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Empty Sources of Growth Accounting, and Empirical Replacements à la Kaldor with Some Beef

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Abstract Standard sources of growth accounts are empty of content because they rely on neoclassical production theory. Rather, analysis can be based on productivity growth equations derived either from NIPA accounting conventions or algebraic identities. These complementary schemes impose valid restrictions on growth rates of the wage rate, profit rate, capital, labor, and their respective average productivities. A Solow-type growth model based on proper accounting can be shown to converge. Detailed results differ markedly from those of the standard model. Alternative, essentially Kaldorian supply-and demand-based alternatives to sources of growth based on a familiar output growth vs. productivity growth diagram with constant employment growth contours added in look like a useful alternative to the mainstream models.

By now, it should be widely recognized that aggregate production functions have negligible empirical content. As Felipe and Fisher (2003) observe, "… the *relationship* $GDP = F(K, L)$ between aggregate output (*GDP*) and aggregate inputs (K, L) used in theoretical and applied macroeconomic work does not have, in general, a meaningful interpretation. This implies that the statement that there must be some connection between aggregate output and aggregate inputs, and that this is what the aggregate production function shows, has no theoretical basis." (p. 248, emphasis in original).

In this note, we take this point for granted, and illustrate it in terms of growth accounting. We use *accounting decompositions* from the national income and product accounts (NIPA) and

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algebraic identities to investigate how average labor and capital productivity growth rates change over time, along with convergence properties of growth models based on such indicators. As will be seen the empirical distinction between the two approaches tends to blur because of arbitrary assumptions that are required to compute the labor share, particularly in developing countries. However, the accounting decompositions can carry along distributive information which algebraic identities applied only to real quantities ignore. Where they overlap, both methods produce largely the same results.

As the paper is organized, section 1 reviews growth accounts based on NIPA decompositions while section 2 takes up algebraic identity accounting. Section 3 is devoted to a comparison of the two methods. Section 4 looks at sources of growth and section 5 analyzes convergence of an accounting-based version of the Solow (1956) growth model. Sections 6 and 7 present supply- and demand-based Kaldorian growth models which in our eyes are more realistic than the Solow formulation preferred by the mainstream.

1. NIPA-based Growth Accounting

We assume that real output or value-added (X) and capital stock (K) measures exist, the former constructed by double deflation of GDP from current value national income and product accounts, and the latter generated by perpetual inventory procedures from real gross fixed capital formation estimates in the NIPA. Employed labor (L) estimates are also assumed to be available.[1]

For present purposes, we also assume that real value-added (at factor cost) can be decomposed in the form

$$X = \omega L + rK \tag{1}$$

with ω as an index of the real wage and r the ex post rate of profit. In practice, decompositions of the form (1) can be difficult to construct from available data, especially in developing countries. Real value-added is ideally estimated by a double deflation technique which says nothing directly about wages and profits. In poor countries the dominant flow in value-added is "proprietors' incomes" (peasants, small informal urban enterprises) which does not naturally split into labor and capital components. In a country with a real GDP per capita on the order of a few thousand

[1] Felipe and Fisher (2003) argue that estimates of quantities like K and L can be quite problematical but for any sort of quantified macroeconomics it is impossible to work without them.

dollars the wage bill might be 20-30% of GDP and profits of “formal sector” enterprises 10-20% at most. Different procedures are used to allocate the remainder between labor and capital, without being entirely convincing.

Setting such data problems aside until section 3 and using continuous time for simplicity, let the growth rate “X-hat” of X be given by $\hat{X} = (dX/dt)/X = \dot{X}/X$, etc. We define average levels of labor and capital productivity as

$$\varepsilon_L = X/L \tag{2L}$$

and

$$\varepsilon_K = X/K \tag{3L}$$

with respective growth rates

$$\xi_L = \hat{\varepsilon}_L = \hat{X} - \hat{L} \tag{2G}$$

and

$$\xi_K = \hat{\varepsilon}_K = \hat{X} - \hat{K} \quad . \tag{3G}$$

The observed labor share of output at any time is $\psi = \omega L / X = \omega / \varepsilon_L$; the capital share is $1-\psi$. Broadly following Shaikh (1974), one can logarithmically differentiate (1) to get

$$\hat{X} = \psi(\hat{\omega} + \hat{L}) + (1-\psi)(\hat{r} + \hat{K}) \quad . \tag{4}$$

Let

$$\xi(\psi) = \hat{X} - \psi\hat{L} - (1-\psi)\hat{K} \tag{4G}$$

be total factor productivity growth (TFPG) as usually defined. Then (4) and (4G) show that

$$\xi(\psi) = \psi\hat{\omega} + (1-\psi)\hat{r} \quad . \tag{5}$$

At any time $\xi(\psi)$ is a flow of “surplus” (or real output growth) that must be split between growth of real wages and profits (recall the profit surge accompanying the speedup of productivity growth in the American economy in the late 1990s). One can also use (4) to show that

$$\xi(\psi) = \psi\xi_L + (1-\psi)\xi_K \tag{6}$$

so that TFPG is a weighted average of the growth rates of average labor and capital productivities. Subtracting (6) from (5) gives

$$\psi(\hat{\omega}-\xi_L)+(1-\psi)(\hat{r}-\xi_K)=0 \quad , \quad (7G)$$

a cost-side restriction on observed growth rates of average productivities which we make use of below.

