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A NOTE ON THE THEORY OF DEMAND-LED GROWTH

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Abstract: this paper presents a demand-led growth model where an exogenous investment function drives capital accumulation through a Bernoulli differential equation. In such framework investment generates savings through changes in capacity utilization and/or income distribution, making economic growth totally demand-led. Taking a Structuralist perspective, the model is purposefully made to be consistent with different Keynesian closures for the investment function, as well as with different assumptions about savings' adjustment to investment.

Keywords: Effective Demand, Growth, Keynesian

JEL classification: E1 and E12

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1 – Introduction

Mainstream growth models usually follow Say's Law and, accordingly, emphasize the supply side of income growth through some sort of growth accounting. Assumptions about technological progress, returns to scale, and level of aggregation vary across models but, in general, mainstream authors share the Neoclassical view that capitalist economies tend to fully utilize all factors of production in the long run. In such framework there is no fundamental role for aggregate demand since, from the start, it is assumed that savings generate investment (1). In contrast to these savings-driven models, Keynesian models usually follow the principle of effective demand and, therefore, emphasize sources of aggregate demand rather than input combinations necessary to attend aggregate demand. Hence, in Keynesian models growth is a demand-led process where investment generates savings through changes in capacity utilization and income distribution.

In line with the principle of effective demand and building upon the Structuralist approach of Taylor (1991), this paper presents a one-good model of investment-driven growth to illustrate how autonomous expenditures can generate income growth, as well as how investment can generate savings at the macroeconomic level. In the next section I present the basic structure of the model and show how an autonomous investment function sets the long run rate of growth under the assumptions usually made in Keynesian models. The two routes of savings' adjustment to investment are discussed in the third section, where I also make some comments on the Keynesian arguments usually associated with each type of adjustment. The fourth section contains a brief description of the impact of capacity utilization and income distribution on the rate of profit and, in the last section, I conclude the analysis with some Structuralist comments on the results of this paper.

2 – A model of demand-led growth

To simplify the analysis I will consider a one-good closed economy without government (2). Assuming that there is no depreciation we can obtain the following Harrod-Domar equation from the accounting identity between income and expenditure:

(1) $g_K = su/v$

where g_K is the growth rate of capital, s is the average propensity to save, u is the rate of capacity utilization and v is the potential capital output ratio (3).

According to the Principle of Effective Demand causality runs from the left to the right in equation 1, that is, g_K is the independent variable in the Harrod-Domar equation. Since the potential capital-output ratio is usually a constant parameter in Keynesian models, any adjustment of savings to investment has to come through changes in capacity utilization and/or income distribution in these models (4). Following this technological assumption and totally differentiating equation 1 in relation to time, we obtain the following description of the dynamics of capital accumulation:

(2) $\dot{g}_K = \frac{1}{v}\left(\dot{s}u + s\dot{u}\right)$

To obtain the time path of capital accumulation we have to include the growth rate of capital in the right-hand side of equation 2. Such substitution can be easily done since by definition the growth rate of income and the change in capacity utilization can be expressed as follows:

(3) $g_Y = g_I - g_S$

(4) $\dot{u} = u(g_Y - g_K)$

where g_I is the growth rate of investment, g_S is the growth rate of the average propensity to save, and g_Y is the growth rate of income (5).

Substituting 3 into 4 and then the result into 2 we observe that:

(5) $\dot{g}_K = \frac{1}{v}\left[\dot{s}u + su(g_I - g_S - g_K)\right]$

Now note that, by definition, $\dot{s} = sg_S$ and $g_K = su/v$. Thus, equation 5 can be rewritten as:

(6) $\dot{g}_K = g_I g_K - g_K^2$

Equation 6 is a Bernoulli differential equation in g_K and, consequently, it describes the dynamics of capital accumulation solely in terms of some given initial conditions and the growth rate of investment (6). In fact, the particular route through which investment generates savings has no direct impact on the mechanics of equation 6. The pace of capital accumulation is fully determined by the exogenous behavior of investment, that is, it is totally demand-led.

