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Debt-Equity Cycles in the 20th Century:

Empirical Evidence and a Dynamic Keynesian Model

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Abstract Long-term cycles in the equity-capital (*q*) and debt-capital (λ) ratios exist in the US, UK, and Japan. They follow a broadly clockwise pattern in the first two economies and counterclockwise in Japan. The data as constructed satisfy flows of funds for loans and a standard equation for the return to equity, which boil down to differential equations for λ and *q*. Under appropriate restrictions on the Jacobian, this system generates long-term predator-prey cycles. The clockwise variant arises when capital stock growth is "debt-led" and the debt ratio is "equity accelerated." "Debt-burdened" growth and "equity-decelerated" debt produce counterclockwise oscillations. A transcritical bifurcation involving the "required" return to equity in response to shifting "fundamentals" adds realistic equity price dynamics to both variants.

This paper is about long run, macro level cycles involving business debt and equity. The cycles appear in the financial data for the US, UK, and Japan. Evidence that they exist is presented in section 1, using numbers obtained mostly from the Flow of Funds Accounts for these countries.

To a first approximation, cyclical dynamics of the form the data demonstrate can arise from a system of two differential equations based on a Wicksellian financial framework with balance sheets set out in section 2. A bare bones Keynesian model for short-term macro equilibrium and demand-driven capital stock growth follows in section 3. In the model, the flow of new loans is set by firms' needs to finance investment. Outstanding credit determines the level of financial system liabilities ("money") held by households.

Section 4 is about the differential equations themselves. One is for the change in the ratio of the value of corporate equity to capital stock and follows from the standard intertemporal arbitrage equation for the return to holding shares. The other is for the ratio of debt to capital and follows from the corporate sector's flow of funds. In section 5, it is shown that under appropriate conditions, positive feedback of the equity price into its own change over time and indirectly into the debt-capital ratio can generate long-run predator-prey dynamics with contrasting clockwise and counterclockwise oscillations as demonstrated by the data for the US and UK on one hand and Japan on the other.

However, the model as based on two "slow" differential equations for securities-capital ratios suffers from a problem with a turning point. When cycles are clockwise, for example, it predicts that at the trough the equity-capital ratio starts to rise because the increase in share prices outstrips the growth rate of capital, while at the top the ratio begins to fall because capital grows more rapidly than prices. This scenario is consistent with the data at a cyclical low, but not at a peak where asset prices continue to grow more rapidly than capital and then begin to decrease "on their own" (with credit and capital falling in train). Based on Keynes's ideas about the cycle, a simple bifurcation involving the "required" return to equity as it responds to "fundamentals" is set up in section 6 to allow the model to approximate the data better. The bifurcation has "faster" dynamics than the equations for the equity-capital and debt-capital ratios, and shows how the rapid ascents and descents of the former can take place. Section 7 summarizes the main results.

1. Debt-Equity Cycles: Stylized Facts

Over the past 50-100 years, six secular debt-equity cycles appear in data for the US, UK, and Japan.

Post-WWII data for the US and UK (Figures 1 and 2ab respectively) demonstrate clockwise cycles between the value of equity outstanding and the value of business debt, both normalized by the value of the capital stock as an indicator of the size of the sector. In the diagrams we plot the ratio of net debt of non-financial corporations to the value of their capital stock on the horizontal axis against the ratio of outstanding equity to capital (a standard measure of Tobin's q) on the vertical.[1] For the postwar period we observe two clockwise cycles in both economies: a first oscillation ends in 1979 after hitting a bull market peak in 1967. The years 1979-1980 mark the start of the next cycle, which goes into a downswing two decades later with the 1999-2000 crash following the 1990s boom. In Figure 1 for the US and Figure 2b for the UK, the highest value of q in the second cycle is substantially above the maximum in the first, and the run-up to the peak during the 1990s is more rapid.

It can be seen visually that stock prices lead the cycles, or in more allegorical terms the value of equity is a prey variable with debt as a predator. Seemingly, optimistic expectations are fuelled by the rise in the price of equity, and in turn higher equity prices are sustained by better access to loans.