Note that (7G) implies that

$$\xi_K-\hat{r}<0, \xi_L-\hat{\omega}>0 \quad \text{or} \quad \xi_K-\hat{r}>0, \xi_L-\hat{\omega}<0 \quad (8)$$

when the economy is not at a steady state with constant factor shares. One factor share must be rising while the other falls. But growth of a real factor payment cannot differ from its corresponding productivity growth rate forever. Either distributive growth cyclicality or endogenous cessation or reversal of the divergent trends in (8) is required from the accounting. Such restrictions are often ignored in growth theory.

2. Algebraic Identity Accounting

Now we drop real wage and profit rates for the moment to focus on time series for *X, K,* and *L.* The key point for this section is that these data provide three degrees of freedom. Levels and growth rates of average labor and capital productivity are defined above. We will be mainly interested in how these five level and growth rate variables interact in a three-dimensional data space. Although equations (2L-G) and (3L-G) as stated are definitions they can also be interpreted in a behavioral sense. For example, one may have a theory of labor productivity growth and assume that growth of output is driven by effective demand. Then from (2G) labor force growth will be a derived variable, $\hat{L}=\hat{X}-\xi_L$, even though time series on observed $\hat{L}$ and $\hat{X}$ must be used to infer ξ_L.

In what follows, we focus on *exact* relationships among the five variables. Insofar as, say, productivity indicators are derived from moving averages, etc. imposed on time series for *X, K,* and *L* then the system will have more degrees of freedom "created" by the filters. Similarly, discrete time data will not conform to equations such as (2G) and (3G) exactly. Inference based upon such artificially imposed noise doesn't make a lot of sense.

Consider the algebraic identity $K/L=K/L$. Dividing numerator and denominator on the left-hand side by $X \neq 0$ and substituting from (2L) and (3L) gives a level equation of the form

$$\varepsilon_L / \varepsilon_K = K / L \quad . \tag{9L}$$

The growth rate version

$$\xi_L - \xi_K = \hat{K} - \hat{L} \tag{9G}$$

follows directly.

Equations (9L) and (9G) impose one restriction on four variables. In a system with three degrees of freedom they basically say that all four variables *L, K,* ε_L, ε_K along with their four growth rates cannot vary independently.[2] Also, average productivity of labor will rise in comparison to average productivity of capital when the capital-labor ratio goes up.

Another purely algebraic identity is $\hat{X} = \alpha\hat{X} + (1-\alpha)\hat{X}$ with α "fixed at a point in time" but free to jump between them. For the discussion to follow it makes sense to assume that α lies between zero and one. Adding and subtracting $\hat{L}$ and $\hat{K}$ from the appropriate terms on the right-hand side and substituting from (2G) and (3G) gives

$$\hat{X} = \alpha(\hat{L} + \xi_L) + (1-\alpha)(\hat{K} + \xi_K) = \xi(\alpha) + \alpha\hat{L} + (1-\alpha)\hat{K} \tag{10}$$

with

$$\xi(\alpha) = \hat{X} - \alpha\hat{L} - (1-\alpha)\hat{K} \quad . \tag{11}$$

The parallel with (4G) is obvious, except that the former is derived from NIPA cost decompositions with ψ as the observed labor share while (11) follows from the algebraic identity (10) for an arbitrary α. The "residual" $\xi(\alpha)$ will fit the data for any value of α we care to choose, with the effects of any change in weighting factors α and $1-\alpha$ being just offset by a change in $\xi(\alpha)$. As discussed below, in the mainstream literature ξ_L and ξ_K are often called "factor-augmenting" rates of technical progress in the "sources of growth" equation (10) but in reality they are just artifacts of algebraic identity accounting.