Translating equation 6 into a phase diagram, it is straightforward that g_K is stable at zero and g_I. Thus, if we assume a positive growth rate of investment, then g_I is the stable root of this Bernoulli equation, as shown in figure 1 (7).

Figure 1: phase diagram of g_K when $g_I > 0$.

Solving equation 6 we obtain the following time path for g_K:

(8) $g_{K,t} = \dfrac{g_I}{1 + g_I C e^{-g_I t}}$

where C is the constant of integration. Setting t=0 we obtain C in terms of some given initial conditions:

(9) $C = \dfrac{1}{g_{K,0}} - \dfrac{1}{g_{I,0}}$

Thus, provided that $g_{K,0}$ and $g_{I,0}$ are both greater than zero, the growth rate of capital will converge to the exogenous growth rate of investment (8). The difference between $g_{K,0}$ and $g_{I,0}$ determines whether $g_{K,t}$ converges to $g_{I,0}$ from above or below, as shown in figure 2.

Figure 2: time path of g_K when $g_{K,0}$ and $g_{I,0}$ are greater than zero.

Note that, given a permanent change of g_I in figure 2, g_K will tend to the new value of g_I in the long run, provided that this new value is not negative. If the new g_I turns out to be negative, then zero becomes the stable root of the Bernoulli equation and, consequently, the economy tends to stagnate in the long run.

From the previous paragraphs we can see that equation 6 gives us a general Keynesian description of the dynamics of capital accumulation. In other words, equation 6 is a direct result of two traditional Keynesian assumptions, a Leontief production function and an exogenous investment function, and it gives us a flexible and dynamic description of the principle of effective demand. Nevertheless, in order to have a complete theory of demand-led growth, it is necessary to complement equation 6 with a Keynesian theory of investment (or autonomous expenditures). For instance, if we take Minsky's (1982) perspective and define g_I as a result of "animal spirits" and "financial fragility", we obtain a financial theory of demand-led growth. Alternatively, if we take the perspective of Bowles, Gordon, and Weisskopf (1986) and make g_I a technological and institutional component of a "Social Structure of Accumulation", we obtain a Structuralist theory of demand-led growth. Hence, the principle of effective demand is consistent not only with different Keynesian closures to equation 6, but also with different hypothesis about how investment generates savings, to which we now turn.

3 – Capacity utilization, income distribution and savings

In the Keynesian approach to demand-led growth one can find two extreme positions regarding how investment generates savings in the long run, in the words of Garegnani (1992, pp.48-9).

> "(…) savings can be generated by investment along two entirely different routes, and it is the route which is being postulated that separates the two Keynesian positions. The first route is by lowering the real wage thereby raising the normal rate of profits and, other things being equal, the proportion of profits in national income. The second route (…) is by raising the level of output together with the corresponding productive capacity, without any need to change the real wage and the normal rate of profits."

The first Keynesian route is usually associated with the demand theory of income distribution of Kaldor (1955/56), and it stems from the Kaleckian assumption that the propensity to save out of profits is greater than the propensity to save out of wages. In terms of the Harrod-Domar equation presented earlier, the Kaldorian argument can be summarized in one proposition, namely: the profit share of income increases (decreases) whenever capacity utilization is above (below) its "natural" long-run level (9). This natural level of capacity utilization is usually defined in terms of profit optimization by firms but, in fact, it is sufficient to define it tautologically as the rate of capacity utilization at which income distribution is stable (10). If such natural level exists and is stable, and if the "distributive" assumption mentioned earlier holds, then investment will generate savings only through changes in income distribution in the long run.

Assuming that the growth rate of investment is not affected by changes in capacity utilization and/or income distribution, let us see how the first Keynesian position can be illustrated from the results of section 2. Restricting the analysis to positive values of g_K and g_I, it is straightforward that, if the rate of capacity utilization does not change, then the growth rate of the propensity to save corresponds to the difference between g_I and g_K. Thus, in terms of a graph of g_K against *s*, the first Keynesian position implies that the economy tends to the horizontal line given by g_I in the long run, as shown in figure 3.

Figure 3: dynamics of g_K and *s* outside the long-run equilibrium according to the first Keynesian position.

