

# Credit Risk and Sustainable Debt: A Model and Estimations of Why the Euro is Stable in the Long-Run\*

Willi Semmler<sup>†</sup>      Peter Wöhrmann

June 2001

## Abstract

Sustainable debt has become a key issue in rating of private as well as sovereign debtors. The problem of how to estimate sustainable debt has also been at the center of debate over the Asian 1997-1998 financial crisis. If the external value of the currency depends on the external debt of a country it is necessary to estimate the creditworthiness of the country. This paper studies credit risk and sustainable debt in the context of an intertemporal model. For a dynamic growth model with an additional equation for the evolution of debt we demonstrate of how to compute sustainable debt and creditworthiness. The model is estimated by employing time series data for the core countries of Euroland. The computations show that Euroland has large external assets. Using time series methods the sustainability of external debt (assets) is estimated for those core countries of Euroland. Those estimations show that the Euro will be a stable currency in the long run.

---

\*I want to thank Sabine Guschwa for assistance in the empirical part of the paper. I also want to thank for financial support the Ministry of Education, Science and Technology of the State Northrhine-Westfalia, participants of a workshop of the European Macro Group at the Universities Aix-Marseille II and Jerome Henry from the ECB in Frankfurt.

<sup>†</sup>Center for Empirical Macroeconomics, Bielefeld and New School University, New York, E-mail: wsemmler@wiwi.uni-Bielefeld.de

# 1 Introduction

Sustainable debt has become the key issue in credit rating. Economic agents (households, firms, government and countries) are creditworthy as long as the present value of their income does not fall short of the liabilities that the agents face. Credit rating firms continuously evaluate the creditworthiness of debtors. Debt sustainability and creditworthiness was at the root of the Asian financial crisis. A credit crisis in fact can trigger a currency and financial crisis and large output losses.<sup>1</sup> In this paper we want to study and evaluate credit risk in the context of a dynamic economic model. More specifically we want to study borrowing capacity, creditworthiness and credit risk in the context of the model and estimate the model with Euroland time series data.

In order to simplify matters we do not employ a stochastic version of the dynamic model but rather employ a deterministic framework.<sup>2</sup> Yet, our study might still be important for issues of credit risk and for risk management that have kept the attention of the financial economists since the Asian financial crisis and that have recently been discussed in many empirical contributions. Here we do not extensively discuss the diverse empirical variables and methods to evaluate credit risk and to compute default risk of bonds (see Benninga 1998, ch. 17). Those methods are very useful in practice but have only little connection to a theory of credit risk and theoretical measures of creditworthiness.

Measuring credit risk is also important in risk management and the value at risk approach. The latter approach works with expected volatility of asset prices (for a survey, see Duffie and Pan 1997). Although our study has implications for credit risk analysis in empirical finance literature and risk management our approach is more specifically related to the literature that link credit market and economic activity in the context of intertemporal models. In recent times this link has been explored in numerous papers.

In a first type of papers, mostly assuming perfect credit markets, it is assumed that, roughly speaking, agents can borrow against future income as long as the discounted future income, the wealth of the agents, is no smaller than the debt that agents have incurred. In this case there is no credit risk whenever the non-explosiveness condition holds. Positing that the agents can borrow against future income the non-explosiveness condition is equivalent to the requirement that the intertemporal budget constraint holds for the agents. Formally, the necessary conditions for optimality, derived from the Hamiltonian equation, are often employed to derive the dynamics of the

---

<sup>1</sup>See the work by Milesi-Ferreti and Razin (1996,1997)

<sup>2</sup>A stochastic version can be found in Sieveking and Semmler (1999).

state variables and the so called transversality condition is used to provide a statement on the non-explosiveness of the debt of the economic agents. Models of this type have been discussed in the literature for households, firms, governments and small open economies (with access to international capital markets).<sup>3</sup>

In a second type of papers, and also often in practice, assuming credit market imperfections, economists presume that borrowing is constrained. Frequently, borrowing ceilings are assumed which are supposed to prevent agents from borrowing an unlimited amount. Presuming that agents' assets serve as collateral a convenient way to define the debt ceiling is then to assume the debt ceiling to be a fraction of the agents' wealth. <sup>4</sup> It has also been pointed out that banks often define debt ceilings for their borrowers, see Bhandari, Haque and Turnovsky (1990).