3. Differences between ψ and α

As noted above, there is a real data problem posed by the fact that GDP (at factor cost) in practice splits into three components: labor remunerations, profits of incorporated enterprises,

[2] When *L, K,* ε_L, and ε_K satisfy (9L), X will follow from (2L) or (3L).

and incomes of the “self-employed,” “entrepreneurs,” “proprietors,” or some other equally ill-defined group. In his magisterial study of *Modern Economic Growth*, Kuznets (1966) was well aware of the problem. Gollin (2002) rather belatedly goes over some of the same ground. Some sort of imputation is needed if incomes of the self-employed are to be split between returns to “capital” and “labor” Both Kuznets and Gollin in effect suggest imputing average employee compensation to the self-employed labor force to get an overall labor share.[3]

There are at least two problems with any such correction as applied by the mainstream. One is that it can alter labor shares by a substantial amount. In a study discussed in more detail below, Young (1995) presents share data for four rapidly growing Asian economies in the period 1966-1990. Meanwhile, Soon and Ong (2001) give shares of labor remuneration in value-added at factor cost in the late 1990s. The numbers go as follows:

	Soon-Ong	Young
Hong Kong	0.52	0.63
Korea	0.47	0.70
Singapore	0.50	0.51
Taiwan	0.55	0.74

The remuneration shares were presumably lower (and not constant) in the period Young considers, so the corrections are large, and moreover subject to fairly arbitrary adjustments in their computation. It is hard to be convinced.

Second, Kuznets was careful to embed his imputations within the income-generation side of the national accounts. Mainstream studies rarely do that, preferring to focus only on equations like (4G). Without an appropriate cost-side correction redefining labor force and capital stock

[3] An alternative is to impute a rate of return to “entrepreneurial capital” and derive self-employment labor income as a residual. Kuznets does it both ways for a small sample of developed countries, and finds large differences in results but says the labor allocation makes more sense because “… the predominant majority of entrepreneurs are self-employed workers … and the major portion of their income is derived from labor” (p. 180). But the fact that both imputations give dissimilar results is disquieting: “….differences among countries and periods may well reflect genuine differences in the labor-property income composition of entrepreneurial income….” (p. 179).

growth to fit the imputations, just re-labeling some part of self-employment as “wages” is not very helpful.

The conclusion, perhaps, is that in setting up growth models one should work with the “observed” labor share ψ and carry along its distributive implications. But in work on empirical growth analysis, it is essential to recognize that estimating ψ is subject to potentially large errors. Over a range of tens of percentage points ψ resembles a rather arbitrarily specified α .

4. Sources of Growth

We can use the foregoing results to investigate sources of growth. In line with the foregoing discussion, we focus on equations (10)-(11) but much of what is said applies to (4G) and (6) as well. There is *no* presumption that an aggregate production function and associated marginal productivity conditions underlie either set of equations.

A first observation is relevant to questions of model causal structures or “closure.” For example the “Harrod-Domar” model of the 1960s and the “AK” model of the 1990s (which are essentially the same wine in different vintage bottles or goatskins as pointed out by Kurz and Salvadori, 1998) both set *K* and ε_K exogenously. As a consequence either employment or labor productivity has to be endogenous (endogeneity of the latter is of course characteristic of new growth theory). Similarly if *L* and ε_L are exogenous (or determined by additional equation(s) as discussed below) then either capital or capital productivity has to be endogenous.

Most sources of growth studies treat *L and K* as pre-determined in an econometric sense. They wrap ε_L and ε_K into $\varepsilon(\alpha)$ for some value of α and thereby try to apportion growth between factor accumulation and “technical progress.” A famous example is Young’s (1995) study of four East Asian “Dragons” which argued that their rates of TFPG were not high in comparison to other economies. The inference drawn was that their rapid output expansion was basically due to high rates of employment and capital stock growth, especially the latter. Young’s numbers are summarized in Table 1.

Table 1 here

Evidently such conclusions will sensitive to the choice of α. The "observed" labor share ψ is the usual selection but as discussed above there is a high degree of arbitrariness in assigning it a value. Are Young's numbers for α any more plausible than something 0.1 higher or lower? Why should they remain stable over a quarter-century of rapid structural change? The accounting in (10) and (11) works for *any* α so that the case for choosing a particular value is not compelling.

Moreover, Young's results are basically driven by (9G). All four dragons had quite respectable rates of labor productivity growth and their capital stock grew more rapidly than employment. As a consequence, their growth rates of capital productivity *had to be negative* as shown in the table. Their modest values of $\xi(\alpha)$ are a direct consequence of definitional identities as combined in (9G), 10, and (11) for the values of α presented in the table. Other values between zero and one would make $\xi(\alpha)$ either strongly positive or negative. A similar observation applies to the commonly held view that Asian growth has been "inefficient" because it is associated with falling average capital productivity. The purported inefficiency is the consequence of a "theorem of accounting," neither more nor less.

More generally, (9L) and (9G) capture the fundamental neoclassical notion that an increase in the capital-labor ratio "should be" associated with a rise in labor productivity in comparison to capital productivity. One is reminded of Becker's (1962) ancient observation that demand curves tend to have negative slopes because that is the slope of the budget constraint.