Figure1: Postwar debt-equity cycles in the US.

Figures 2a and 2b: Postwar debt-equity cycles in the UK.

A good description of the macroeconomics for the US can be found in Figure 3, which shows changes in net claims among the major NIPA sectors (new net borrowing is measured on the vertical axis). During the 1990s, negative net financial claims for both business and households are on the rise, with the government and rest of the world sector supporting their accumulation of debt. Business net borrowing also rises in the 1960s in the run-up to the peak of the earlier cycle, and then drops off sharply in the 1970s after equity prices begin to fall. A similar collapse in borrowing by corporations (this time accompanied by households) also shows up after 1999. In other words, predator-prey dynamics is observed in both cyclical upswings and downswings.

Figure 3: NIPA-based financial needs of the government, foreign sector, household and rest of the private sectors - real dollars of 1996.

Normalizing equity and debt by the value of capital stock leads to diagrams in which cycles are easy to visualize. However, in both economies there appears to be a long-term tendency for the debt-capital and possibly the equity-capital ratio to rise. Whether these trends persist in future decades of course remains to be seen.

Data are less readily available for the pre-WWII period, but at least in the US the same cyclical pattern emerges from benchmark years for which Goldsmith and Lipsey (1963) calculated values of business capital, equity, and debt.

The lower diagram in Figure 4 is based on their estimates, which satisfy the relationship $PK = P_eE + L$ (where PK = corporate capital stock K at a current price index P, P_eE = outstanding corporate equity with P_e as an equity price index and E the corresponding volume, and L = outstanding corporate debt) thereby unrealistically setting observed corporate net worth to zero. The upper diagram instead uses the S&P 500 as the indicator of equity valuation. In both versions a clockwise cycle between the early 1900s and the 1930s emerges, with the former years probably representing the high point of a cyclical upswing during Mark Twain's "Gilded Age" in the late 1800s.

Figure 4: Prewar debt-equity cycles in the US.

Japan provides an interesting contrast to the US and UK, as shown in Figure 5 where we plot the outstanding debt to capital ratio against the ratio of an equity price index to capital stock in the upper diagram and against the ratio of outstanding value of equity to capital stock in the lower. Japan's characteristic financial structure is reflected in its relatively high values of the debt-capital ratio (between 0.7 and 1.4), in contrast with levels well below one for the US and UK. When accumulation of debt is on the rise, the equity to capital ratio goes down. Vice versa, when the ratio of debt to capital stock is decreasing, the stock market follows an upward trend. In Japan, it seems that the debt-capital ratio leads the equity-capital ratio. Several small cycles also show up - the first in the second half of 1950s, the second during 1970s and the last in the late 1980s. However, they are far from being as definite as in the US and UK and do not detract from the general tendency for Japanese debt to be the prey variable and equity the predator.

Figure 5: A postwar debt-equity cycle in Japan.

The observed pattern may also point to differences in the institutional framework of Japan versus the US or the UK. Between the 1950s and 1970s, Japan is rapidly catching up with the US and the UK by pursuing an aggressive industrial policy aimed at building strong corporations. The stock market is not developed, so high rates of investment must be supported by accumulation of debt. Thereafter comes the famous "bubble" as portfolios shift from debt toward equity and then the bust with the debt-capital ratio still in decline.

In all three countries, the cycles show up in the balance sheets for the non-financial corporate sector, and reflect changes in the structure of assets and liabilities as driven by financial market dynamics. In the next section we describe an accounting framework for a debt-equity model.

2. The Accounting Basis for a Debt-Equity Cycle Model

Our specification builds upon Lavoie and Godley (2001-2002). It presupposes that the main institutional sectors of the economy – households, business, and the financial sector or "banks" – satisfy their balance sheet and flows of funds restrictions at all times. The balance sheets are set out in Table 1, in which in addition to terms previously defined, M is the money supply and Ω_h and Ω_f stand for levels of net worth of households and business firms respectively.