A third type of literature also assumes credit market imperfections but employs endogenous borrowing cost such as in the work by Bernanke and Gertler (1989, 1994). State dependent borrowing cost has been associated with the financial accelerator theory. Often here one presupposes only a one period zero horizon model and then it is shown that due to endogenous change of net worth of firms, as collateral for borrowing, credit cost is endogenous. For potential borrowers their credit cost is inversely related to their net worth. In parallel other literature has posited that borrowers may face a risk dependent interest rate which is assumed to be composed by a market interest rate (for example, an international interest rate) and an idiosyncratic component determined by the individual degree of risk of the borrower. Various forms of the agent specific risk premium can be assumed.<sup>5</sup>

Recent extensions of the third type of work have been undertaken by embedding credit market imperfections and endogenous borrowing cost more formally in intertemporal models such as the standard stochastic growth model, see Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1998), Cooley and Quadrini (1998) and Krieger (1999). Some of this literature has dealt also with borrowing constraints of heterogenous agents (households, firms) in a intertemporal general equilibrium framework. Although in our paper we stress intertemporal behavior of economic agents in the context of a growth model, we here will not deal with the case of heterogenous agents.

---

<sup>3</sup>For a brief survey of such models for households', firms' and governments or countries, see Blanchard and Fischer (1989, ch.2) and Turnovsky (1995).

<sup>4</sup>The definition of debt ceilings have become standard, for example, in a Ramsey growth model for small open economies; see, for example, Cohen and Sachs (1986) and Barro, Mankiw and Sala-i-Martin (1995).

<sup>5</sup>The interest rate as a convex function of the default risk of the borrower is posited by Bhandari, Haque and Turnovsky (1990), Rauscher (1990) and Turnovsky (1995).

Another type of important recent work has focused on sovereign debtors. Initiated by the Asian financial crisis 1997-1998 a large number of papers has concentrated on external debt, currency crisis and financial crisis, see, for example, the papers by Krugman (1999), Mishkin (1998) and Miller and Stiglitz (1999) and have employed the aforementioned theory of imperfect capital markets.

We present a dynamic model with credit market that can be perceived as holding true for single agents or a country. In fact the set up of the model is undertaken in a way that reflects a country borrowing from abroad. Although we think that our approach is very appropriate to model and study credit risk and sustainable debt for indebted countries – possibly leading up to a currency crisis as in the case of the Asian countries in 1997-98 – because of a lack of data we do not pursue an empirical test for those countries. Empirically we estimate instead the sustainability of foreign debt (assets) of Euroland where we have sufficient time series data available. This might also give us an answer about the future stability of the Euro. The remainder of this paper is organized as follows. In section 2 we set up the model and transform it into an estimable form. Section 3 estimates the model with time series data from Euroland. Section 4 undertakes an econometric debt sustainability test for time series data of Euroland. Section 5 concludes the paper.

## 2 The Dynamic Model

Next, we want to give a formal presentation of our model. In a contract between a creditor and debtor there are two problems involved. The first pertains to the computation of debt and the second to the computation of the debt ceiling. The first problem is usually answered by employing an equation of the form

$$\dot{B}(t) = \theta B(t) - f(t), \quad B(0) = B$$

where  $B(t)$  is the level of debt<sup>6</sup> at time  $t$ ,  $\theta$  the interest rate determining the credit cost and  $f(t)$  the net income of the agent. The second problem can be settled by defining a debt ceiling such as

$$B(t) \leq C, \quad (t > 0)$$

---

<sup>6</sup>Note that all subsequent state variables are written in terms of efficiency labor along the line of Blanchard (1983).

or less restrictively by

$$\sup_{t \geq 0} B(t) < \infty$$

or even less restrictively by the aforementioned transversality condition

$$\lim_{t \rightarrow \infty} e^{-\theta t} B(t) = 0.$$

The ability of a debtor to service the debt, i.e. the feasibility of a contract, will depend on the debtors source of income. Along the line of intertemporal models with borrowing and lending<sup>7</sup> we model this source of income as arising from a stock of capital  $k(t)$ , at time  $t$ , which changes with investment rate  $j(t)$  at time  $t$  through

$$\dot{k}(t) = j(t) - \sigma(k(t)), \quad k(0) = k.$$

In the subsequent model  $\sigma$  will be constant, reflecting depreciation rate of capital, productivity and population growth. In our general model both the capital stock and the investment are allowed to be multivariate. As debt service we take the net income from the investment rate  $j(t)$  at capital stock level  $k(t)$  minus some minimal rate of consumption.<sup>8</sup> Hence

$$\dot{B}(t) = \theta B(t) - f(k(t), j(t)), \quad B(0) = B$$

where  $\theta B(t)$  is the credit cost. Note that the credit cost is not necessarily a constant factor (a constant interest rate). We call  $B^*(k)$  the creditworthiness of the capital stock  $k$ . The problem to be solved is how to compute  $B^*$ .