With regard to non-mainstream economics, if one were to write (9L) in the form $\varepsilon_L = \varepsilon_K (K/L)$ and *assume* that $\varepsilon_K = \bar{\varepsilon}_K (K/L)^{-\eta}$ with η between zero and one and $\bar{\varepsilon}_K$ as a scaling factor, then one gets

$$\varepsilon_L = \bar{\varepsilon}_K (K/L)^{1-\eta} \qquad (12)$$

which is basically Kaldor's (1957) "technical progress function." In effect, Kaldor was presenting a slightly disguised identity.

An immediate question is how well it fits the data. Arc-elasticity calculations for three countries in Table 1 suggest that η is around 0.45 or 0.5 with the fourth (Hong Kong) giving a value of 0.15. On the other hand, Table 2.8 in Foley and Michl (1999) presents growth rates for labor, capital, and their respective productivities for 20 selected periods in six countries since 1820 based on data from Maddison (1995).[4] Growth of the capital-labor ratio is positive in all periods. However, capital productivity growth is positive in eight of them, implying a negative η (or a positive trend in $\bar{\varepsilon}_K$). When $\xi_K < 0$, arc-elasticity values for η all lie below 0.5. Insofar as η can be viewed as a proxy for the exponent on labor from a Cobb-Douglas production function (a common neoclassical reinterpretation of Kaldor), it appears closer to the share of remunerations in GDP than to revised estimates of the labor share à la Kuznets, Gollin, and Young.

5. Convergence

Unsurprisingly, our identities also shed light on the steady state analysis that informed the "convergence debates" of the 1990s. The crux of this section is that convergence in savings-driven growth models à la Solow (1956) occurs when there is no aggregate production function – a fact that is well-known (Taylor, 2004) but ignored by the mainstream. Proper identity accounting (now relying on ψ as opposed to α) is all that is required. It gives results that differ from those from models with production functions.

The standard specification (Mankiw, 1995) incorporates a stock of "effective" labor which at time *t* can be expressed as $Z(t)L(t)$. Both *K* and $Z(t)L(t)$ are assumed to be fully employed (Say's law is enforced), determining the level of output and wage and profit rates from an aggregate production function $X = F(ZL, K)$ with the usual marginal productivity conditions. The augmentation factor *Z* and the labor force grow at exogenous rates.

Capital stock accumulation follows from national saving, set as a fraction of output. One can thereby set up a differential equation for $k = K / ZL$ which converges to a steady state under the usual assumptions. At the steady state the profit rate and the output-capital ratio ε_K are constant so that technical progress is Harrod-neutral.

[4] Needless to say, the numbers satisfy (9G).

The Felipe-Fisher critique assures us that this model makes no sense. What we can do is use our NIPA-based accounting conventions to replace it with something empirically defensible. Because there is no other source, in practice the time trend of the augmentation factor *Z* must be inferred from the observed growth rate of average labor productivity ξ_L. So in line with our emphasis on accounting identities we tentatively set $Z = f(\varepsilon_L)$ and proceed. It will be shown below that only a function *f* with a unit elasticity can be consistent with a steady state. In other words *f* has to be a simple proportionality relationship.

Under typical Say's Law assumptions, capital accumulation is driven by available saving in a closed economy,

$$\dot{K} = sX - \delta K \qquad (13)$$

or

$$\hat{K} = s\varepsilon_K - \delta \qquad (14)$$

with *s* as the national rate of saving and δ the depreciation rate.

Let *k* be capital per unit of effective labor, $k = K / f(\varepsilon_L)L$. The growth rate of effective labor is $\phi\xi_L + \hat{L}$ with ϕ being the elasticity of *f* so that we get

$$\dot{k} = k(\hat{K} - \phi\xi_L - \hat{L})$$

in which ξ_L and $\hat{L}$ are exogenous (we are getting close to the limit on degrees of freedom imposed by (9G)). Substitution shows that

$$\dot{k} = s\frac{\varepsilon_L}{f(\varepsilon_L)} - (\delta + \phi\xi_L + \hat{L})k$$

In this expression ε_L changes over time at the postulated growth rate ξ_L. The only way to avoid a term with a time trend multiplying *s* (thereby ruling out the existence of a steady state) is to set $f(\varepsilon_L) = A\varepsilon_L$ with *A* constant. Similarly, we must have $\phi = 1$ if the "pure" productivity growth indicator ξ_L is desired on the right-hand side. The growth equation boils down to

$$\dot{k} = (s / A) - (\delta + \xi_L + \hat{L})k \quad . \qquad (15)$$

which as shown in Figure 1 reliably converges to a steady state value of *k* because

$$d\dot{k} / dk = -(\delta + \xi_L + \hat{L}) < 0. \tag{16}$$

Figure 1 here

Moreover, $k = K / f(\varepsilon_L)L = K / A\varepsilon_L L = K / A(X / L)L = K / AX$ so that the steady state expression emerging from (15) with $\dot{k} = 0$ takes the familiar form

$$s\varepsilon_K = \delta + \xi_L + \hat{L} \tag{17}$$

The bottom line is that with a specification of labor productivity growth consistent with the data, a savings-driven economy will converge to a steady state described by (17), with no aggregate production function required to support the dynamics. The speed of convergence is faster than in the usual story because the term on the right-hand side of (16) is not multiplied by the labor share.