Given the natural level of capacity utilization *u**, it is straightforward that the long-run equilibrium is on the intersection of the horizontal line given by g_I and the Harrod-Domar line given by *u** and *v*, as shown in figure 4 below. In other words, given an exogenous growth rate of investment, the average propensity to save associated with any particular long-run position follows residually from the slope of the Harrod-Domar line.

Figure 4: long-run position and the Harrod-Domar line according to the first Keynesian position.

According to the first Keynesian position a change in the growth rate of investment moves the economy along the Harrod-Domar line of figure 4 in the long run, but nothing prevents a temporary change in capacity utilization in the short run. To illustrate this, consider a permanent increase in the growth rate of investment in a graph of g_K against *s*, as depicted in figure 5. If the short-run adjustment of savings comes solely through an increase in capacity utilization, then the economy moves from point A to B, without any change in income distribution. As the higher level of economic activity leads to an increase in the income-share of profits, the average propensity to save increases and the economy moves from point B to point C, returning to the normal level of capacity utilization (11).

Figure 5: short and long-run adjustment of savings according to the first Keynesian position

The dynamics of income growth can be inferred from the behavior of capacity utilization in figure 5, that is:

(i) in the first phase (from point A to point B) income grows at the same rate as investment and, consequently, faster than capital; and

(ii) in the second phase (from point B to point C) the fall in the average propensity to save makes income grow slower than investment and, therefore, slower than capital.

Turning to the second Keynesian route, the basic idea is that capitalist economies do not need to revert to a natural level of capacity utilization in the long run. According to this position income distribution is the invariant factor in the long run, that is, technology and social conflict determine income distribution in the long run, with no direct influence coming from the level of economic activity. The basic inspiration for this interpretation comes from Sraffa's (1960) analysis, where the determination of relative prices is shown to be analytically separable from the determination of quantities in the long run position of capitalist economies (12). To illustrate this in terms of the model of section 2, note that, if *s* is constant, then both *u* and g_K increase whenever g_I is greater than g_K. Thus, in terms of a graph of g_K against *u*, the second Keynesian position implies that the economy tends to the horizontal line given by g_I in the long run, as shown in figure 6.

Figure 6: dynamics of g_K and u outside the long-run equilibrium according to the second Keynesian position.

As we did in the previous case, assume that the growth rate of investment is not affected by changes in capacity utilization and income distribution. If both the real wage and the potential capital-output ratio are constants (a stable Harrod-Domar line), then the long-run adjustment of savings to investment comes from an increase in the level of economic activity, provided that the increase in g_I does not make the economy hit maximum capacity utilization, as shown in figure 7.

Figure 7: long-run adjustment of savings according to the second Keynesian position.

Note that, in the same vein that the first Keynesian position does not rule out temporary changes in capacity utilization, the second Keynesian position does not rule out temporary changes in income distribution. In terms of figure 7, this means that the

Harrod-Domar line can rotate in any direction in the short run, but it will ultimately return to the long-run slope given by technology and social conflict.

4 – Capacity utilization, income distribution, and the rate of profit

In the previous section we analyzed the adjustment of savings to investment without any explicit mention of how changes in capacity utilization and income distribution affect the rate of profit. Our discussion of income distribution was focused on the profit share of income, which is one component of the rate of profit. To see this note that, by definition, the rate of profit on fixed capital r can be expressed as:

$$(10)\quad r = \frac{\pi u}{v}$$

where π is the profit share of income.

From equation 10 we see that the adjustment of savings to investment can result in a change in the rate of profit, provided that the changes in capacity utilization and income distribution do not cancel each other. Hence, according to the two Keynesian positions analyzed earlier, the adjustment of savings comes either through π or u and, consequently, an increase (decrease) in the growth rate of investment leads to an increase (decrease) in the rate of profit. In short, the rate of profit on fixed capital is demand determined according to both Keynesian positions (13).

The previous result is consistent with the demand theory of income distribution of the first Keynesian position, but it does not fit so well into the “classical” theory of income distribution of the second Keynesian position. Hence, if capacity utilization is demand-determined in the long run, the same holds for the rate of profit on fixed capital. A stable profit share of income does not necessarily mean a stable rate of profit and, consequently, the effective rate of profit can be permanently different from the normal rate of profit derived from the "classical" core (technology and social conflict) (14).

