali, J., M. Gertler and J.D. Lopez-Salido (2003): "Markups, Gaps, and the Welfare Costs of Business Fluctuations," mimeo, Deptartment of Economics, New York University.

[30] Gali, J. J.D. Lopez-Salido and J. Vallés (2007): "Understanding the Effects of Government Spending on Consumption," Journal of the European Economic Association, 5, 227-270.

[31] Gali, J. and O. Blanchard (2005): "Real Wage Rigidities and the New Keynesian Model", mimeo, Department of Economics, MIT.

[32] Gong, G. and W. Semmler (2006): "Stochastic Dynamic Macroeconomics - Theory and Empirical Evidence," Oxford/New York: Oxford University Press.

[33] Hall, R.E. (2005): "Employment Fluctuations with Equilibrium Wage Stickiness," American Economic Review, 95, 50-66.

[34] Hamilton, J.D. (1989): "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," Econometrica, 57, 357-84.

[35] Ilzetzki, E., E. G. Mendoza, and C. A. Vegh (2009): "How Big are Fiscal Multipliers? New Evidence from New Data," http://www.voxeu.org/index.php?q=node/4036.

[36] Kaldor, N. (1985). Economics Without Equilibrium (The Arthur M. Okun Memorial Lectures). M. E. Sharpe.

[37] King, R. G. and S. T. Rebelo (1999): "Resusciting Real Business Cycles," in: Handbook of Macroeconomics, Volume I, J. B. Taylor and M. Woodford (eds.), Amsterdam/New York: Elsevier Science.

[38] King, R. G. and A. L. Wolman (1999): "What should the Monetary Authority Do when Prices are Sticky?," in: Monetary Policy Rules, edited by J. Taylor, Chicago: The University of Chicago Press.

[39] Koop, G., M. Pesaran and S. Potter (1996): "Impulse Responses in Nonlinear Multivariate Models," Journal of Econometrics, 74, 119-147.

[40] Krolzig, H.-M. (1997): "Markov-Switching Vector Autoregressions: Modelling, Statistical Inference and Application to Business Cycle Analysis," Lecture Notes in Economics and Mathematical Systems, 454, Berlin: Springer-Verlag.

[41] Krolzig, H.-M., M. Marcellino and G. Mizon (2002): "A Markov–switching Vector Equilibrium Correction Model of the UK Labour Market," Empirical Economics, 27, 233-254.

[42] Malinvaud, E. (1978): The Theory of Unemployment, Oxford: Oxford University Press.

[43] Malinvaud, E. (1994): Diagnosing Unemployment, Cambridge: Cambridge University Press.

[44] Mankiw, N. G. (1985): "Small Menu Cost and Larger Business Cycles, a Macroeconomic Model of Monoply," Journal of Quaterly Economics, 100, 529-539.

[45] Mankiw, N.G., J. Rotemberg and L. Summers (1985): "Intertemporal Substitution in Modern Macroeconomics," Quarterly Journal of Economics, 100, 225-251.

[46] Merz, M. (1999): "Heterogenous Job-Matches and the Cyclical Behavior of Labor Turnover," Journal of Monetary Economics, 43, 91-124.

[47] Meyers, R. J. (1968): "What Can We Learn from European Experience, in Unemployment and the American Economy?", A. M. Ross (ed.), New York: John Wiley & Sons.

[48] Mittnik, S. and P. Zadrozny (1993): "Asymptotic Distributions of Impulse Responses, Step Responses, and Variance Decompositions of Estimated Linear Dynamic Models," Econometrica, 61, 857-87.

[49] Mountfort and Uhlig 2005 ??????

[50] Okun, A. (1962): "Potential GNP: Its Measurement and Significance," Proceedings of the Business and Economic Statistics Section, American Statistical Association.

[51] Perron, P. (1989): "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis," Econometrica, 57, 1361-1401.

[52] Perotti, R. (2005): "Estimating the Effects of Fiscal Policy in OECD Countries," Proceedings, Federal Reserve Bank of San Francisco.