With regard to employment and output, we have $k = 1 / A\varepsilon_K$ so that (17) becomes

$$s / Ak = \delta + \xi_L + \hat{L} \quad .$$

An increase in *s* at steady state means that *k* has to rise in proportion. The capital/effective labor and output/effective labor ratios increase or the economy becomes richer. The relevant elasticity is one, higher than the typical value of one-half coming from a neoclassical specification (Romer, 2001).

In sum, Solow-style growth accounting can be made consistent with accounting identities and under appropriate assumptions there is convergence to a steady state. However, the detailed results differ from the standard pattern and have nothing to do with the properties of an aggregate production function.

Turning to distribution, as mentioned above the neoclassical model has constant values of *r* and ε_K at the steady state. If *k* is initially "low" then ω will be low as well while *r* will be high. As the model converges, (7G) will be satisfied with $\hat{\omega} - \xi_L > 0$ and $\hat{r} - \xi_K < 0$ until a steady state with $\hat{\omega} = \xi_L$ and $\hat{r} = \xi_K = 0$ is attained. This is an example of endogenous trend cessation as discussed above.

What happens in the looser specification of this section? From the arguments above, we have $\xi_K = -\hat{k} < 0$ while k is rising. We can thereby restate (7G) as

$$\psi(\hat{\omega} - \xi_L) + (1-\psi)(\hat{r} + \hat{k}) = 0 \quad . \tag{18}$$

This equation constrains any theory of distribution we choose to apply to the system. If the real wage is initially low we certainly need $\hat{r} < \hat{k}$ if there is to be any chance for convergence to a steady state. A falling rate of profit as in the standard specification may not be required but r certainly cannot grow "too fast." Moreover, if one had independent theories of $\hat{\omega}$ and $\hat{r}$ then ξ_L would have to be endogenous in (18). Outside the highly stylized neoclassical framework, the functional income distribution in growth models has been scarcely explored.

6. Supply-Driven Models

The favored empirical approach to economic growth treats employment as predetermined and derives capital accumulation from either an aggregate saving equation like (13) or a Ramsey-style dynamic optimization exercise (which we omit). The existence of an aggregate production function is taken for granted. The foregoing arguments suggest that this line of analysis leads nowhere.

Nevertheless, supply-based studies of growth are likely to remain of interest, not in a never-never land of steady states but rather over periods of a few decades as in Young's work and Table 1. Here we suggest two specifications to pursue, respectively incorporating Kaldor's models of technical change circa late 1950s and late 1960s.

The first starts out from the Harrod-Domar-AK model mentioned above, with predetermined $\hat{L}$ (and thereby L at any point in time) and accumulation equation (14). Instead of holding the output/capital ratio ε_K constant, however, we may just as well follow Kaldor (1957) in setting $\varepsilon_K = \bar{\varepsilon}_K (K/L)^{-\eta}$ so that "in the medium run" labor productivity comes from (12). Our rough-and-ready calculations reported above suggest that this may be a useful approximation – at least when capital productivity is not trending upward!

However, it does have at least one drawback, shared by the traditional production function. For a given rate of accumulation, a higher labor force growth rate means that K/L will rise more slowly, holding down productivity. In other words, an economy with a more rapidly growing population will be poorer. But then *negative* employment growth should work wonders! Because it is hard to find anyone who believes that a shrinking, aging population in, say, Japan will have such beneficial effects a model which makes them a central plank should perhaps be taken with a grain of salt.[5]

The latter-day Kaldorian approach can be seen as giving the labor productivity equation (2G) pride of place in combination with a technical progress function of the form proposed by Verdoorn (1949) and Okun (1962),

$$\xi_L = \bar{\xi}_L + \gamma\hat{X} \tag{19}$$

in which the productivity trend term $\bar{\xi}_L$ could be affected by human capital growth, industrial policy, international openness, population growth, and other factors. Given $\hat{L}$, $\hat{K}$ from (8), and ξ_L from (10), the growth of capital productivity ξ_K follows from (9G).

This model can be analyzed in terms of Figure 2, sketched verbally but not actually drawn by Kaldor in his 1966 Inaugural Lecture (published in Kaldor, 1978). To the traditional diagram we have added "Employment growth contours" with slopes of 45 degrees.[6] Each one shows combinations of the output growth rate ($\hat{X}$) and labor productivity growth rate (ξ_L) that hold the employment growth rate ($\hat{L} = \hat{X} - \xi_L$) constant. Employment growth is more rapid along contours further to the SE. As will be seen, the contours can be blended with the other schedules in various ways.