The effects of capacity utilization and income distribution on the effective rate of profit call our attention to the possible feedback of savings' adjustment into the growth rate of investment. Hence, if the latter is positively affected by the effective rate of profit, then a positive (negative) demand shock can result in an explosive increase (decrease) in the pace of capital accumulation similar to Harrod's (1939) cumulative deviations from "warranted growth". In fact, I adopted a completely exogenous investment function in the previous sections not only to emphasize the leading role of effective demand, but also to exclude unstable dynamics from the analysis. An explosive and rigid accelerator does not seem to be a good description of the dynamics of real capitalist economies and, consequently, the assumption of an exogenous investment function was purposefully made in order to leave the model of section 2 open to different qualitative analysis of investment decisions.

5 – Conclusion

From the previous sections we can conclude that based on the Keynesian assumptions of a Leontief production function and an exogenous investment function, one can obtain a well-defined representation of demand-led growth. However, the two Keynesian positions regarding savings' adjustment to investment reflect extreme assumptions about the cyclical behavior of income distribution and, consequently, there is no reason to expect a capitalist economy to behave according to either one of these two ideal cases. Taking a Structuralist perspective, the model of section two was purposefully constructed to be consistent with different closures for the investment function, and the analysis of section three was oriented to show the alternative routes through which investment can generate savings. Since institutions and technology have a great influence on the form of the investment function and the adjustment of savings, these two features should not be expected to be invariant across countries or through time. Hence, the principle of effective demand gives us not a closed theory of demand-led growth but only the direction of causality between expenditure and income which, nevertheless, allows Keynesian economists to extend growth analysis to areas ignored by the supply-side perspective of mainstream models.

Notes:

1. For a presentation of mainstream growth theory see the textbook of Barro and Sala-i-Martin (1995). A Structuralist analysis of mainstream and non-mainstream growth theories can be found in Taylor (1996).
2. Including net exports and government expenditures does not alter the dynamics of capital accumulation. An extension to the case with other autonomous expenditures besides investment is presented in the appendix.
3. Let Y = income, Y* = potential income, C = consumption, I= investment, and K = capital. If we define C=cY, s=(1-c), u=Y/Y*, v=(K/Y*), and I=ΔK, then equation 1 follows directly from Y=C+I.
4. Keynesian models are usually based on a Leontief production function, what means that firms can accumulate excess capital. As pointed out by Dutt (1990, p.12), "(…) the asymmetry that allows excess capital to exist, but the labor output ratio to be fixed technologically, results from the fact that labor is hired (…) and capital is not." Note also that, in terms of equation 1, a constant v means that u has a maximum at $1/v$.
5. Equations 3 and 4 come from the fact that Y= (1/s)I and u = v(Y/K).
6. In the general case g_I represents the growth rate of any autonomous expenditure (autonomous consumption, exports, government final expenditures, etc.). However, as long as the determination of g_I remains exogenous to g_K, the dynamics of capital accumulation are not significantly different from the simple case illustrated by equation 6. An alternative formulation would be to separate the investment function into an endogenous and an exogenous component, along the lines done by Hicks (1950). However, as pointed by Trezzini (1995), this usually comes with a rigid

accelerator and, consequently, leaves no role for a residual adjustment of savings via changes in capacity utilization. For a Sraffian interpretation of Hicks's "supermultiplier" see Serrano (1995) as well as Trezzini's (1998) critique. As shown in the appendix, the dynamics of Hick's supermultiplier can be represented through a Ricatti differential equation.