Table 1: Balance Sheets underlying the debt-equity model

The household balance sheet is consistent with the flow of funds relationship Household Savings = Change in Money Holdings + Change in Equity Holdings at Current Prices (or $S_h = \dot{M} + P_e\dot{E}$ where a "dot" over a variable signifies its derivative with respect to time) and the statement that Change in Net Worth = Savings + Capital Gains ($\dot{\Omega}_h = S_h + \dot{P}_e E$). The capital gains can be substantial: "Between 1994 and the first quarter of 2000, the market capitalization of shares held by households snowballed from $4 trillion to $12.2 trillion" (Brenner 2002). Moreover, American households, so well described by Keynes (1936) as constituting a "stock-minded public…where a rising stock-market may be an almost essential condition of a satisfactory condition to consume", opted for a dramatic change in their saving behavior with the personal saving rate decreasing from 8.7% in 1992 to -0.12% in 2000 (Brenner 2002). These linkages are built into the macro model presented below.

For the corporate sector, the balance sheet is consistent with the flow of funds accounts of the United States, which allow for non-zero corporate net worth.[2] For purposes of modeling, we assume that the corporate sector's assets include the value of capital stock alone, while liabilities comprise net debt and equity at market value, with net worth as the balancing item which can take either sign. As we have seen, debt-equity cycles in the US and UK reveal increases in the ratios of both debt and equity to capital stock during the boom which points to a positive relationship between the propensity to borrow/lend and stock market dynamics. As discussed in section 6, the theoretical underpinnings of this argument are Keynesian in essence. They were clearly stated by Minsky (1975) in his analysis of the investment and financial instability. He says that "whenever, as a result of an improvement in confidence and credit, the leveraging of investment increases, the owners of the inherited stock of capital assets, whose liability structure is compatible with a previous stage of confidence, find themselves with an unused margin of 'borrowing power'" (p. 120). During the last boom in the US, non-financial corporations financed 22% of their investment in fixed assets through new borrowing (Brenner 2002, p. 200).

For the financial sector's balance sheet we use a simple Wicksellian "banking" formula: Loans = Bank Assets = Bank Liabilities = Money Supply. We assume that "loans create money" in the sense that banks satisfy loan demand coming from the non-financial business sector's flow of funds and that the household balance sheet adjusts to absorb the corresponding increase in the money supply. This specification is compatible with the view expressed by scholars such as Kindleberger (2000) that "endogenous" expansion of net credit often underlies financial cycles.

On the basis of the balance sheets, we next set out a short-term macro model along broadly Post-Keynesian lines. It will be used to fill in the details of differential equations for the debt-capital ratio $\lambda = L/PK$ and the equity-capital ratio $q = P_e E / PK$.

3. A Short-term Macro Model

The pre-determined real interest rate on both money and loans is *j*. From Table 1, firms can have non-zero net worth and the value of their capital stock, *PK,* is exhausted by their outstanding loans *L*, the value of their equity $P_e E$, and their net worth Ω_f .

Total corporate net income is $(r - j\lambda)PK$ with *r* as the profit rate that can also be written as $r = \pi PX / PK = \pi u$ where $u = X/K$ is the ratio of output *X* to capital and π is the profit share. We treat *u* as an indicator of the level of economic activity and assume that π stays constant.[3] If

s_f is the saving rate of firms, then their saving as scaled to the value of capital is $s_f(r - j\lambda)$.

Their other sources of funds are new borrowing $\dot{L}/PK = \lambda\hat{L}$ (with $\hat{L} = \dot{L}/L$) and issuance of equity. A working hypothesis is that they finance a share χ of their capital formation $g = I/K$ with new shares, so that $P_e\dot{E}/PK = \chi g$.[4] Firms' flows of funds thus become

$$s_f(r - j\lambda) + \lambda\hat{L} - (1 - \chi)g = 0 \qquad (1)$$

A Post-Keynesian twist in this equation comes from the term for the growth of financial credit, $\lambda\hat{L}$. The profit rate r and growth rate g are determined on the real side of the model, so the change in the supply of bank loans $\lambda\hat{L}$ has to be endogenous to allow firms to carry through their investment plans.