Figure 2 here

The one of interest for now combines a "Kaldor-Verdoorn" schedule representing (19) with a predetermined employment growth rate along one of the contours, as at point A, with $\hat{X}$ determined endogenously (ignore the "Output growth" schedule for the moment). If employment

[5] Of course one could always get around the problem by making ε_L an increasing function of both K/L and $\hat{L}$, but this begins to look like adding epicycles.

[6] The 45-degree slope comes from the absence of a "relative price term" between the real indexes X and L in (1L).

growth were faster, say along the contour passing through point B, then $\hat{X}$ would increase as well.

By how much will $\hat{X}$ go up in response to a higher $\hat{L}$? With $0 < \gamma < 1$, the Kaldor-Verdoorn curve has a shallower slope than the employment growth contour, cutting it from above. An upward shift in $\bar{\xi}_L$ means that $\hat{X}$ and ξ_L rise by the same amount. Faster employment growth requires a move to a contour further SE and so both $\hat{X}$ and ξ_L rise, the former by a larger amount so that $\hat{L}$ can in fact accelerate.[7] But growth in output per worker does *increase* as a function of $\hat{L}$, just reversing the Japan story mentioned above (along with myriad papers in the economic demography literature saying that the way to raise per capita income growth is to reduce population growth).

Finally, the models just discussed suggest that plots over time of labor productivity vs. the capital-labor ratio or output will show the former increasing, seemingly as a function of the latter two variables. As Foley and Michl (1999) point out, such plots in reality show "fossil production functions" along which the "…history of past techniques appears to trace out a production function, but in fact is just the fossil record of past technology" (p. 124). It is always wise to bear such cautions in mind when applying modeling tricks to numbers like those in Table 1.

7. A Demand-Driven Growth Model

For many if not most developing and transition economies in the recent period, a predetermined "full employment" labor force does not make a lot of sense. This observation suggests another use of Figure 2. We can combine Kaldor-Verdoorn with an Output growth schedule which makes $\hat{X}$ depend on ξ_L, letting employment growth be determined along one of its contour lines as at point D. (One could also think of combining Output growth with a given level of $\hat{L}$, dropping the Kaldor-Verdoorn schedule and making ξ_L "endogenous" as at point C. This specification mimics much New Growth Theory but for brevity we do not elaborate here.)

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[7] In algebraic terms the system solves as $\hat{X} = (\bar{\xi}_L + \hat{L})/(1-\gamma)$ and $\xi_L = (\bar{\xi}_L + \gamma\hat{L})/(1-\gamma)$.

In a developing country context, one might reasonably take effective demand or available foreign resources as binding restrictions on $\hat{X}$. With such a growth rate closure, effects on employment of shifts in the two schedules become of interest. The employment growth rate is higher for combinations of $\hat{X}$ and ξ_L values lying below the contour running through D than at the point itself, and lower for combinations above. As discussed below, faster overall productivity growth in the sense of an upward shift of the Kaldor-Verdoorn schedule could reduce $\hat{L}$ due to "labor-shedding." This case is illustrated in Figure 2, with its relatively steep Output growth schedule which means that $\hat{X}$ is *in*sensitive to ξ_L. An outward shift in the Output growth curve (for example, due to more rapidly growing aggregate demand and/or more availability of foreign exchange) would speed up job creation.

Insofar as increased employment growth is a policy objective, it may or may not transpire depending on how the schedules shift. Evidence reported in Taylor (2001, 2005) and elsewhere suggests that external liberalization in many developing countries in the 1980s and 1990s was associated with faster productivity than demand growth (especially in traded goods sectors), leading to reductions in $\hat{L}$.

So how does one model the effects of labor productivity growth on aggregate demand?[8] An illustrative specification focuses on changes of the *observed* labor share ψ .In other contexts, some other distributive variable may be more important than ψ but the principles underlying alternative specifications would be the same as those utilized here.[9]

If we start with an aggregate demand equation of the form $X = C + I + E$ with the new symbols taking their usual meanings, and assume that $C = [1 - s(\psi)]X$, then we have

[8] External constraints can be modeled in a "gap" model framework (Taylor, 1994), taking into account foreign aid, capital movements, and shifts in the terms of trade. Using gap-based and other counterfactual methodologies, Taylor and Rada (2003) show that output growth rates in the late 20th century in sub-Saharan Africa and Latin America might have been substantially higher if the debt crisis and adverse terms-of-trade shocks "had not happened."