7. From a long-run perspective a positive g_I does not seem to be an unrealistic assumption about capitalist economies.
8. Note also that, for the dynamics of g_K to have a reasonable economic meaning, we have to add that both I and K are positive at time zero.
9. Alternatively we can say the growth rate of labor productivity exceeds the growth rate of the real wage when the economy is above its natural level of economic activity and vice versa.
10. Note that this natural rate of capacity utilization is analytically similar to the mainstream NAIRU.
11. If the economy responds with profit squeeze instead of forced savings, then the average propensity to save decreases in the long run, requiring an additional increase in capacity utilization for savings to adjust to investment (the Harrod-Domar line rotates clockwise in terms of figure 5). Note that, in addition to alternative Keynesian closures, equation 6 is also consistent with the Marxist closure of Goodwin (1967), where investment is determined endogenously by the predator-prey interaction of capacity utilization and the real wage.
12. According to Eatwell and Milgate (1983, p.16): "The determination of relative prices in a market economy does not involve the simultaneous determination of the size and

composition of output. Prices and quantities are determined by separable, though interacting forces."

13. To obtain a constant long-run rate of profit, one can assume a counter-cyclical profit share (profit squeeze) and that changes in π and u always balance out. Alternatively, as done by Serrano (1995), one can keep the Sraffian assumption of a constant π and assume that u is also constant through a rigid accelerator. As it will be shown in the appendix, the latter strategy preserves the classical determination of the rate of profit by restricting the scope of effective demand.

14. Note that, in spite of this result, Garegnani (1992) argues that the normal rate of profit is a meaningful concept because of its influence on investment decisions. A complete analysis of Garegnani's theory of investment is beyond the scope of this paper. The interested reader should refer to Garegnani (1983).

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Appendix: Hicks's supermultiplier

Instead of an exogenous investment function consider the following endogenous specification:

(A.1) $I = v\dot{Y}$

As pointed by Trezzini (1998) equation A.1 implies two assumptions, namely, perfect foresight and normal utilization of capacity at time zero (the "initial-condition" hypothesis). Assuming that there is a positive autonomous component in aggregate demand *A*, we have that:

(A.2) $Y = cY + v\dot{Y} + A$

Rewriting equation A.2 in terms of the growth rate of income g_Y:

(A.3) $g_Y = \frac{s - a}{v}$

where *a* is the ratio of autonomous demand to income (A/Y). Note that, since *a*>0, the growth rate of income is necessarily smaller than the "warranted" rate *s/v*. If both *s* and *v* are constants through time, then we can obtain the following differential equation in g_Y from equation A.3:

(A.4) $\dot{g}_Y = \left(\frac{s}{v} + g_A\right)g_Y - g_Y^2 - \frac{s}{v}g_A$

where g_A is the growth rate of autonomous expenditures. Equation A.4 is a Ricatti differential equation in g_Y with two particular solutions: g_A and *s/v*. The time-path of g_Y is given by:

(A.5) $g_{Y,t} = g_A + \left(\frac{s}{v} - g_A\right) \Big/ \left[1 + C\left(\frac{s}{v} - g_A\right) e^{-\left(\frac{s}{v} - g_A\right)t}\right]$

where C is the constant of integration.

Assuming that the economy is somewhere between the particular solutions of equation A.4 at time zero, it is straightforward from equation A.5 that g_Y will converge to g_A (*s/v*) if g_A is greater (smaller) than *s/v*. In other words, provided that the economy starts above the lower particular solution, the higher particular solution of equation A.4 sets the long-run value of g_Y. If the economy starts below the lower particular solutions, then g_Y falls indefinitely through time, what does not seem to be a good description of the dynamics of capitalist economies.

If the economy is somewhere between the two particular solutions of equation A.4 at time zero, then the natural question is what is higher, g_A or *s/v*? Since we assumed that *A*>0 at any point in time, g_Y is necessarily below *s/v* at any point in time. Thus, we have that $g_A \leq g_Y \leq s/v$ when t=0, what means that the economy tends to the warranted rate of growth in the long run.

From above we can conclude that including a rigid accelerator does not allow a substantial long-run role for effective demand, at least not as in the model of section 2. Changes in g_A affect the path of g_Y but, since the latter has an upper boundary at *s/v*, in the long run we cannot avoid a convergence to the warranted rate of growth or an explosive fall in income growth (what ultimately leads to zero income). According to the principle of effective demand it seems more reasonable not to constraint the behavior of investment and let *s* and *u* be the adjusting variables in the long run, as done in sections 2 and 3.