If there is a constant credit cost (interest rate), then as is easy to see  $B^*(k)$  is the present value of  $k$

$$B^*(k) = \underset{j}{Max} \int_0^{\infty} e^{-\theta t} f(k(t), j(t)) dt - B(0) \quad (1)$$

$$s.t. \quad \dot{k}(t) = j(t) - \sigma(k(t)), \quad k(0) = k_0 \quad (2)$$

$$\dot{B}(t) = \theta B(t) - f(k(t), j(t)), \quad B(0) = B_0 \quad (3)$$

---

<sup>7</sup>Prototype models used as basis for our further presentation can be found in Blanchard (1983), Blanchard and Fischer (1989) or Turnovsky (1995).

<sup>8</sup>In the subsequent analysis of creditworthiness we can set consumption equal to zero. Any positive consumption will move down the creditworthiness curve. Note also that public debt for which the Ricardian equivalence theorem holds, i.e. where debt is serviced by a non-distortionary tax, would cause the creditworthiness curve to shift down.

The more general case is, however, that  $\theta$  is not a constant. As in the theory of credit market imperfections we generically may let  $\theta$  depend on  $k$  and  $B$ .<sup>9</sup> Employing a growth model in terms of efficiency labor<sup>10</sup> we can use the following net income function that takes account of adjustment cost of capital

$$f(k, j) = k^\alpha - j - j^\beta k^{-\gamma} \quad (4)$$

where  $\beta > 0, \alpha > 0, \gamma > 0$  are constants.<sup>11</sup> In the above model  $\sigma > 0$  captures as aforementioned both a constant growth rate of productivity as well as a capital depreciation rate and population growth.<sup>12</sup> Blanchard (1983) used  $\beta = 2, \gamma = 1$  to analyze optimal indebtedness of a country (see also Blanchard and Fischer 1989, ch. 2).

Note that in the model (1)-(3) we have not used utility theory. As shown in Sieveking and Semmler (1998) the model (1)-(3) exhibits, however, a strict relationship to a growth model built on a utility functional, for example, such as<sup>13</sup>

$$Max \quad \int_0^\infty e^{-\theta t} u(c(t)) dt \quad (5)$$

$$s.t. \quad \dot{k}(t) = j(t) - \sigma(k(t)), \quad k(0) = k. \quad (6)$$

$$\dot{B}(t) = \theta B(t) - f(k(t), j) - c(t), \quad B(0) = B \quad (7)$$

with the transversality condition

$$\lim_{t \rightarrow \infty} e^{-\theta t} B(t) = 0 \quad (8)$$

---

<sup>9</sup>The more general theory of creditworthiness with state dependent credit cost,  $\theta(k, B)$  is provided in Semmler and Sieveking (1998). Note that instead of relating the credit cost inversely to networth, as in Bernanke, Gertler and Gilchrist (1998), one could use the two arguments,  $k$  and  $B$ , explicitly.

<sup>10</sup>The subsequent growth model can be viewed as a standard RBC model where the stochastic process for technology shocks is shut down and technical change is exogenous occurring at a constant rate. Moreover, a debt equation, as in (3) is added. In Bernanke, Gertler and Gilchrist (1998) networth is the second state equation. In fact, it can be shown that their use of the second state equation is equivalent to our equ. (3) except for the use of adjustment cost in our model.

<sup>11</sup>Note that the production function may  $k^\alpha$  may have to be multiplied by a scaling factor. For the analytics we leave it aside.

<sup>12</sup>For details, see Blanchard (1983).

<sup>13</sup>For details, see Blanchard (1983).

which often turns up in the literature<sup>14</sup> among the "necessary conditions" for a solution of a welfare problem such as (5)-(8). In Sieveking and Semmler (1998) it is shown that the problem (5)-(8) can be separated into two problems. The first problem is to find optimal solutions that generate the present value of net income flows and the second problem is to study the path of how the present value of net come flows is consumed. There are also conditions discussed under which such separation is feasible. The separation into those two problems appear to be feasible as long as the evolution of debt does not appear in the objective function. If such separation is feasible we then only need to be concerned with the model (1)-(3). Yet instead of maximizing a utility functional the present value of a net income function is maximized.

In fact in this case we only need to focus on the maximization problem (1)-(2). It can be solved by using the necessary conditions of the Hamiltonian. So we maximize

$$\begin{aligned} \underset{j}{Max} \int_0^{\infty} e^{-\theta t} f(k(t), j(t)) dt \\ s.t. \quad (2) \end{aligned}$$

The Hamiltonian for this problem is

$$\begin{aligned} H(k, x, j, \lambda) &= \max_j H(k, x, j, \lambda) \\ H(k, x, j, \lambda) &= \lambda f(k, j) + x(j - \sigma k) \\ \dot{x} &= \frac{-\partial H}{\partial k} + \theta x = (\sigma + \theta) x - \lambda f_k(k, j) \end{aligned}$$

We denote  $x$  as the co-state variable in the Hamiltonian equations and  $\lambda$  is equal to 1.<sup>15</sup> The function  $f(k, \cdot)$  is strictly concave by assumption therefore there is a function  $j(k, x)$  which satisfies the first order condition of the Hamiltonian

---

<sup>14</sup>See, for example, Bhandari, Hague and Turnovsky (1990). In our framework the equivalent transversality condition will be

$$\sup_{t \geq 0} B(t) < \infty$$

<sup>15</sup>For details of the computation of the equilibria in the case when one can apply the Hamiltonian, see Semmler and Sieveking (1999), appendix.

$$f_j(k, j) + x = 0 \quad (9)$$

$$j = j(k, x) = \left( \frac{x-1}{k^{-\gamma} \cdot \beta} \right)^{\frac{1}{\beta-1}} \quad (10)$$

and  $j(.,.)$  is uniquely determined thereby. It follows that  $(k, x)$  satisfy

$$\dot{k} = j(k, x) - \sigma k \quad (11)$$

$$\dot{x} = (\sigma + \theta)x - f_k(k, j(k, x)) \quad (12)$$

The isoclines can be obtained by the points in the  $(k, x)$  space for  $\beta = 2$  where  $\dot{k} = 0$  satisfies

$$x = 1 + 2\sigma k^{1-\gamma} \quad (13)$$

and where  $\dot{x} = 0$  satisfies

$$x_{\pm} = 1 + \vartheta k^{1-\gamma} \pm \sqrt{\vartheta^2 k^{2-2\gamma} + 2\vartheta k^{1-\gamma} - 4\alpha\gamma^{-1}k^{\alpha-\gamma}} \quad (14)$$

where  $\vartheta = 2\gamma^{-1}(\sigma + \theta)$ . Note that the latter isocline has two branches.

If the parameters are given the steady state – or the steady states, if there are multiple ones – can be computed and the local and global dynamics studied. We scale the production function by  $a$ <sup>16</sup> We employ the following parameters:  $\alpha = 1.1$ ,  $\gamma = 0.3$ ,  $\sigma = 0.15$ ,  $\theta = 0.1$ ,  $\beta = 2$ .

For those parameters, using the Hamiltonian approach, Figure 1 depicts the isoclines (13) - (14) showing two positive candidates for equilibria.

[Figure 1 about here]

The two equilibrium candidates are: (HE1):  $k^* = 1.057$ ,  $x^* = 1.3$  and (HE2):  $k^{**} = 0.21$ ,  $x^{**} = 1.1$ . The two candidates are numerically obtained by using a nonlinear equation solver.<sup>17</sup> Since the second branch of (14) does not intersect with (13) we have left it aside. We also want to note that the equilibrium candidate  $(k^*, x^*)$  is a saddle whereas  $(k^{**}, x^{**})$  represents a repeller. There is a third equilibrium candidate which is  $k = 0$ . We want to remark that there is the possibility of multiple equilibria in such a model with nonlinear adjustment cost of capital. Yet, this problem will not be pursued here further.<sup>18</sup>

<sup>16</sup>In the following numerical example we have multiplied the production function by  $a = 0.3$ .

<sup>17</sup>The nonlinear equation solver from the software package GAUSS is employed.

<sup>18</sup>For a detailed treatment of such a model with multiple equilibria, see Semmler and Sieveking (1998).

### 3 Estimating the Parameters of the Model

Next, we want to take our growth model with adjustment cost of investment to the data. We will use quarterly data for Euroland. We were able to generate time series data for the relevant variables for most of the core countries of Euroland. For the purpose of the parameter estimation we have to transform our dynamic equations into estimable equations. By presuming the above (1)-(3) version where in the debt equation only a constant credit cost factor enters we can employ the Hamiltonian equation. This is in the case of Euroland justified, since there are likely to be not severe idiosyncratic risk components in the interest rate for Euroland. We can transform the system (11)-(12) into estimable equations and employ time series data on capital stock and investment – all expressed in efficiency units – to estimate the involved parameter set.<sup>19</sup>

Substituting the optimal investment rate (10) into (11) we get the following two dynamic equations

$$\dot{k} = \left( \frac{x-1}{k^{-\gamma} \cdot \beta} \right)^{\frac{1}{\beta-1}} - \sigma k \quad (15)$$

$$\dot{x} = (\sigma + \theta)x - \alpha k^{\alpha-1} - j^{\beta} \gamma k^{(-\gamma-1)} \quad (16)$$

Next, we transform the above system (15)-(16) into observable variables so that we obtain estimable dynamic equations.

From (15) we obtain

$$\hat{k} = j/k - \sigma \quad (17)$$

with  $\hat{k} = \dot{k}/k$

Note that from (9) we can get

$$x = 1 + \beta j^{\beta-1} k^{-\gamma} \quad (18)$$

From this we can take the time derivative and obtain

$$\dot{x} = (\beta(\beta-1)j^{\beta-2}k^{-\gamma}) \cdot \dot{j}$$

Setting the above equation equal to (16) we have

---

<sup>19</sup>Estimable equations for a version with a state dependent credit cost  $H(k, B)$  would predict a slightly different paths for optimal investment and capital stock. Such an approach, however, appears to be more cumbersome to estimate.

$$(\beta(\beta - 1)j^{\beta-2}k^{-\gamma}) \cdot \dot{j} = (\sigma + \theta)x - \alpha k^{\alpha-1} - j^{\beta}\gamma k^{(-\gamma-1)}$$

Thus

$$\dot{j} = \frac{(\sigma + \theta)x - \alpha k^{\alpha-1} - j^{\beta}\gamma k^{(-\gamma-1)}}{\beta(\beta - 1)j^{\beta-2}k^{-\gamma}} \quad (19)$$

or

$$\hat{j} = \frac{\dot{j}}{j} = \left( \frac{(\sigma + \theta)x - \alpha k^{\alpha-1} - j^{\beta}\gamma k^{(-\gamma-1)}}{\beta(\beta - 1)j^{\beta-2}k^{-\gamma}} \right) / j \quad (20)$$

Substituting (18) into (20) we get as estimable equations in observable variables (17) and (20) which depend on the following parameter set to be estimated.

$$\varphi = (\theta, \sigma, \beta, \gamma, \alpha, a)$$

The estimation of the above parameter set is undertaken by aggregating capital stock and investment for the core countries of Euroland. The data are quarterly data from 1978.1 - 1996.2. Although aggregate capital stock data starting from 1970.1 are available, we apply our estimation to the period 1978.1 - 1996.2, since the European Monetary System has been introduced in 1978 whereby the exchange rates between the countries were fixed within a band. This makes the across country aggregation of capital stock and investment feasible. The aggregate capital stock series is for gross private capital stock and the investment series is total fixed investment. Both are taken from OECD data base (1999). The series for gross capital stock and investment represent aggregate real data for German, France, Italy, Spain, Austria, Netherland and Belgium. Since we are employing a model in efficiency labor each countries time series for capital stock and investment are scaled down by labor in efficiency units measured by the time series  $L_t = L_0 e^{(n+g_y/l)t}$  where  $n$  is average population growth and  $g_y/l$  average productivity growth. As to our estimation strategy we employ NLLS estimation and use a constrained optimization procedure.<sup>20</sup> The results are shown in Table 1.

Table 1: Parameter estimates for Euroland (1978.1-1996.2)

$\theta$	$\sigma$	$\beta$	$\gamma$	$\alpha$	$a$
0.035	0.092	0.312	0.116	0.385	3.32

<sup>20</sup>The estimations were undertaken in GAUSS for which the constrained optimization procedure recently provided by GAUSS was used.

The parameters obtained from historical data are quite reasonable.<sup>21</sup> Overall one can observe that the adjustment cost of investment are not very large since the exponents  $\beta$  and  $\gamma$  are small.

Using the estimated parameters one can again compute through (15) - (16) the steady states for the capital stock. Doing so numerically it turns out that for our parameter estimates of Table 1 the steady state is unique and we obtain a  $k^* = 37.12$  which coincides roughly with the mean of the historical series of the capital stock for Euroland. This gives a steady state of net income of  $f(k, j) = 8.799$ , computed from (4) at the steady state of  $k^* = 37.12$ . Moreover, for the present value of the net income at the steady state we obtain  $V(k^*) = 244.4193$ .

Using the estimated parameters figure 2 shows the computed output, investment (including adjustment cost of investment) and the net income.

[Figure 2 about here]

Figure 2: Net income, investment (including adjustment cost) and output

As the figure 2 shows, since we are using the aggregate variables in efficiency units, the output in efficiency units tends to be stationary and the net income moves inversely to investment (the latter including adjustment cost).

Finally, note that with those parameter estimates given in Table 1 we also can now easily compute the present value outside the steady state and thus the critical debt curve by using the above Hamiltonian method.<sup>22</sup> Since, however, there is no external debt of Euroland but rather external assets (as shown in the next section), the result of such an exercise will not be very instructive. The balance sheets of banks and firms, as discussed in Krugman (1999) and Mishkin (1998), will presumably show no sign of deterioration, since Euroland has net claims, negative debt, vis-a-vis the rest of the world. Our methods to compute present value of net income could, however, be fruitfully undertaken for other countries with external debt and balance sheets of banks and firms deteriorating.<sup>23</sup> Note, however, that the above method gives us only asymptotic results, i.e. if  $t \Rightarrow \infty$ . Next, for Euroland we pursue another method – for finite number of observations – to compute the sustainability of external debt or assets.

---

<sup>21</sup>We want to note that standard errors could not be recovered since the Hessian in the estimation was not non-negative definite.