Households' consumption is assumed to depend on income and wealth. Again scaled to the capital stock, their income ξ_h comprises wage payments $(1 - \pi)u$, interest $j\lambda$ that firms pay to the financial sector which is then transferred to households via payments on their deposits, and distributed profits $(1 - s_f)(r - j\lambda)$. Because $(1 - \pi)u = u - r$, we have that $\xi_h = u - s_f(r - j\lambda)$. For future reference, note that household income rises when the corporate debt burden $j\lambda$ goes up.

If we use q as a convenient indicator of wealth held in the form of equity, then household consumption γ_h (scaled to PK) becomes $\gamma_h = (1 - s_h)\xi_h + \phi q$ where s_h is the household saving rate and the parameter ϕ measures the wealth effect. As discussed above, households use their saving to acquire new bank deposits $\dot{M}/PK$ and new equity χg issued by firms. Their flows of funds (scaled to the value of capital) thus become

$$s_h[u - s_f(r - j\lambda)] - \phi q - \chi g - \lambda\hat{M} = 0 \qquad (2)$$

Because $L = M$ and $\hat{L} = \hat{M}$ from the Wicksellian financial system's balance sheet, accounting consistency ensures that households boost their net worth by picking up the flow of new deposits that new bank lending creates.

Ignoring depreciation, the growth rate of the capital stock permitted by available saving, g^s, is the sum of (1) and (2),

$$g^s = s_h u + \alpha(r - j\lambda) - \phi q \quad (3)$$

in which $\alpha = s_f(1 - s_h)$. By reducing the saving of firms a bigger debt burden $j\lambda$ cuts into g^s but the effect is attenuated by the $(1 - s_h)$ term which reflects the boost to household income from $j\lambda$ mentioned above.

Post Keynesian investment functions of the sort estimated by Fazzari and Mott (1986-87) and Ndikumana (1999) emphasize cash-flow considerations. If the interest burden $j\lambda$ increases, firms are likely to cut back on capital formation g^i. For symmetry with the saving function (3) it is convenient to make g^i depend on *q,* and we also carry an accelerator term βu in the output/capital ratio or "capacity utilization:"

$$g^i = g_0 + \beta u + \eta q - \psi j\lambda \quad . \quad (4)$$

The short-term macro equilibrium condition is $g^i - g^s = 0$, or

$$g_0 + (\eta + \phi)q + (\alpha - \psi)j\lambda - (\alpha\pi + s_h - \beta)u = 0 \quad . \quad (5)$$

Solving for *u* in (5) we obtain the expression:

$$u = \frac{g_0 + (\alpha - \psi)j\lambda + (\eta + \phi)q}{\Delta} \quad (6)$$

with $\Delta = \alpha\pi + s_h - \beta$.

The usual short-run stability condition is $\Delta > 0$, or saving rises more than investment in response to an increase in *u*. Assuming that this condition is satisfied, note the ambiguous effect of $j\lambda$ on *u.* A bigger debt burden reduces investment demand through the coefficient $-\psi$ but also cuts into overall saving through the coefficient $\alpha = s_f(1 - s_h)$ as discussed above. If the saving effect dominates, $\alpha > \psi$, effective demand can be said to be "debt-led." Otherwise, it is "debt-burdened." The remaining term in (5) involves *q*. Through both investment and saving effects, a higher *q* increases the level of economic activity. How *q* itself gets determined is taken up below.

Capital stock growth is set from the demand side in this model (as opposed to the supply of saving as preferred in neoclassical formulations). To get a final expression for *g* we plug (6) into the investment demand function (4):

$$g = g_0[\frac{s_h + \alpha\pi}{\Delta}] + [\frac{\beta}{\Delta}\alpha - (1 + \frac{\beta}{\Delta})\psi]j\lambda + [\eta(1 + \frac{\beta}{\Delta}) + \phi\frac{\beta}{\Delta}]q \quad (7)$$

The relation between the growth rate of capital and the debt-capital ratio is not as straightforward as is the case for u. The sign of the partial derivative of g with respect to λ depends on the expression $\frac{\beta}{\Delta}\alpha - (1 + \frac{\beta}{\Delta})\psi$. If this term is positive, we have debt-led capital stock growth; otherwise g is debt-burdened.

Some rough numbers may help clarify the distinction. Taking into account taxes, imports, etc., "leakage" parameters for the US economy are on the order of $s_f = 0.6$ and $s_h = 0.2$, making $\alpha = 0.48$. For $\pi = 0.25$ and $\beta = 0.2$, we get $\Delta = 0.12$ and $\beta / \Delta = 1.6667$. For these numbers, $\frac{\beta}{\Delta}\alpha - (1 + \frac{\beta}{\Delta})\psi > 0$ so long as $\psi < 0.3$, a condition that estimated investment functions suggest is likely to be satisfied. Although this sort of back of the envelope calculation is far from definitive, it does show that debt-led growth is a clear possibility.

4. Differential Equations for the Debt-Capital and Equity-Capital Ratios

Whether growth is debt-led or –burdened becomes of interest in discussing the evolution of the equity-capital ratio *q.* But since it follows directly from the short-run model, we first take up the dynamic behavior of λ. From the business sector's flow of funds (1), a differential equation can be written as

$$\dot{\lambda} = (s_f j - g)\lambda + (1 - \chi)g - s_f \pi u \quad . \quad (8)$$

Is this equation locally stable in the sense that $\partial\dot{\lambda} / \partial\lambda < 0$? With regard to the lead term $(s_f j - g)\lambda$, real interest rates can exceed growth rates for extended periods of time (for example, during the last two to three decades of the 20^{th} century), but since $s_f < 1$, $s_f j - g$ can be negative. There are also effects of λ on u and g to be considered. From the discussion above, they can take either sign. To keep possible outcomes of the model within limits, we simply assume local stability in (8).

Turning to the effects on $\dot{\lambda}$ of q, in (8) note that an increase in q will raise u and g from (7) and (8) respectively. The former shift increases corporate saving $s_f \pi u$ and thereby reduces new borrowing $\dot{\lambda}$. More investment, however, increases the need to borrow. To see which effect dominates we can use the expression

$$\frac{\partial \dot{\lambda}}{\partial q} = \frac{[-s_f \pi + \beta(1-\chi-\lambda)](\eta+\phi)}{\Delta} + (1-\chi-\lambda)\eta \quad . \tag{9}$$

We can say that there is an "equity-accelerated" debt ratio if $\partial \dot{\lambda} / \partial q > 0$, and an "equity-decelerated" ratio otherwise. The most important term is $-s_f \pi + \beta(1-\chi-\lambda)$. With $\chi < 0$ and λ a fraction, it can be positive if the accelerator term β is relatively strong. On the other hand, if λ is in the range of one or greater as in Japan, we can have $\partial \dot{\lambda} / \partial q < 0$.

The next step is to bring in a differential equation for *q.* Following Diamond (2000) and ultimately Gordon (1962) the return to equity depends via intertemporal arbitrage on capital gains and dividends in a formula such as

$$\rho P_e E = \dot{P}_e E + D \tag{10}$$

in which ρ is the return and *D* is the level of dividend payments. It is convenient to set $D = \delta PK$ and for the moment we hold ρ constant as a "required" return.[5] On these hypotheses we have that

$$\rho = \hat{P}_e + \frac{\delta}{P_e E / PK} = \hat{P}_e + \frac{\delta}{q}$$

which immediately provides an equation for the growth rate of the equity price (independent of the corporate debt position λ),

$$\hat{P}_e = \rho - \frac{\delta}{q} \quad . \tag{11}$$

Because $q = P_e E / PK$, we have $\partial \hat{P}_e / \partial P_e > 0$ in (11). The asset return equation (10) incorporates positive feedback of the equity price into its own growth rate, as is well known.