If one wants to include other autonomous expenditures than investment while preserving a flexible capacity utilization, the Harrod-Domar equation of section 2 has to be modified to:

(A.7) $$g_K = \frac{(s-a)u}{v}$$

Despite the change, the pace of capital accumulation is still described by the Bernoulli equation presented in section 2. The difference lies in the dynamics of income growth, which now depends on two exogenous growth rates, g_I and g_A. In order to make the latter the only exogenous force driving economic growth, one can express the dynamics of g_I in terms of a stable accelerator as, for instance:

(A.8) $$\dot{g}_I = \beta(g_Y - g_I)$$

where $\beta>0$.

Figure 1

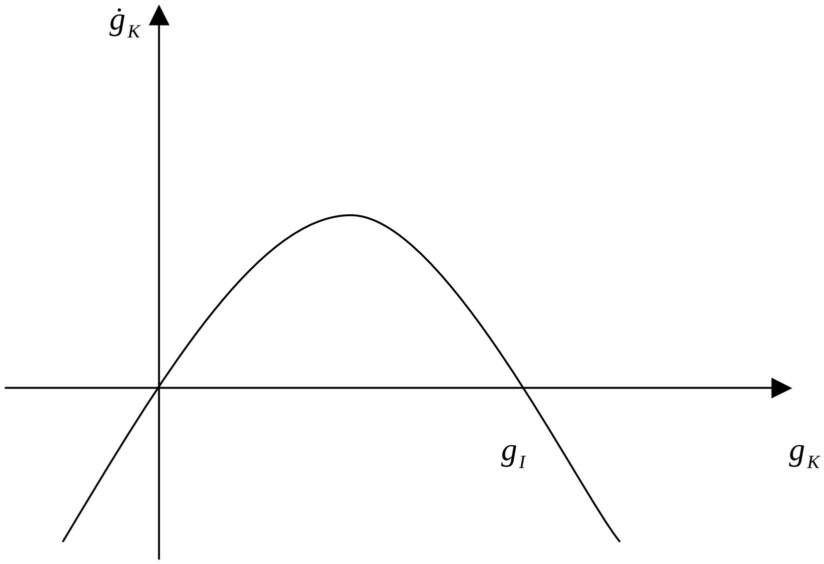
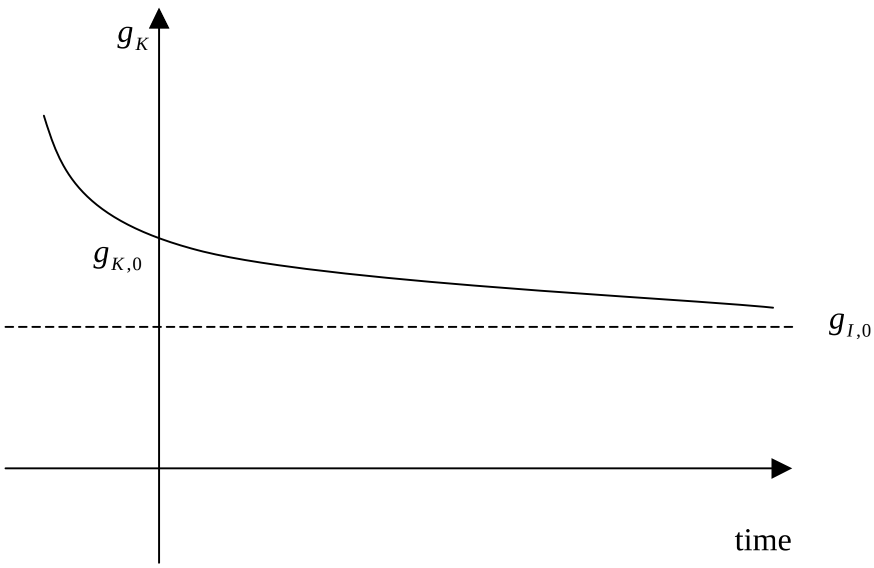
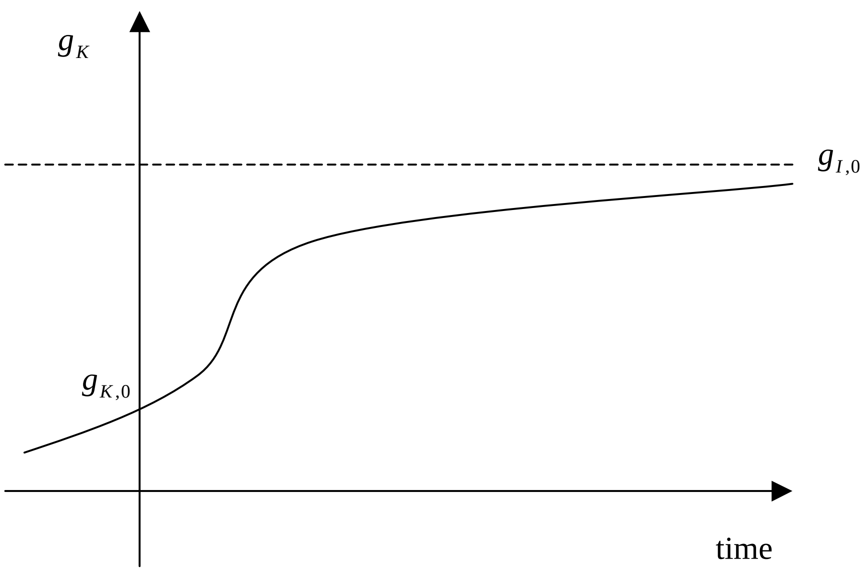