<sup>22</sup>For a method of how to compute the present value outside the steady state, see Semmler and Sieveking (1998).

<sup>23</sup>Of course, one would have to consider also the exchange rate regime under which the country borrows and in particular the fact whether the country (banks, firms) borrows in foreign currency. In this case a exchange rate shock will exacerbate the deterioration of the balance sheets, see Mishkin (1998) and Krugman (1999).

## 4 Testing Sustainability of External Debt

Next, following Flood and Garber (1980) and Hamilton and Flaven (1986) a NLLS estimate for the sustainability of external debt can be designed for a finite number of observations. Similarly to the computation of the capital stock and investment for our core countries of Euroland we have computed the trade account, the current account and the net foreign assets of those core countries for the time period 1978.1-1998.1. Since we want to undertake sustainability tests for certain regimes, we have computed monthly observations. In our computation we had to eliminate the trade among the Euroland-countries.<sup>24</sup> We consider the time series for the entire period 1978.1-1998.12 and in addition subdivide the period into two periods 1978.1-1993.12. and 1994.1-1998.12. The break in 1994 makes sense since the exchange rate crisis of September 1992 has led to a reestablishment of new exchange rates with a wider band in 1993. Thus, the sustainability tests will be undertaken for those two subperiods.

In a discrete time version the external debt (or not foreign assets) can be computed as follows. Starting with initial debt  $B_0$  one can compute in a discrete time way the stock of debt as follows. By assuming a constant interest rate we have

$$B_t = (1 + r_{t-1})B_{t-1} - TA_t \quad (21)$$

where  $TA_t$  is the trade account and  $B_{t-1}$  the stock of foreign debt at period  $t-1$  and  $r_{t-1}$  the interest rate. As interest rate we took the Libor rate. The initial stock of foreign debt  $B_0$  for 1978.1 has been estimated. This way, the entire time series of external debt and trade account could be generated.

From equ. (21) we can develop a discrete time sustainability test. For reason of simplicity let us assume a constant interest rate. Equ. (21) is then a simple first order difference equation that can be solved by recursive substitution forward leading to

$$B_t = \sum_{i=t+1}^N \frac{TA_i}{(1+r)^{i-t}} + \frac{(1+r)^t B_N}{(1+r)^N} \quad (22)$$

In the equ. (22) the second term must go to zero if the intertemporal budget constraint is supposed to hold. Then equ. (23) means that the current value

---

<sup>24</sup>A similar attempt to compute external debt of countries and regions, following a similar methodology as suggested above, has been recently undertaken by Lane and Milesi-Ferreti (1999). Their results for the Euroland core countries show similar trends as our computation. Their results are, however, less precise since they do not eliminate intra-Euroland trade.

of debt is equal to the expected discounted future trade account surplus

$$B_t = E_t \sum_{i=t+1}^{\infty} \frac{TA_i}{(1+r)^{i-t}} \quad (23)$$

Equivalent to requiring that equ. (23) must be fulfilled is that the following condition holds

$$E_t \lim_{N \rightarrow \infty} \frac{B_N}{(1+r)^N} = 0 \quad (24)$$

The equation is the usual transversality condition or no-ponzi game condition as discussed in section 2.

If the external debt is constrained not to exceed a constant,  $A_0$ , on the right hand side of (22), we then have

$$B_t = E_t \sum_{i=t+1}^{\infty} \frac{TA_i}{(1+r)^{i-t}} + A_0(1+r)^t \quad (25)$$

The NLLS test proposed by Flood and Garber (1980) and Hamilton and Flaven (1986) and Greiner and Semmler (1998) can be modified for our case. It reads:

$$TA_t = b_1 + b_2 TA_{t-1} + b_3 TA_{t-2} + b_4 TA_{t-3} + \varepsilon_{2t} \quad (26)$$

$$B_t = b_5(1+r)^t + b_6 + \frac{(b_2b+b_3b^2+b_4b^3)TA_t}{(1-b_2b-b_3b^2-b_4b^3)} + \frac{(b_3b+b_4b^2)TA_{t-1}}{(1-b_2b-b_3b^2-b_4b^3)} + \frac{(b_4b)TA_{t-2}}{(1-b_2b-b_3b^2-b_4b^3)} + \varepsilon_{1t} \quad (27)$$

We want to note, however, that following Wilcox (1989) it might be reasonable to compute trade account surplus and debt series as discounted time series. We have also undertaken the computation of those discounted time series by discounting both the trade account and the external debt with an average interest rate and performed the above (26)-(27) sustainability test.

For Euroland it turns out that it has foreign assets instead of external debt. Figure 3 shows the undiscounted and discounted time series for external assets of Euroland.

[Figure 3 about here]

Figure 3: Undiscounted and discounted net foreign assets

Table 2 reports test results for both types of time series for the entire time period 1978.1-1998.12.

Table 2: Sustainability Test of Net Foreign Assets of Euroland, 1978.1-1998.12

Param	undiscounted		discounted	
	Estim	t-stat	Estim	t-stat
$b_1$	0.00	0.00	0.00	0.0
$b_2$	0.76	0.05	0.53	0.04
$b_3$	0.45	-0.02	0.37	0.02
$b_4$	-0.51	-0.05	-0.06	-0.07
$b_5$	-0.07	-1.20	-0.002	-0.04
$b_6$	0.0051	0.06	-0.064	-0.88

Table 3 reports our estimation results for subperiods again for both undiscounted and discounted trade account and debt service. The results of the estimation of the coefficients as to the relevance of non-sustainability of foreign assets for Euroland are not very conclusive. The coefficient  $b_5$ , which is the relevant coefficient in our context, has the correct sign but is always insignificant.

Next we compute the estimate (26)-(27) for the two subperiods. Table 3 reports the results for undiscounted and discounted variables respectively.

Table 3: Sustainability Test of Net Foreign Assets for Euroland, 1978.1-1993.12 and 1994.1-1998.12

	undiscounted				discounted			
	1978.1-1993.12		1994.1-1998.12		1978.1-1993.12		1994.1-1998.12	
$b_1$	0.00	0.00	0.001	0.53	0.00	0.00	0.001	0.39
$b_2$	0.423	0.04	0.308	0.16	0.498	0.04	0.365	0.14
$b_3$	0.338	0.02	-0.200	-0.10	0.401	0.03	-0.266	-0.09
$b_4$	0.048	0.01	-0.318	-0.19	-0.062	-0.01	-0.548	-0.22
$b_5$	-0.042	-0.72	-0.203	-25.86	0.018	0.30	-0.061	-13.08
$b_6$	-0.025	-0.32	0.277	17.61	-0.086	-1.07	0.056	5.91

As can clearly be seen from the coefficients  $b_5$  both the undiscounted as well as discounted trade time series show that there has been a rapid built-up of net foreign assets of Euroland that do not seem to be sustainable. Our tests imply there is a build-up of foreign assets that particularly occurred after the currency crisis 1992/1993.

## 5 Conclusions

We have shown that sustainable debt in models with borrowing and lending may typically be state constrained. In order to control credit risk the lender needs to know the debt capacity of the borrower at each point in time. This knowledge seems to be necessary if one wants to move beyond an one period debt contract. We explore the problem of critical debt and creditworthiness by applying the Hamiltonian. Imposing a ceiling of borrowing may lead to a loss of welfare if the ceiling is set to low. Moreover, in some instances it may be necessary for the borrower to first increase debt in order to decrease it. On the other hand, if the ceiling is set to high the non-explosiveness condition may not hold and creditworthiness may be lost.

We want to note that there are, of course, nowadays numerous empirical approaches to control for credit risk by approximating sustainable debt by empirical indicators.<sup>25</sup> Our attempt was, however, to show how one can compute sustainable debt based on a dynamic economic model without making reference to the numerous indicators for credit risk that rating companies refer to.

The empirical application of our model is to an open economy problem where a country borrows from abroad. To estimate our model with time series data we have managed it to transform the dynamic model into an empirical model so that it can be taken to the data. Given the parameter estimates we can, for actual economies, compute the borrowing capacity and debt ceiling of actual debtor countries. We have also shown that one can compute the sustainability of debt (net assets) for actual economies by using time series methods. This was undertaken for the core countries of Euroland. As it turns out the result for Euroland is that Euroland does not have external debt but rather owns net assets vis-a-vis the rest of the world (if anything it is the net foreign assets that have become, since the middle of 1990's,  $x$ , unsustainable). According to this computation the Euro as currency and its external value should be rather stable in the long run. There will be no perception of insolvency of Euroland by foreign asset holders or currency trades. Moreover, due to the large net foreign assets for Euroland there are sufficient foreign currency reserves that the European Central Bank could use in case of a currency run. Thus, there will be no danger of a currency crisis for Euroland.

---

<sup>25</sup>In a series of papers Milesi-Ferretti and Razin (1996, 1997) have addressed the empirical issue of how to obtain proxies for measuring sustainable debt.