[53] Portier, F. and L. A. Puch (2004): "The Welfare Cost of Business Cycles in an Economy with Nonclearing Markets," Working Paper, University of Toulouse.

[54] Rabanal, P. and J. Gali (2005): "Technology Shocks and Aggregate Fluctuations: How Well Does the RBC Model Fit Postwar U.S. Data?" IMF Working Paper 04/234.

[55] Ramey, V.A., (2009): "Governemnt Spendign Shocks: It's all in the Timing," http://www.econ.ucsd.edu/ vramey/research/IdentifyingGovt.pdf.

[56] Ramey, V. A., Shapiro, M. D. (1998): "Costly Capital Reallocation and the Effects of Government Spending," Carnegie–Rochester Conference Series on Public Policy, Elsevier, 48, 145-194.

[57] Romer, C. and R. Bernstein (2009): "The Job Impact of the American Recovery and Reinvestment Plan," http://otrans.3cdn.net/5593e8ecbd339d074_l3m6bt1te.pdf.

[58] Rotemberg, J. (1982): "Sticky Prices in the United States," Journal of Political Economy, 90, 1187-1211.

[59] Rotemberg, J. and M. Woodford (1995): "Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets," in: Frontiers of Business Cycle Research, T. F. Cooley (ed.), Princeton: Princeton University Press.

[60] Rotemberg, J. and M. Woodford (1996): "Real Business-Cycle Models and the Forecastable Movements in Output," American Economic Review, 86, 71-89.

[61] Rotemberg, J. and M. Woodford (1999): "Interest Rate Rules in an Estimated Sticky Price Model," in: Monetary Policy Rules, J. Taylor (ed.), Chicago: The University of Chicago Press.

[62] Schmidt–Grohe, S. (2001): "Endogenous Business Cycles and the Dynamics of Output, Hours and Consumption," American Economic Review, 90, 1136-1159.

[63] Semmler, W. and G. Gong (2009): "Macroeconomics with Non-Clearing Labor Market," forthcoming in: Festschrift for Bertram Schefold.

[64] Shimer, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," American Economic Review, 95, 25-49.

[65] Smets, F. and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Baysian DSGE Approach," American Economic Review, 97, 586-606.

[66] Solow, R. (1979): “Another Possible Source of Wage Stickiness,” Journal of Macroeconomics, 1, 79-82.

[67] Solow, R. (1997): Learning from ”Learning by Doing”: Lessons for Economic Growth. Palo Alto: Stanford University Press.

[68] Tobin, J. (1993): “Price Flexibility and Output Stability: An Old Keynesian View,” Journal of Economic Perspectives 7, 45-65.

[69] Tong, H. (1978): “On a Threshold Model,” in: Pattern Recognition and Signal Processing, C. H. Chen (ed.), Amsterdam: Kluwer.

[70] Tong, H. (1983): “Threshold Models in Non–linear Time Series Analysis,” New York: Springer–Verlag.

[71] Tsay, R.S. (1998): “Testing and Modelling Multivariate Threshold Models,” Journal of the American Statistical Association, 93, 1188-1202.

[72] Uhlig, H. (1999): “A Toolkit for Analysing Nonlinear Stochastic Models Easily,” in: R. Marimon and A. Scott (eds.), Computational Methods for the Study of Dynamic Economics, Oxford: Oxford University Press.

[73] Uhlig, H. (2004): “Macroeconomics and Asset Markets: Some Mutual Implications,” Working Paper, Department of Economics, Humboldt University Berlin.

[74] Uhlig, H. (2009): “Some Fiscal Calculus”, mimeo, Department of Economics, University of Chicago.

[75] Walsh, L. E. (2002): “Labor Market Search and Monetary Shocks,” Working Paper, Department of Economics, University of California, Santa Cruz.

[76] Woodford, M. (2003): Interest Rate and Prices, Princeton: Princeton University Press.

[77] Woodford, M. (2010): “Simple Analytics of the Governemnt Expenditure Multiplier,” mimeo, Department of Economics, Columbia University.