Figure 2





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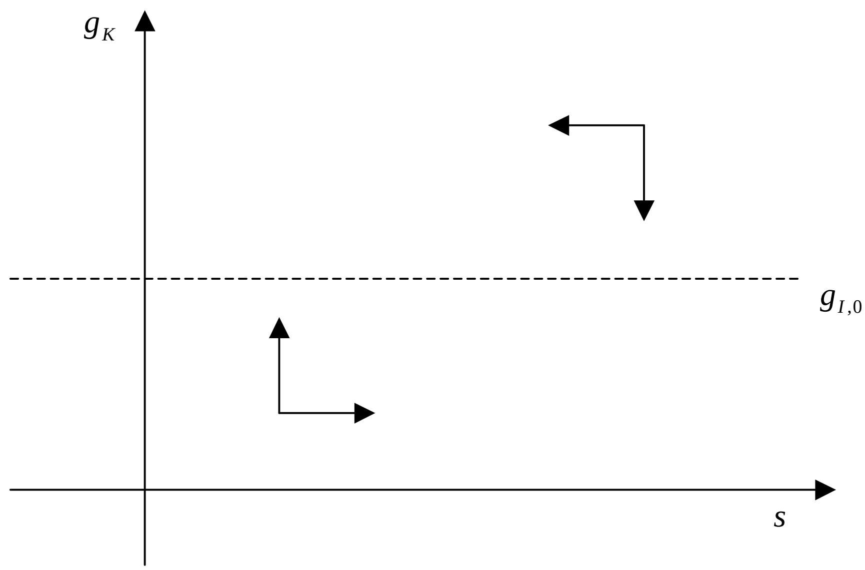