## References

- [1] Barro, R.J., N.G. Mankiw and X. Sala-i-Martin (1995), "Capital Mobility in Neoclassical Models of Growth", *American Economic Review*, vol. 82, no1:103-116.
- [2] Bernanke, B. and M. Gertler (1989), "Agency Costs, Net Worth and Business Fluctuations." *American Economic Review*, vol. 79, pp. 14-31
- [3] Bernanke, B., and M. Gertler (1994), "The Financial Accelerator and the Flight to Quality". *Review of Economics and Statistics*, vol. 78, no. 1, pp. 1-15
- [4] Bernanke, B., Gertler, M. and S. Gilchrist (1998), "The Financial Accelerator in a Quantitative Business Cycle Framework", NBER working paper, no 6455, forthcoming in J. Taylor and M. Woodford (eds), *Handbook of Macroeconomics*, Amsterdam, North-Holland.
- [5] Benninga, S. (1998), "Financial Modeling", The MIT Press, Cambridge,
- [6] Bhandary, J. S., N. U. Haque and S. J. Turnovsky (1990), "Growth, External Debt, and Sovereign Risk in a Small Open Economy", *IMF Staff Papers*, vol. 37, no.2 International Monetary Fund.
- [7] Blanchard O.J. (1983) "Debt and Current Account Deficit in Brazil" in P.A. Armella, R. Dornbush and M. Obstfeld, eds., *Financial Policies and the World Capital Market: The Problem of Latin American Countries*, Chicago: University of Chicago Press, pp.187-197.
- [8] Blanchard, O.J. and S. Fischer (1989), *Lectures on Macroeconomics*, Cambridge: MIT Press.
- [9] Carlstrom, C.T. and T.S. Fuerst (1997), Agency Cost, Net Worth and Business Fluctuations: A Compatible General Equilibrium Analysis, *The American Economic Review*, December, pp: 893-910
- [10] Cohn. D. and J. D. Sachs (1986), "Growth and External Debt under Risk of Debt Repudiation", *European Economic Review*, vol.30, no 3:1138-54.
- [11] Cooley, T. and V. Quadrini (1998) "Monetary Policy and the Financial Decisions of Firms", University of Rochester, mimeo
- [12] Duffie, D. and J. Pan (1997), "An Overview of Value at Risk", *Journal of Derivatives*, vol. 4 (3): 7-49

- [13] Flood, R.P. and P.M. Garber (1980) "Market Fundamentals Versus Price Level Bubbles: The First Tests". *Journal of Political Economy*, 88: 745-770
- [14] Fleming, W. H. and R.W. Rishel (1975): *Deterministic and Stochastic Optimal Control*. Springer Berlin–Heidelberg–New York.
- [15] Greiner, A. and W. Semmler (1998): "An Inquiry into the Sustainability of German Fiscal Policy: Some Time-Series Tests", *Public Finance Review*, vol. 27, no. 2, March, pp: 220-236
- [16] Hamilton, J. and M.A. Flaven (1986) "On the Limitations of Government Borrowing: A Framework for Empirical Testing". *American Economic Review*, vol. 76, (4): 808-819
- [17] Krieger, S. (1999), "Bankruptcy Costs, Financial Constraints and the Business Cycle", Yale University, mimeo.
- [18] Krugman, P. (1999), "Balance Sheets, The Transfer Problem and Financial Crises", MIT, mimeo.
- [19] Lane, P. and G.M. Milesi-Ferretti (1999) "The External Wealth of Nations. Measures of Foreign Assets and Liabilities for Industrial and Developing Countries", IMF, mimeo.
- [20] Milesi-Ferretti, G.M. and A. Razin (1997), Sharp Reduction in Current Account Deficits: An Empirical Analysis. NBER working paper no 6310.
- [21] Milesi-Ferretti, G.M. and A. Razin (1996), Current Account Sustainability: Selected East Asian and Latin American Experiences. NBER working paper no 5791, Cambridge.
- [22] Miller, M. and J. Stiglitz (1999) "Bankruptcy Protection against Macroeconomic Shocks", mimeo, The Worldbank.
- [23] Mishkin, F.S. (1998) "International Capital Movements, Financial Volatility and Financial Instability", NBER working paper No. 6390, Cambridge.
- [24] Semmler, W. and M. Sieveking (1998) "The Use of Vector Field Analysis for Studying Debt Dynamics", paper presented at the 1998 North American Meeting of the Econometric Society, January 1998, Chicago, discussion paper Nr. 379, Dept. of Economics, University of Bielefeld

- [25] Semmler, W. and M. Sieveking (1999), "Critical Debt and Debt Dynamics", *Journal of Economic Dynamics and Control*, vol. 24, no 5-7, March 2000.
- [26] Sieveking, M. and W. Semmler (1998), "The Optimal Value of Consumption", Dept. of Economics, University of Bielefeld, mimeo
- [27] Sieveking, M. and W. Semmler (1999), "Credibility", Dept. of Mathematics, University of Frankfurt, mimeo.
- [28] Turnovsky, S. J. (1995), *Methods of Macroeconomic Dynamics*, Cambridge: MIT Press.
- [29] Wilcox, D.W. (1989) "The Sustainability of Government Deficits: Implications of the Present-Value Borrowing Constraint". *Journal of Money, Credit and Banking* 21 (3): 291-306.