[9] For example, in Russia the relative price of energy is of crucial importance, while in other countries an increase in the agricultural terms of trade may stimulate overall demand via income effects (as in Turkey with its landed peasantry) or hold it down (as in India with its large proportion of impoverished landless laborers from whom food is the predominant component of demand). Both these relative prices will be influenced by sectoral rates of productivity growth.

$$X = (I + E) / s(\psi) \quad . \tag{20}$$

As in Kaldor (1957) a higher labor share will reduce the overall savings rate *s* if saving rates from profit incomes exceed those from wages (an empirical truism).

Evidently,

$$\hat{X} = \lambda \hat{I} + (1-\lambda)\hat{E} - \hat{s} \tag{21}$$

with $\lambda = I/(I+E)$.

It is reasonable to postulate that investment demand is stimulated by faster output growth and held back by falling profitability if ψ goes up:

$$\hat{I} = \hat{I}_0 + \phi_X \hat{X} - \phi_\psi \hat{\psi} \tag{22}$$

with $\hat{I}_0$ as a trend rate of growth of investment demand and both ϕ_X and ϕ_ψ being positive.

Omitting a trend term for simplicity, export sales may be cut back by higher domestic demand as well as by higher unit labor costs,

$$\hat{E} = -\theta_X \hat{X} - \theta_\psi \hat{\psi} \quad . \tag{23}$$

For the reasons mentioned above the saving rate decreases with the labor share,

$$\hat{s} = -\sigma\hat{\psi} \quad . \tag{24}$$

The labor share itself changes in response to trend growth in the real wage $\hat{\omega}$ and labor productivity growth.

Combining equations (20)-(24) gives a reduced form expression for $\hat{X}$,

$$\hat{X} = \frac{\hat{I}_0}{1-\lambda\phi_x + (1-\lambda)\theta_x} + \frac{\lambda\phi_\psi + (1-\lambda)\theta_\psi - \sigma}{1-\lambda\phi_X + (1-\lambda)\theta_X}(\xi_L - \hat{\omega}) = \chi\hat{I}_0 + \beta(\xi_L - \hat{\omega}) \quad . \tag{25}$$

As discussed above, in Figure 2 (25) is an Output growth equation which can be crossed with the Kaldor-Verdoorn equation (19) to determine $\hat{X}$, ξ_L, and (along a labor contour) $\hat{L}$. Note that the complicated coefficient β can exceed or be less than one. It will be large when investment and exports respond strongly and saving is insensitive to changes in $\hat{\psi}$,and investment is strongly crowded in and exports are not strongly crowded out by $\hat{X}$. These are basically conditions for aggregate demand to be "profit-led."[10]

Detailed comparative dynamic results when (19) and (25) are solved together are:

$$\hat{X} = \frac{\chi\hat{I}_0 + \beta(\bar{\xi}_L - \hat{\omega})}{1-\beta\gamma} \quad , \tag{26}$$

$$\xi_L = \frac{\gamma\chi\hat{I}_0 + \bar{\xi}_L - \beta\gamma\hat{\omega}}{1-\beta\gamma} \quad , \tag{27}$$

and

$$\hat{L} = \hat{X} - \xi_L = \frac{(1-\gamma)\chi\hat{I}_0 + (\beta-1)\bar{\xi}_L - (1-\gamma)\beta\hat{\omega}}{1-\beta\gamma} \quad . \tag{28}$$

We have already assumed that $\gamma < 1$ and for the Output growth and Kaldor-Verdoorn curves to cross in the stable configuration of Figure 2 it must be true that $1-\beta\gamma > 0$. Faster trend growth of investment increases all three growth rates.[11] By reducing profitability faster real wage growth makes them fall. The effect of a faster trend rate of productivity growth $\bar{\xi}_L$ is ambiguous. From (28), it only stimulates employment growth when $\beta > 1$ or the economy is profit-led. In terms of Figure 2, $d\xi_L / d\hat{X} = 1/\beta < 1$ along the output growth schedule so that it is less steep than the employment growth contours. Because in practice many developing countries appear to have wage-led effective demand (Taylor, 2004), this result can be problematical in terms of providing enough jobs to satisfy potential labor force growth.

More generally, looking at the institutional and historical forces underlying shifts in these curves can be an illuminating method for studying medium-term growth. A long-run caveat is that eventually real wage growth is likely to keep up with labor productivity growth so that the term

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[10] See Taylor (2004) for more details. The inner workings of Kaldorian growth models very often hinge on whether effective demand is wage- or profit-led.

[11] Faster trend growth of exports and slower trend growth of saving would have the same effects.

$\xi_L - \hat{\omega}$ in (17) vanishes and $\hat{X}$ will not be affected by productivity increases, but over periods of a decade or three this reservation is not likely to be relevant.[12]

8. Conclusions

In summary:

Standard sources of growth accounting is empty of content because it depends upon neoclassical production theory. Rather, growth analysis must be based on productivity equations that can be derived either from NIPA accounting conventions or algebraic identities. These complementary schemes do impose valid restrictions on growth of the wage rate, profit rate, capital, labor, and their respective average productivities.