Figure 4

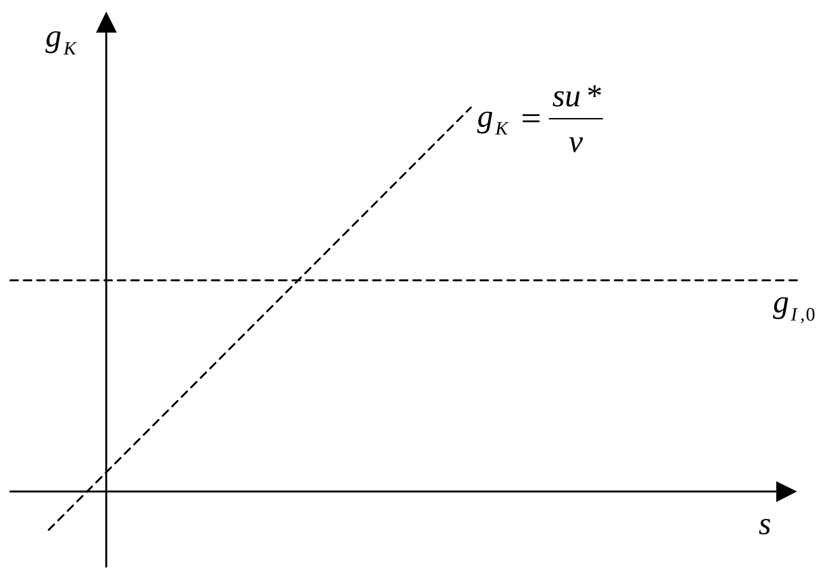

Figure 5:

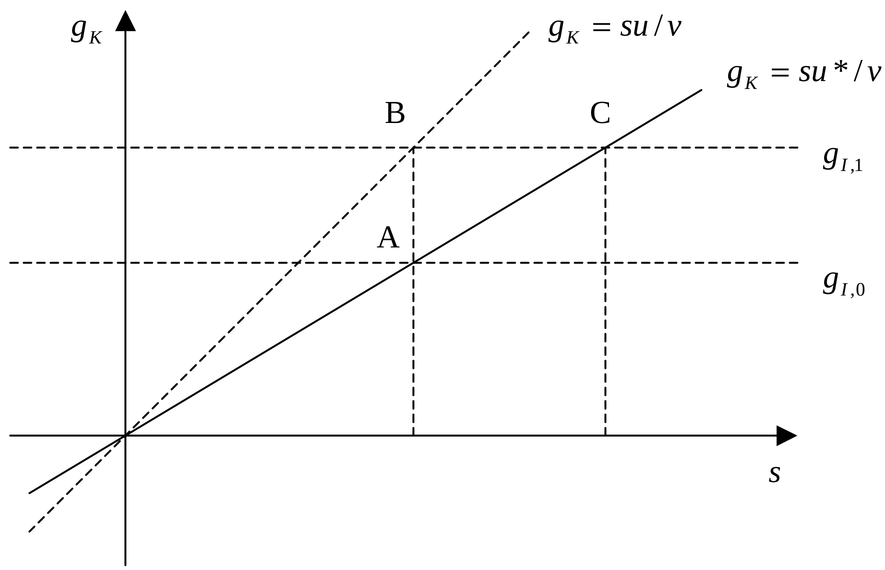

Figure 6

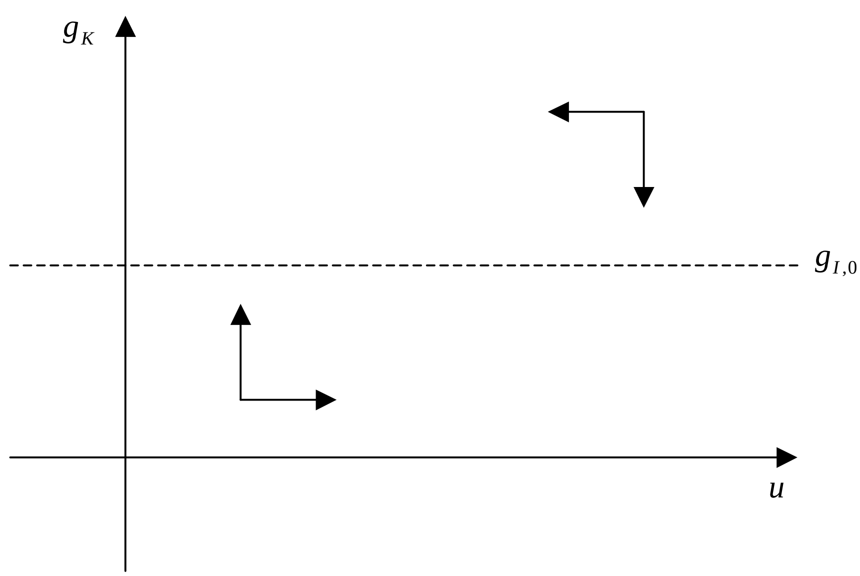

Figure 7: