

# Anthropogenic climate change and abatement in a multi-region world with endogenous growth

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## **Abstract**

This paper studies the effects of global warming in a descriptive model of endogenous growth with multiple regions. It is assumed that deviations from the global surface temperature, which result from an increase in greenhouse gas emissions negatively affect aggregate output and the marginal product of capital. The paper derives optimal abatement ratios in the non-cooperative world and for the cooperative case assuming that the growth rate is an endogenous variable.

Keywords: climate change, greenhouse gases, economic growth, abatement, multi-region world

JEL: E60, O41, Q28

# 1 Introduction

According to the Intergovernmental Panel on Climate Change (IPCC) it is very likely<sup>1</sup> that the 1990s was the warmest decade and 1998 the warmest year since 1861 (IPCC, 2001, p. 26) and the warming of the earth still continues. A possible reason for this phenomenon may be the emission of man-made greenhouse gases (GHGs), like carbon dioxide ( $CO_2$ ) or methane ( $CH_4$ ), which has drastically increased in the 20th century and still continues to rise leading to higher concentrations of GHGs in the atmosphere. Higher GHG concentrations can generate a rise in the average global surface temperature and can make extreme weather events more likely. Further, it is likely that statistically significant increases in heavy and extreme weather events have occurred in many mid- and high latitude areas, primarily in the Northern Hemisphere.

In the economics literature numerous studies analyze the impact of environmental degradation on economic growth using endogenous growth models (for a survey see e.g. Smulders, 1995, or Hettich, 2000). Generally, these studies are rather abstract because they intend to derive general results. It is assumed that economic activities lead to environmental degradation and, as a consequence, reduce utility and/or production possibilities. The goal of these studies often is to analyze how public policy affects environmental conditions as well as the growth rate and welfare of economies.

On the other hand, there exist a great many analyses which study the effects of global warming within general equilibrium models. These models take as a starting point a general equilibrium framework and integrate a more or less elaborate climatic module into this framework (see e.g. Deke et al., 2001, Hackl and Pruckner, 2002, Kemfert, 2001, Nordhaus, 1994, 2000, Stern, 2006, Tol 2001, 2003, and for a survey IPCC, 1996, 2007). The goal of these contributions, then, is to calculate costs of abatement and to confront these costs with the benefits going along with less GHG emissions in order to evaluate the effects of climate policies.

Our model in this paper basically represents an extension of the DICE94 framework presented by Nordhaus (1994). The major extension of our framework consists in allowing for endogenous growth implying that environmental policy does not only affect the level of eco-

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<sup>1</sup>Very likely (likely) means that the level of confidence is between 90 – 99 (66 – 90) percent.

conomic variables but also the long-run growth rate.<sup>2</sup> Since an important stylized fact of market economies is that sustained per capita growth can be observed without a tendency for declining growth rates, it seems necessary to incorporate that aspect in a model dealing with climate change and to evaluate climate policies within this framework. Thus, we want to bring together models of endogenous growth and models dealing with changes in the climate on earth.

In this paper we pursue a neoclassical approach which assumes that losses can be expressed in terms of GDP. This implies that the environment does not have an intrinsic value for mankind or that the loss can be substituted by consuming an additional amount of goods. This neoclassical approach may be seen as problematic since the possible value of an unpolluted environment in the future may not yet be known. Further, destroying large parts of the environment and species without even knowing of their existence may be extremely costly since its potential can never be exploited. Taking these arguments seriously would require a different approach where the destruction of the environment just equals its regenerative capacity so that the constancy of the environmental quality is a constraint which has to be fulfilled in solving an economic problem, like welfare maximization for example. For the problem of global warming this would mean that the GHG concentration remains at its pre-industrial level which, however, is not a realistic assumption. So allowing for a framework where the GHGs exceed their pre-industrial level seems to be more appropriate. Nevertheless, the problem of estimating future costs going along with environmental damages, whether caused by global warming or by other activities of man, is a difficult one and should be addressed with care. More details concerning the problem of estimating costs associated with global warming are given in Azar and Schneider (2003) and in Gerlagh and Papyrakis (2003).

The rest of the paper is organized as follows. In section 2 we present our general descriptive growth model. Further, we model GHG emissions and changes in average surface temperature using a simple energy balance model (EBM). Section 3 studies optimal abatement spending and the resulting growth rates in a non-cooperative world. Section 4 analyzes the cooperative world and section 5, finally, concludes the paper.

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<sup>2</sup>However, we do not model technical progress. For effects of technical progress in the DICE94 model see e.g. Haurie (2003) or Popp (2003).

## 2 A descriptive model of endogenous growth

We assume that aggregate production in region  $i$ ,  $i = 1, \dots, n$ , takes place according to the following per capita production function (see e.g. Barro and Sala-i-Martin, 1995, p. 39).

$$Y_i(t) = A_i K_i(t) D_i(T(t) - T_o), \quad (1)$$

with  $Y_i(t)$  per capita production in region  $i$ ,  $A_i$  a positive constant,  $K_i(t)$  a composite of human and physical capital.  $D_i(T(t) - T_o)$  is the function giving the decline in aggregate per capita production in country  $i$  resulting from deviations of the actual temperature from the pre-industrial temperature,  $T_o$ .<sup>3</sup> The assumption of a continuous function  $D_i(T(t) - T_o)$  is only justified provided the increase in the average surface temperature does not exceed a certain threshold. This holds because for higher increases of the temperature catastrophic events may occur going along with extremely high economic costs which are difficult to estimate. Just one example is the break down of the Gulf Stream which would dramatically change the climate in Europe. Therefore, the analysis assuming a function like  $D_i(\cdot)$  only makes sense for temperature increases within certain bounds.

We should also like to point out that  $AK$  models are very sensitive with respect to the parameters. However, we do not intend to make calibrations but we want to get insights into the structure of the model and to see how certain climate policies affect economies qualitatively. This should be kept in mind when interpreting the results in the next sections.

As to the function  $D_i(T - T_o)$  we assume that it is continuously differentiable and that it satisfies<sup>4</sup>

$$D_i(T - T_o) \begin{cases} = 1, & \text{for } T = T_o \\ < 1, & \text{for } T > T_o, \end{cases} \quad (2)$$

with derivative

$$\frac{\partial D_i(\cdot)}{\partial T} \equiv D'_i(\cdot) < 0. \quad (3)$$

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<sup>3</sup>It should be mentioned that the damage of the temperature increase is given by  $1 - D_i(\cdot)$ .

<sup>4</sup>In the following we delete the time argument  $t$ .

The per capita capital accumulation function is given by<sup>5</sup>

$$\dot{K}_i = A_i K_i D_i(\cdot) (1 - c_i - \tau_{B,i}) - (\delta_i + n_i) K_i, \quad (4)$$

with  $c_i$  the consumption share in region  $i$  and  $\tau_{B,i}$  the abatement share.  $n_i \in (0, 1)$  is the population growth rate in region  $i$  and  $\delta_i \in (0, 1)$  is the depreciation rate of capital.

It should be noted that we take as a starting point the Solow-Swan approach with a given consumption and saving share. We do this because we want to focus on effects resulting from climate changes which affect production as modelled in equations (1)-(3) and, therefore, neglect effects resulting from different preferences. From equations (1) and (4) we see that the gross marginal product of private capital, which equals the interest rate in our economy, is equal to  $A_i D_i(\cdot)$  and that deviations from the pre-industrial temperature affect both the level of production as well as the growth rate of capital and production.

Concerning GHG emissions we assume that these are a by-product of production and expressed in  $CO_2$  equivalents. So, emissions are a function of per capita output relative to per capita abatement activities. This implies that higher production goes along with more emissions for a given level of abatement spending. This assumption is frequently encountered in environmental economics (see e.g. Smulders, 1995). It should also be mentioned that the emission of GHGs does not affect production directly but only indirectly by raising the concentration of GHGs in the atmosphere which affects the climate of the Earth and which leads to a higher surface temperature and to more extreme weather situations.

Formally, emissions in region  $i$  are described by

$$E_i = \left( \frac{a_i Y_i}{B_i} \right)^{\gamma_i} = \left( \frac{a_i}{\tau_{B,i}} \right)^{\gamma_i}, \quad (5)$$

where  $B_i$  is per capita abatement with  $B_i = \tau_{B,i} Y_i$ .  $\gamma_i > 0$  and  $a_i > 0$  are positive constants. The parameter  $a_i$  can be interpreted as a technology index describing how polluting a given technology is. For large values of  $a_i$  a given production (and abatement) goes along with high emissions implying a relatively polluting technology and vice versa.

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<sup>5</sup>The dot over a variable gives the derivative with respect to time.

The concentration of GHGs,  $M$ , evolves according to the following differential equation

$$\dot{M} = \beta_1 \sum_{j=1}^n E_j - \mu M, M(0) = M_0. \quad (6)$$

where  $\mu$  is the inverse of the atmospheric lifetime of  $CO_2$ . As to the parameter  $\mu$  we assume a value of  $\mu = 0.1$  and  $\mu = 0.005$ .<sup>6</sup>  $\beta_1$  captures the fact that a certain part of GHG emissions are taken up by oceans and do not enter the atmosphere. According to IPCC  $\beta_1 = 0.49$  for the time period 1990 to 1999 for  $CO_2$  emissions (IPCC, 2001, p. 39).

Next, we describe the interrelation between economic activities and the change in the average global surface temperature. The simplest method of considering the climate system of the earth is in terms of its global energy balance which is done by so-called energy balance models (EBM). According to an EBM the change in the average surface temperature on earth is described by<sup>7</sup>

$$\frac{dT(t)}{dt} c_h \equiv \dot{T}(t) c_h = S_E - H_E(t) - F_N(t) + \beta_2 (1 - \xi) 6.3 \ln \frac{M}{M_o}, T(0) = T_0, \quad (7)$$

with  $T(t)$  the average global surface temperature measured in Kelvin<sup>8</sup> (K),  $c_h$  the heat capacity<sup>9</sup> of the earth with dimension  $J m^{-2} K^{-1}$  (Joule per square meter per Kelvin)<sup>10</sup> which is considered a constant parameter,  $S_E$  is the solar input,  $H_E(t)$  is the nonradiative energy flow, and  $F_N(t) = F \uparrow (t) - F \downarrow (t)$  is the difference between the outgoing radiative flux and the incoming radiative flux.  $S_E$ ,  $H_E(t)$  and  $F_N(t)$  have the dimension Watt per square meter ( $W m^{-2}$ ).  $F \uparrow$  follows the Stefan-Boltzmann equation which is

$$F \uparrow = \epsilon \sigma_T T^4, \quad (8)$$

with  $\epsilon$  the emissivity which gives the ratio of actual emission to blackbody emission. Blackbodies are objects which emit the maximum amount of radiation and which have  $\epsilon = 1$ . For the

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<sup>6</sup>The range of  $\mu$  given by IPCC is  $\mu \in (0.005, 0.2)$ , see IPCC1, 2001, p. 38.

<sup>7</sup>This part follows Roedel, 2001, chap. 10.2.1 and chap. 1. See also Henderson, 1987, and Gassmann, 1992. A more complex presentation can be found in Harvey, 2000.

<sup>8</sup>273 Kelvin are 0 degree Celsius.

<sup>9</sup>The heat capacity is the amount of heat that needs to be added per square meter of horizontal area to raise the surface temperature of the reservoir by 1K.

<sup>10</sup>1 Watt is 1 Joule per second.

earth  $\epsilon$  can be set to  $\epsilon = 0.95$ .  $\sigma_T$  is the Stefan-Boltzmann constant which is given by  $\sigma_T = 5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ . Further, the ratio  $F \uparrow / F \downarrow$  is given by  $F \uparrow / F \downarrow = 109/88$ . The difference  $S_E - H_E$  can be written as  $S_E - H_E = Q(1 - \alpha_1)\alpha_2/4$ , with  $Q = 1367.5 \text{ W m}^{-2}$  the solar constant,  $\alpha_1 = 0.3$  the planetary albedo, determining how much of the incoming energy is reflected by the atmosphere and  $\alpha_2 = 0.3$  captures the fact that a part of the energy is absorbed by the surface of the Earth.

The effect of emitting GHGs is to raise the concentration of GHGs in the atmosphere which increases the greenhouse effect of the Earth. This is done by calculating the so-called radiative forcing which is a measure of the influence a GHG, like  $CO_2$  or  $CH_4$ , has on changing the balance of incoming and outgoing energy in the Earth-atmosphere system. The dimension of the radiative forcing is  $\text{W m}^{-2}$ . For example, for  $CO_2$  the radiative forcing, which we denote as  $F$ , is given by

$$F \equiv 6.3 \ln(M/M_o), \quad (9)$$

with  $M$  the actual  $CO_2$  concentration,  $M_o$  the pre-industrial  $CO_2$  concentration and  $\ln$  the natural logarithm (see IPCC, 2001, p. 52-53).<sup>11</sup> For other GHGs other formulas can be given describing their respective radiative forcing and these values can be converted in  $CO_2$  equivalents.  $\beta_2$  in (7) is a feedback factor which captures the fact that a higher  $CO_2$  concentration affects for example atmospheric water vapour which has effects for the surface temperature on Earth.  $\beta_2$  is assumed to take values between 1.1 and 3.4. The parameter  $\xi$ , finally, captures the fact that  $\xi = 0.3$  of the warmth generated by the greenhouse effect is absorbed by the oceans which transport the heat from upper layers to the deep sea. In equilibrium, i.e. for  $\dot{T} = 0$ , (7) gives a surface temperature of about 288.4 Kelvin which is about 15 degree Celsius for the pre-industrial GHG concentration, i.e. for  $M = M_o$ .

The heat capacity of the Earth,  $c_h$ , is largely determined by the oceans since most of the Earth's surface is covered by seawater. Therefore, the heat capacity of the oceans is used as a proxy for that of the earth.  $c_h$  is then given by  $c_h = \rho_w c_w d 0.7$ , with  $\rho_w$  the density of seawater ( $1027 \text{ m}^{-3} \text{ kg}$ ),  $c_w$  the specific heat of water ( $4186 \text{ J kg}^{-1} \text{ K}^{-1}$ ) and  $d$  the depth of the mixed

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<sup>11</sup>The  $CO_2$  concentration is given in parts per million (ppm).

layer which is set to 70 meters. The constant 0.7 results from the fact that 70 percent of the Earth are covered with seawater. Inserting the numerical values, assuming a depth of 70 meters and dividing by the surface of the earth gives  $c_h = 0.1497$ .

Setting  $\beta_2 = 1.1$  and assuming a doubling of  $CO_2$  implies that in equilibrium the average surface temperature rises from 288.4 to 291.7 Kelvin, causing a temperature increase of about 3.3 degree Celsius. This is in the range of IPCC estimates<sup>12</sup> which yield increases between 1.5 and 4.5 degree Celsius as a consequence of a doubling  $CO_2$  concentration (IPCC, 2001, p. 67).

Summarizing this discussion the EBM can be rewritten as

$$\dot{T}(t) c_h = \frac{1367.5}{4} 0.21 - 0.95 \left( 5.67 \cdot 10^{-8} \right) (21/109) T^4 + 4.851 \ln \frac{M}{M_o}, T(0) = T_0. \quad (10)$$

The economy is completely described by equations (4), (10) and (6), with emissions given by (5).

### 3 The non-cooperative world

In this section we analyze the non-cooperative world or the Nash equilibrium. Each region maximizes utility resulting from per capita consumption where we assume a logarithmic utility function. Thus, the optimization problem in each region  $i = 1, \dots, n$  is given by

$$\max_{\tau_{B,i}} \int_0^{\infty} e^{-\rho_i t} \ln(c_i A_i K_i D_i(\cdot)) dt \quad (11)$$

subject to (6), (10) and (4) with  $c_i A_i K_i D_i(\cdot) = C_i$  per capita consumption.  $\ln$  denotes the natural logarithm and  $\rho_i$  is the discount rate.

To find the optimum we construct the current-value Hamiltonian which is

$$\begin{aligned} H_i(\cdot) = & \ln(c_i A_i K_i D_i(\cdot)) + \lambda_{1,i} \left( \beta_1 \sum_{j=1}^n \left( \frac{a_j}{\tau_{B,j}} \right)^{\gamma_j} - \mu M \right) + \lambda_{2,i} \left( k_1 - k_2 T^4 + k_3 \ln \frac{M}{M_o} \right) + \\ & + \lambda_{3,i} (A_i K_i D_i(\cdot) (1 - c_i - \tau_{B,i}) - (\delta_i + n_i) K_i), \end{aligned} \quad (12)$$

with  $k_1 \equiv c_h^{-1} 0.21 \cdot 1367.5/4$ ,  $k_2 \equiv c_h^{-1} 0.95 (5.67 \cdot 10^{-8}) (21/109)$  and  $k_3 \equiv 4.851 c_h^{-1}$ .  $\lambda_{k,i}$ ,  $i = 1, 2, 3$ , denote the shadow prices of  $M$ ,  $T$  and  $K_i$  in region  $i$  respectively and  $E_i = a_i^{\gamma_i} Y_i^{\gamma_i} B_i^{-\gamma_i}$  are emissions. Note that  $\lambda_{1,i}$  and  $\lambda_{2,i}$  are negative while  $\lambda_{3,i}$  are positive.

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<sup>12</sup>IPCC results are obtained with more sophisticated Atmosphere-Ocean General Circulation Models.

The necessary optimality conditions are obtained as

$$\frac{\partial H_i(\cdot)}{\partial \tau_{B,i}} = \lambda_{1,i} \beta_1 (-\gamma_i) a_i^{\gamma_i} \tau_{B,i}^{-\gamma_i-1} - \lambda_{3,i} A_i K_i D_i(\cdot) = 0, \quad (13)$$

$$\dot{\lambda}_{1,i} = (\rho_i + \mu) \lambda_{1,i} - \lambda_{2,i} k_3 M^{-1} \quad (14)$$

$$\dot{\lambda}_{2,i} = \rho_i \lambda_{2,i} - D'_i(\cdot)/D_i + \lambda_{2,i} k_2 4 T^3 - \lambda_{3,i} A_i K_i D'_i(\cdot) (1 - c_i - \tau_{B,i}) \quad (15)$$

$$\dot{\lambda}_{3,i} = (\rho_i + \delta_i + n_i) \lambda_{3,i} - K_i^{-1} - \lambda_{3,i} A_i D_i(\cdot) (1 - c_i - \tau_{B,i}). \quad (16)$$

Further, the limiting transversality condition  $\lim_{t \rightarrow \infty} e^{-\rho_i t} (\lambda_{1,i} M + \lambda_{2,i} T + \lambda_{3,i} K_i) = 0$  must hold.

From (13) we get the optimal abatement activities (as a ratio to GDP) in each region as

$$\tau_{B,i}^o = \left( \frac{\beta_1 (-\lambda_{1,i}) \gamma_i a_i^{\gamma_i}}{\lambda_{3,i} A_i K_i D_i(\cdot)} \right)^{1/(1+\gamma_i)} \quad (17)$$

(17) shows that  $\tau_{B,i}^o$  is the higher the more polluting the technology in use is, which is modelled in our framework by the coefficient  $a_i$ . This means that economies with less clean production technologies have a higher optimal abatement share than economies with a cleaner technology. However, this does not mean that economies with a cleaner technology have higher emissions. This holds because, on the one hand, the higher abatement share may not be high enough to compensate for the more polluting technology. On the other hand, the second-best pollution tax rate also depends on  $\lambda_{1,i}$ ,  $\lambda_{3,i}$  and  $K_i$ . Further, from the expression for  $\tau_{B,i}^o$  one realizes that the higher the absolute value of the shadow price of the GHG concentration,  $|\lambda_{1,i}|$ , the higher the abatement share has to be set.

In the following we will confine our investigations to the balanced growth path (BGP). A BGP is defined as follows<sup>13</sup>

**Definition** *A balanced growth path (BGP) is a path such that  $\dot{T} = 0$ ,  $\dot{M} = 0$  and  $\dot{K}/K = C_1$  hold, with  $M \geq M_o$  and  $C_1 > 0$  a positive constant.*

This definition contains several aspects. First, we require that the GHG concentration and the temperature must be constant along a BGP. This is a sustainability aspect. Second, the

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<sup>13</sup>In the following, steady state is used equivalently to balanced growth path.

growth rate of per capita capital is constant over time. It should be noted that this implies that the growth rates of per capita GDP and of per capita consumption are constant, too, and equal to that of capital. Third, we only consider balanced growth paths with a GHG concentration which is larger than or equal to the pre-industrial level. This requirement is made for reasons of realism. Since the GHG concentration has been rising monotonically over the last decades it is not necessary to consider a situation with a declining GHG concentration.

To gain further insight into our model we use numerical calculations and we consider three regions. Two relatively highly developed regions where one region is producing with a relatively clean technology and the other uses a relatively polluting technology. One may think of the European OECD countries as the first region and of the USA as the second region. The third region is given by low income countries with a technology which is more polluting than the other two regions. We set  $a_1 = 3.75 \cdot 10^{-4}$ .  $a_2$  is double as large as  $a_1$ , i.e.  $a_2 = 7.5 \cdot 10^{-4}$ , and  $a_3$  is four times as large as  $a_1$ , i.e.  $a_3 = 0.003$ . These relations reflect about the situation in European OECD countries relative to the USA and relative to low income countries in 1995 (see Nordhaus and Boyer, 2000, table 3.1).  $\gamma_i$ ,  $i = 1, 2, 3$ , is set to one in all three regions, i.e.  $\gamma_i = 1$ ,  $i = 1, 2, 3$ .

As to the function  $D_i(T(t) - T_o)$  we assume the following function

$$D_i = \left(1 + m_i(T - T_o)^2\right)^{-b_i}, \quad m_i, b_i > 0, \quad (18)$$

which fulfills the requirements of (2). The damage caused by a higher GHG concentration is assumed to be the same for the first and second region and about three times as high in the third region for a doubling of GHGs. This is achieved by the following parameter values,  $m_1 = m_2 = 0.0013$ ,  $b_1 = b_2 = 1$  and  $m_3 = 0.0087$ ,  $b_3 = 0.5$ . This implies that an increase of the average surface temperature by 3 degrees as a result of a doubling of GHGs goes along with a damage of about 1.2 percent in regions 1 and 2. A rise of the temperature by about 6 degrees implies a damage of roughly 4.5 percent. For the third region the damage is 4 percent for a 3 degrees increase of the temperature and about 13 percent when the temperature rises by 6 degrees. These values roughly reflect the situation in European OECD countries, in the USA and in low income countries (see Hackl and Pruckner, 2003, table 1).

Damages are not the same in the regions because of differences in the state of development. For example, in developing countries people are prepared worse to possible catastrophes than in developed countries because they cannot afford to invest in preventive measures. Further, poor countries depend more heavily on agriculture and have less means to compensate losses in agricultural production compared to high developed countries so that the consequences of climatic changes are more dramatic in less developed countries.

The subjective discount rate is assumed to be the same in the three regions and we set  $\rho_i = 0.03$ ,  $i = 1, 2, 3$ . We assume the same discount rates in all regions because we want to focus on growth effects resulting from the supply side which is affected by a possible temperature increase and we are not interested in differences resulting from different preferences. If the discount rates were different this would lead to differences in growth rates even if the effects of the temperature increase in the regions were the same and such would complicate the analysis. The population growth rates are assumed to be zero in the first two regions,  $n_1 = n_2 = 0$ , and two percent in the third region,  $n_3 = 0.02$ .

The marginal propensity to consume is set to 80 percent in all three regions,  $c_i = 0.8$ ,  $i = 1, 2, 3$ . The marginal product of capital in the second region is assumed to be larger than in the first region and the latter is larger than in the third region and we set  $A_1 = 0.35$ ,  $A_2 = 0.5$  and  $A_3 = 0.25$ . This implies a higher marginal product of capital in the second region compared to the first and third. A justification for different marginal products can be seen in different levels of technology and in capital transfer constraints.<sup>14</sup> Depreciation rates are set to  $\delta_1 = \delta_2 = 0.04$  in regions 1 and 2 and  $\delta_3 = 0.01$  in region 3. Thus, we acknowledge that depreciation of capital is higher in those regions with higher income.

Defining  $\kappa_i \equiv \lambda_{3,i} \cdot K_i$ , a BGP is given by the solution of the equations.

$$0 = \beta_1 \sum_{j=1}^3 \left( \frac{a_j}{\tau_{B,j}^o} \right) - \mu M \quad (19)$$

$$0 = k_1 - k_2 T^4 + k_3 \ln \frac{M}{M_o} \quad (20)$$

$$0 = \kappa_i \left( \dot{K}_i / K_i + \dot{\lambda}_{3,i} / \lambda_{3,i} \right) \quad (21)$$

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<sup>14</sup>I thank a referee for directing my attention to this point.

$$0 = (\rho_i + \mu) \lambda_{1,i} - \lambda_{2,i} k_3 M^{-1} \quad (22)$$

$$0 = \rho_i \lambda_{2,i} - D'_i(\cdot)/D_i + \lambda_{2,i} k_2 4T^3 - \kappa_i A_i D'_i(\cdot)(1 - c_i - \tau_{B,i}), \quad (23)$$

with  $\tau_{B,i}^o = ((\beta_1(-\lambda_{1,i})a_i)/(\lambda_{3,i}A_iK_iD_i(\cdot)))^{0.5}$ ,  $i = 1, 2, 3$ . (19) follows from (6) and (20) follows from (10). (21) is obtained by combining (4) and (16) and (22) and (23), finally, are obtained from (14) and (15). It should be noted that a constantly rising capital stock goes along with a constantly declining (shadow) price of capita implying that  $\kappa_i$  is constant on a BGP. Solving equations (19)-(23) gives steady state values<sup>15</sup> for the level of GHGs ( $M^*$ ), for the temperature ( $T^*$ ), for the product of the capital stock and its shadow price ( $\kappa^*$ ) and for the shadow prices of GHGs ( $\lambda_1^*$ ) and of temperature ( $\lambda_2^*$ ). These variables, then, give the balanced growth rate in region  $i$  is given by  $g_i \equiv A_i D_i(\cdot)(1 - c_i - \tau_{B,i}) - (\delta_i + n_i)$ , with  $\tau_{B,i}^o$  as above.

As concerns the existence of a BGP Rosen (1965) has derived general conditions such that an N-person game has a unique solution. However, this result cannot be applied to our model because we consider a differential game with ongoing growth. Therefore, we will numerically compute BGPs in our examples below where the computations show that existence and local uniqueness is assured in each case.

In table 1 we give the result of our calculations for the three regions. As to the rate of decay of GHGs we consider two values, namely  $\mu = 0.1$  and  $\mu = 0.005$ .

Table 1. Optimal abatement shares, emissions, balanced growth rates for the three regions as well as average global temperature (non-cooperative case) for  $\mu = 0.1$ . The values in brackets are for  $\mu = 0.005$ .

$\tau_{B,1}^o$	$E_1$	$g_1$	$\tau_{B,2}^o$	$E_2$	$g_2$	$\tau_{B,3}^o$	$E_3$	$g_3$	$T^*$
0.29%	0.131	2.69%	0.38%	0.1949	5.51%	1.36%	0.2214	1.27%	293.1
(0.15%)	(0.25)	(0.5%)	(0.2%)	(0.377)	(2.4%)	(0.7%)	(0.443)	(-0.8%)	(308.8)

For  $\mu = 0.1$  this table shows that the region with the less clean production technology (region 2) has a higher abatement share than the region with the cleaner production technology

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<sup>15</sup>The \* denotes values on the BGP.

(region 1) if damages caused by a rise in the average surface temperature are the same in the two regions. However, this does not mean that emissions in region 2 are smaller than in region 1. So, region 1 has fewer emissions than region 2. This means that the higher abatement share cannot compensate for the less clean production technology.

Taking into account that both the production technology and the damages caused by a rise in GHGs are different (comparing regions 2 and 3) one can see that region 2 spends relatively less for abatement than region 3, 0.4 percent versus 1.4 percent. Further, region 3 has higher emissions than region 2 although the first spends a higher share of GDP for abatement.

With no cooperation GHGs rise by about 2.7 of the pre-industrial level implying an increase in the average global surface temperature of 4.7 degrees Celsius for a decay rate of GHGs of 10 percent and the other parameter values we assume.

Setting  $\mu = 0.005$  the qualitative results remain unchanged. It should also be mentioned that now the model does not produce sustained growth in region 3 because of the high damages going along with the rise in temperature. The temperature increase in this case is about 10 degrees.

In the next section we will compare this result to the outcome in the cooperative world.

## 4 The cooperative world

In the cooperative world the optimization problem of the planner is given by<sup>16</sup>

$$\max_{\tau_{B,i}} \int_0^{\infty} e^{-\rho t} \sum_{j=1}^n w_j \ln(c_j A_j K_j D_j(\cdot)) dt \quad (24)$$

subject to (6) and (4) with  $c_i A_i K_i D_i(\cdot) = C_i$  per capita consumption in region  $i$ .  $\ln$  again denotes the natural logarithm and  $\rho$  is the discount rate.  $w_i$  gives the weight given to region  $i$ .

To find the optimum we construct the current-value Hamiltonian which is now written as

$$H(\cdot) = \sum_{j=1}^n w_j \ln(c_j A_j K_j D_j(\cdot)) + \lambda_4 \left( \beta_1 \sum_{j=1}^n \left( \frac{a_j}{\tau_{B,j}} \right)^{\gamma_j} - \mu M \right) +$$

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<sup>16</sup>We do not call this situation Pareto optimum because in the Pareto optimum the social planner would also determine the savings rate, which is exogenous in our context. Therefore, this solution is in a way second-best.

$$\lambda_5 \left( k_1 - k_2 T^4 + k_3 \ln \frac{M}{M_o} \right) + \sum_{j=1}^n \lambda_{6,j} (A_j K_j D_j(\cdot) (1 - c_j - \tau_{B,j}) - (\delta_j + n_j) K_j), \quad (25)$$

with  $\lambda_4, \lambda_5$  the shadow prices of  $M$  and  $T$  and  $\lambda_{6,i}$  the shadow prices of  $K_i$ . Again,  $\lambda_4$  and  $\lambda_5$  are negative while  $\lambda_{6,i}$  are positive.

The necessary optimality conditions are obtained as

$$\frac{\partial H(\cdot)}{\partial \tau_{B,i}} = \lambda_4 \beta_1 (-\gamma_i) a_i^{\gamma_i} \tau_{B,i}^{-\gamma_i-1} - \lambda_{6,i} A_i K_i D_i(\cdot) = 0, \quad (26)$$

$$\dot{\lambda}_4 = (\rho + \mu) \lambda_4 - \lambda_5 k_3 M^{-1} \quad (27)$$

$$\dot{\lambda}_5 = \lambda_5 \rho + \lambda_5 k_2 4 T^3 - \sum_{j=1}^n w_j D'_j(\cdot) / D_j - \sum_{j=1}^n \lambda_{6,j} A_j K_j D'_j(\cdot) (1 - c_j - \tau_{B,j}) \quad (28)$$

$$\dot{\lambda}_{6,i} = (\rho + \delta_i + n_i) \lambda_{6,i} - w_i K_i^{-1} - \lambda_{6,i} A_i D_i(\cdot) (1 - c_i - \tau_{B,i}). \quad (29)$$

Further, the limiting transversality condition  $\lim_{t \rightarrow \infty} e^{-\rho t} (\lambda_4 M + \lambda_5 T + \sum_{j=1}^n \lambda_{6,j} K_j) = 0$  must hold.

From (26) we get the optimal abatement ratios as

$$\tau_{B,i}^o = \left( \frac{\beta_1 (-\lambda_4) \gamma_i a_i^{\gamma_i}}{\lambda_{6,i} A_i K_i D_i(\cdot)} \right)^{1/(1+\gamma_i)} \quad (30)$$

(30) basically is equivalent to (17) with the exception that the shadow prices are different. This holds because in the cooperative world regions do not optimize separately.

To get further insight we proceed as in the last section. That is we consider three regions, insert numerical values for the parameters and then calculate the corresponding abatement shares, emissions, balanced growth rates as well as the rise in GHGs and in the average global surface temperature. The parameter values are as in the last section, with  $\rho = 0.03$ .

Defining  $\kappa_i \equiv \lambda_{6,i} \cdot K_i$  a BGP is given by the solution of the following system of equations,

$$0 = \beta_1 \sum_{j=1}^3 \left( \frac{a_j}{\tau_{B,j}^o} \right) - \mu M \quad (31)$$

$$0 = k_1 - k_2 T^4 + k_3 \ln \frac{M}{M_o} \quad (32)$$

$$0 = \kappa_i \left( \dot{K}_i / K_i + \dot{\lambda}_{6,i} / \lambda_{6,i} \right) \quad (33)$$

$$0 = (\rho + \mu) \lambda_4 - \lambda_5 k_3 M^{-1} \quad (34)$$

$$0 = \lambda_5 \rho + \lambda_5 k_2 4T^3 - \sum_{j=1}^n w_j D'_j(\cdot)/D_j - \sum_{j=1}^n \kappa_j A_j D'_j(\cdot)(1 - c_j - \tau_{B,j}), \quad (35)$$

with  $\tau_{B,j}^o$  given by (30). Table 2 gives the result assuming equal weight to each region ( $w_1 = w_2 = w_3 = 1$ ).

Table 2. Optimal abatement shares, emissions and balanced growth rates for the three regions as well as average global temperature (cooperative case) for  $\mu = 0.1$ . The values in brackets are for  $\mu = 0.005$ .

$\tau_{B,1}^o$	$E_1$	$g_1$	$\tau_{B,2}^o$	$E_2$	$g_2$	$\tau_{B,3}^o$	$E_3$	$g_3$	$T^*$
0.57%	0.065	2.75%	0.68%	0.11	5.59%	1.9%	0.155	1.41%	290.7
(0.39%)	(0.096)	(0.9%)	(0.5%)	(0.162)	(3.1%)	(1.5%)	(0.196)	(-0.5%)	(305)

Comparing the outcome of the cooperative case with the non-cooperative one it is realized that the rise in GHGs is smaller and, consequently, the increase in the temperature smaller. With  $\mu = 0.1$  GHGs rise by about the factor 1.6 implying an increase in temperature by 2.3 degrees Celsius. This is due to higher abatement shares in the cooperative world and, as a consequence, to smaller emissions in all regions. As to the qualitative results we see that they do not differ from the last section.

It can also be seen that emissions are clearly smaller than in the non-cooperative case. In region 1 emissions are 50 percent smaller, in region 2 44 percent and in region 3 there are 37 percent fewer emissions compared to the non-cooperative world. The reason why emissions in regions 1 and 2 in the cooperative case are much smaller than in the non-cooperative case compared to region 3 is that the shadow price of emissions for regions 1 and 2 in the cooperative case is much higher in absolute values than in the non-cooperative case. This holds because in the cooperative case regions 1 and 2 do not only take into account their own damages but also damages in region 3.

Further, growth rates tend to be larger in the cooperative world. This holds for all three regions and is due to the smaller rise in the average surface temperature. But in region 3, still no positive sustained growth can be observed.

From a qualitative point of view the results remain the same for  $\mu = 0.005$  except for emissions in regions 2 and 3. Now, emissions in region 3 are higher than in region 2. In the non-cooperative case the opposite outcome could be observed. Further, the increase in the average global surface temperature is much larger compared to the case  $\mu = 0.1$ .

In table 3 we study our model assuming that welfare in region 3 gets a weight which is double the weight given to welfare in regions 1 and 2, i.e.  $w_3 = 2w_1 = 2w_2 = 2$ , where we limit our consideration to the case  $\mu = 0.1$ . A possible justification for higher weights can be seen by thinking of the Rawls criterion according to which welfare in an economy is determined by the poorest. Then, one can argue that welfare in the poorest region should receive a higher weight in the world.<sup>17</sup>

Table 3. Abatement shares, emissions and balanced growth rates for the three regions as well as average global temperature ( $w_3 = 2w_1 = 2w_2 = 2$ ,  $\mu = 0.1$ .)

$\tau_{B,1}^o$	$E_1$	$g_1$	$\tau_{B,2}^o$	$E_2$	$g_2$	$\tau_{B,3}^o$	$E_3$	$g_3$	$T^*$
0.66%	0.057	2.71%	0.78%	0.096	5.53%	1.6%	0.19	1.49%	290.9

Table 3 shows that now region 3 has a smaller abatement share and higher emissions if welfare of that region gets a higher weight compared to the case where all three regions get the same weight. The other two regions have higher abatement shares and smaller emissions. As a result, the growth rates in regions 1 and 2 tend to fall while that in region 3 tends to rise.

## 5 Conclusion

In this paper we have extended a basic framework for modelling the interaction between climate change and economic growth by allowing for endogenous growth.

Our analysis showed among other things that countries with more polluting technologies and higher damages should spend a higher share of GDP for abatement but, nevertheless,

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<sup>17</sup>Of course, a strict application of the Rawls criterion would require to maximize welfare of only the poorest region which, however, would not be a cooperative solution.

may still emit more than countries with cleaner technologies and smaller damages. This holds both for the non-cooperative and for the cooperative world. Further, the study showed that countries with higher damages from the temperature increase emit less when damages are equated between regions compared to the solution of the dynamic optimization problem with cooperation among regions.

As to future research, it would be interesting to extend the model by developing a more elaborate endogenous growth model and by incorporating endogenous technical change affecting production and emissions going along with the production process.<sup>18</sup>

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<sup>18</sup>A promising approach working with one representative country is the paper by Gerlagh (2004).

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