A Solow-type growth model based on proper accounting can be shown to converge. But the detailed results differ markedly from those of the standard model.

Alternative, essentially Kaldorian supply-and demand-based alternatives to sources of growth based on a familiar output growth vs. productivity growth diagram with constant employment growth contours added in look like a useful alternative to the mainstream models.

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[12] Convergence dynamics may well be of interest in themselves, since in light of (7G) they are likely to be cyclical, especially when ψ affects both demand injections like investment and exports as well as saving and import leakages.

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Table 1: Output, Factor Input, and Productivity Growth Rates in East Asia, 1966-1990

(%/year)

Country

	Hong Kong	Singapore	South Korea	Taiwan
$\hat{X}$	7.3	8.7	10.3	8.9
$\hat{K}$	8.0	11.5	13.7	12.3
$\hat{L}$	3.2	5.7	6.4	4.9
ξ_K	-0.7	-2.8	-3.4	-3.4
ξ_L	4.1	3.0	3.9	4.0
$\xi(\alpha)$	2.3	0.2	1.7	2.1
Memo item (%)				
α	62.8	50.9	70.3	74.3

Source: Young (1995)

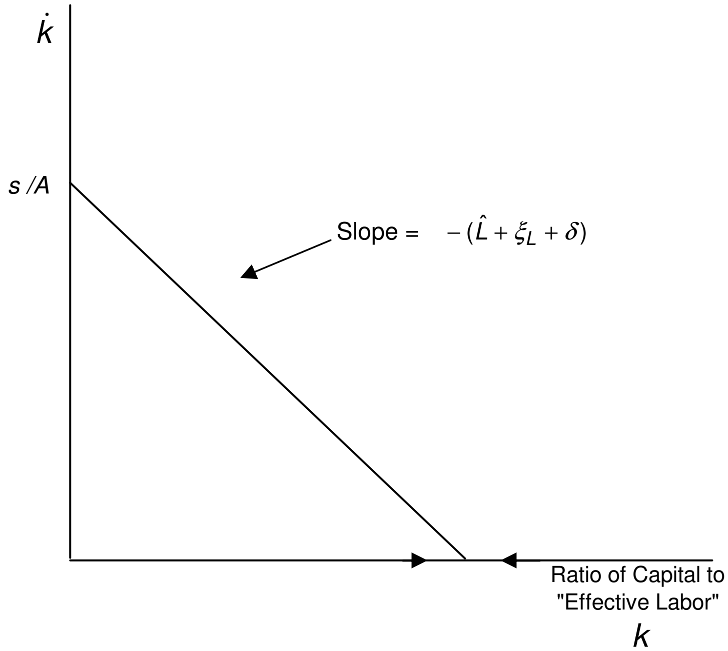

Figure 1: Convergence in a Solow-type model with Harrod-neutral technical progress

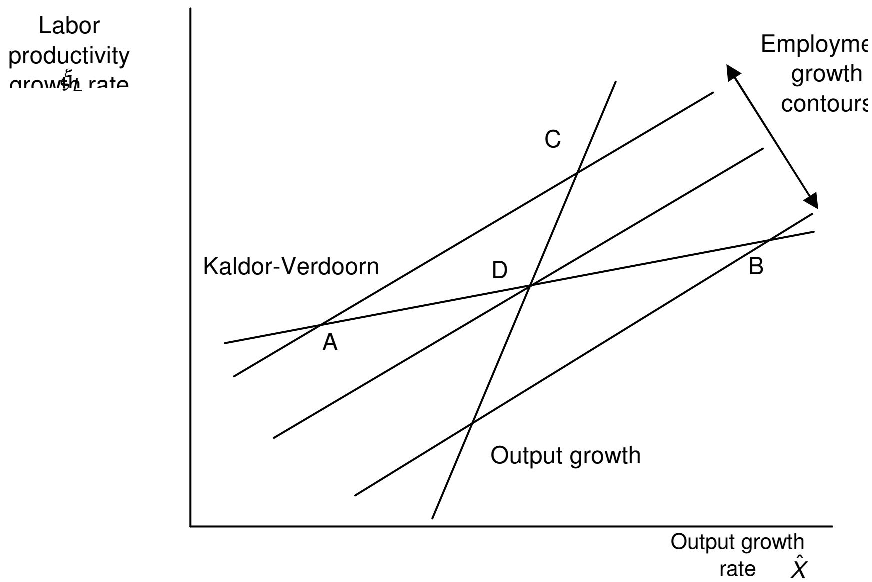

Figure 2: Joint determination of output, labor productivity, and employment growth rates