New issues of equity are $P_e \dot{E} / PK = \chi g$ so that

$$\hat{E} = \frac{\chi g}{q} \quad . \tag{12}$$

Differentiating the definition $q = P_e E / PK$ gives $\dot{q} = q[\hat{P}_e + \hat{E} - (\hat{P} + g)]$. Plugging in (11) and (12) with $\hat{P} = 0$ shows that

$$\dot{q} = (\rho - g)q + \chi g - \delta \quad . \tag{13}$$

The effect of *q* on $\dot{q}$ is given by

$$\partial\dot{q} / \partial q = (\rho - g) + (\chi - q)\partial g / \partial q \quad . \tag{14}$$

In the first term, the real long-run equity return ρ is famously in the range of 7% , higher than any reasonable growth rate. In the second term, χ is small in absolute terms and recently negative in the US. Because $\partial g / \partial q > 0$, we can have $\partial\dot{q} / \partial q < 0$ for sufficiently large values of *q*. The intuition is that the positive feedback of P_e into $\hat{P}_e$ just mentioned makes $\partial\dot{q} / \partial q > 0$ for low values of *q*. But if a higher equity-capital ratio pushes up *g* in a demand-driven growth model by reducing ex ante saving and stimulating investment demand, then with a high enough value of *q*, the derivative $\partial\dot{q} / \partial q$ will change signs and *q* will not tend to infinity.

The effect of λ on $\dot{q}$ comes in through the presence of *g* in equation (12) for $\hat{E}$. It assumes a critical role in the expression

$$\partial\dot{q} / \partial\lambda = (\chi - q)\frac{\partial g}{\partial\lambda} \quad . \tag{15}$$

If capital stock growth is debt-led, the derivative of $\dot{q}$ with respect to λ will be negative since $\frac{\partial g}{\partial\lambda} > 0$ and $\chi - q < 0$. If growth is debt-burdened then we have $\partial\dot{q} / \partial\lambda > 0$.

Along with the contrast between an equity-accelerated and –decelerated debt ratio, this distinction has important implications for the dynamics of *q* and λ over time. For example, if capital stock growth is debt-led, the turning point at the top of a cycle would happen when rising *q* and λ push up *g* enough in (13) to satisfy the condition $g > -(\rho - \delta)/(\chi - q)$ so that $\dot{q} < 0$. In practice, the cyclical turning points in Figures 1-5 are dominated by rises and falls in P_e as opposed to acceleration in *g* (though the latter may also be observed as in the US in the late

1990s).[6] In section 6 we extend the basic model to take such "fast" equity price dynamics into account.

5. Dynamics of the Model

Equations (4) and (13) combine to give a 2 x 2 dynamical system which can generate an equity-debt cycle:

$$\begin{cases} \dot{\lambda} = (s_f j - g)\lambda + (1-\chi)g - s_f r \\ \dot{q} = (\rho - g)q + \chi g - \delta \end{cases} . \tag{16}$$

Its Jacobian is

$$J = \begin{pmatrix} a_{11} = s_f(j - \pi \frac{\partial u}{\partial \lambda}) - g + (1-\chi-\lambda)\frac{\partial g}{\partial \lambda} & a_{12} = -s_f \pi \frac{\partial u}{\partial q} + (1-\chi-\lambda)\frac{\partial g}{\partial q} \\ a_{21} = (\chi - q)\frac{\partial g}{\partial \lambda} & a_{22} = (\rho - g) + (\chi - q)\frac{\partial g}{\partial q} \end{pmatrix} \tag{17}$$

and the slopes of the loci along which $\dot{\lambda} = 0$ and $\dot{q} = 0$ are given by $\frac{\partial q}{\partial \lambda}|_{\dot{\lambda}=0} = -\frac{a_{11}}{a_{12}}$ and $\frac{\partial q}{\partial \lambda}|_{\dot{q}=0} = -\frac{a_{21}}{a_{22}}$ respectively.

Two points should be recalled about the Jacobian. One is that the partial derivative $\partial \dot{q} / \partial q$ can change sign. As noted above, the term a_{22} in the SE corner of the Jacobian is likely to be positive for low values of *q* but then may become negative at some higher value. This switch gives rise to interesting dynamic possibilities. Second, appropriate permutations of an equity-accelerated or –decelerated debt ratio with debt-led or –burdened growth can give rise to cycles. Table 2 summarizes possible scenarios. Two of them are consistent with the stylized facts presented in Section 1. The one in the SW section of Table 2 involves an equity-accelerated debt ratio and debt-led growth. It applies to the US and the UK cases. Figures 6a and 6b depict the movements of the two state variables.

Table 2: Four Possible Scenarios for the two differential equation system

Figures 6a and 6b: Dynamics with an equity-accelerated debt ratio and debt-led growth

Figure 6a presupposes that $\partial\dot{q}/\partial q > 0$. It shows movements of the variables around the steady-state loci along which $\dot{\lambda} = 0$ and $\dot{q} = 0$. Both stationeries are upward sloping curves, with the one for $\dot{q} = 0$ demonstrating local instability and the one for $\dot{\lambda} = 0$ stability. As can be seen from the signs of the entries in the Jacobian in the SW corner of Table 2, the equity-capital ratio is the prey variable feeding positively into itself as well as into the predatory debt-capital ratio. The resulting clockwise cycles will be convergent or divergent depending on whether the trace of the Jacobian is negative or positive.

Figure 6b illustrates the implications of a switch in the sign of a_{22} induced by an increase in *q*. Because the slope of the $\dot{q} = 0$ locus is $-a_{21}/a_{22}$, the sign change gives a stationary schedule with two branches separating at a cusp at which $\partial\dot{q}/\partial q \rightarrow \pm\infty$ as $a_{22} \rightarrow \mp 0$. The branch to the right, with high values of *q* and λ, is locally stable and would give rise to a steady state with high equity and debt ratios unless other dynamic forces intervened. Possibilities are discussed in the following section.

Figures 7a and 7b show what happens when there is an equity-decelerated debt ratio and debt-burdened growth from the NE section of Table 2. The former condition is more likely when λ is high and the latter when there is high household saving and a strong drag on investment demand as business debt rises. This configuration plausibly applies to Japan, and the counterclockwise cycles in the diagrams are consistent with the data plotted in Figure 5.

6. More Volatile Dynamics

What we have found so far is that a simple demand-driven financial growth model can generate cycles of the sort illustrated in section 1. One reason why the model works is that both the flow of funds balance (8) and the differential equation for *q* in (13) rest upon relationships that by construction are satisfied by the data. The model maintains stock-flow consistency with this accounting so it is not surprising that it can produce results compatible with Figures 1-5.

On the other hand, one significant feature of observed real-financial oscillations is left out. As noted above, around a cyclical peak P_e tends to rise and then fall more rapidly than the capital stock – the upper turning point does not occur because K starts growing faster than $P_e E$ as equation (13) would predict. Rather, in Kindleberger's (2000) terminology a share price "mania" may begin, to be followed by a "panic" and then a "crash". The mania is characterized by rapid increases in the price of equity. It presumably continues until retarding factors such as doubts about the sustainability of a high debt-capital ratio and/or a high price-earnings ratio begin to mount. At some point a flight toward liquidity ensues, eventually causing a sharp decrease in equity prices and as a result a crash of the market. But how does the market become bullish in the first place? How does this theory fit our model?

Kindleberger's answer to the first question is that there is a "displacement" (from Figure 1, somewhere around 1990 in the latest US cycle?) that kicks off an asset price spiral, propelled by positive feedback.[7] His appendix presents examples, to which we can add nothing here. We can, however, say something about process, drawing heavily upon Keynes. As Erturk (2001) points out, Keynes sets out similar but not identical descriptions of the financial side of business cycles in his *Treatise on Money* (1930) and *General Theory* (1936).[8]

The latter's discussion is summarized in Chapter 22: the upswing is characterized by expectations-driven shifts upward in the schedule for the marginal efficiency of capital and the consumption function, and a shift downward in liquidity preference. Reversals in these movements (especially for the marginal efficiency of capital) provoke panic and crisis. With an emphasis on the role of the interest rate and shifts in the quality of balance sheets (with "hedge" changing to "speculative" and ultimately "Ponzi" financial positions), Minsky (1975) presents a similar model.

The *Treatise* version focuses more on asset prices. As an upswing begins, there is a decreasing bear position, roughly indicated by a fall in the volume of savings deposits as portfolios switch toward securities – this is a "bull market with a consensus of opinion" which shifts toward a "bull market with a difference of opinion" in which liquidity preference (in *General Theory* language) starts to rise even though asset prices are still increasing. As more and more traders join the bear brigade and portfolio preferences move strongly against equity, the downswing gets going.

So why do the traders change their views? Presumably at the macro level they are responding to shifts in observable "fundamental" indicators such as the debt-capital ratio λ and the current price-earnings ratio $R = P_e / [(r - j\lambda)PK / E] = q / (r - j\lambda)$. Both fundamentals increase with λ and R rises with q as well. In a short hand description, confidence erodes as they approach "critical" or "unsustainable" levels, say $\overline{\lambda}$ and $\overline{R}$ respectively.

In the present set-up, a way to model such shifts in sentiment is through changes in the "required" return to equity ρ, heretofore held constant in equation (13). When the displacement happens and the liquidity preference schedule shifts downward, ρ must rise in response to greater equity demand. For the reasons just mentioned, at some point this process will reverse. A simple formulation is to set

$$\dot{\rho} = \mu(\overline{R} - R, \overline{\lambda} - \lambda)(\rho - \overline{\rho})(\overline{\rho} + \sigma - \rho) \qquad (18)$$

in which the function $\mu(\overline{R} - R, \overline{\lambda} - \lambda)$ is positive when both its arguments are positive. Depending on the channel(s) one wishes to emphasize, it switches sign when one or perhaps both arguments become negative. Since a US/UK-style boom is accompanied by increases in both R and λ, at some point this sign change will occur (analogous Japanese-style dynamics are presumably driven by variables such as R or perhaps q itself).

In a bit more detail, when $\mu > 0$ the right hand side of (18) looks like Figure 8a. A positive value of μ pushes ρ toward an upper bound $\overline{\rho} + \sigma$ where $\overline{\rho}$ is a base level return to equity and σ reflects market exhilaration. The implication is that a high value of ρ makes q rise rapidly until R and/or λ exceeds its crisis level. The equity-capital ratio passes a cyclical turning point, and then drops off quickly as Figure 8b comes into effect. This is the beginning of the panic phase with economic agents rushing to exit the stock market and selling their share holdings en masse. Bankers behave in a similar manner and stop lending, making the crash inevitable. Economic agents, now bears, switch asset demands from shares toward liquidity.

Sooner or later investment drops substantially, slowing the capital stock growth rate enough to set up a turnaround in q as recovery gets underway. If a mechanism like the one in

(18) works strongly enough on the downswing in conjunction with a low $\bar{\rho}$, it could conceivably drive the system from the high-q toward the low-q cyclical attractor in Figure 6b or 7b, prolonging the downswing.[9]

7. Conclusions

In summary,

Long-term oscillations involving the equity-capital and debt-capital ratios (q and λ respectively) exist in data for the US, UK, and Japan. They follow a broadly clockwise pattern in the first two economies and run counterclockwise in Japan.

By construction, the data satisfy the non-financial business sector's flows of funds and a standard asset return equation. These relationships can be formulated as ordinary differential equations for λ and q that can be analyzed in a phase diagram in the (λ, q) plane. Under appropriate restrictions on the Jacobian, the dynamical system can generate long-term predator-prey cycles like those apparent in the data.

A simple demand-driven post-Keynesian model can be set up to rationalize the cycles. The clockwise variant (with q as the prey variable and λ the predator) arises when capital stock growth is "debt-led" along lines suggested by Minsky (1975) and the debt ratio is "equity accelerated." "Debt-burdened" growth and "equity-decelerated" debt can produce counter-clockwise oscillations.

Around a cyclical peak, short-term dynamics of the equity price can be modeled using a transcritical bifurcation involving changes in the "required" return to equity ρ in response to shifting "fundamentals" such as the debt-equity and price-earnings ratios.

Appendix on Sources of Data

1) United States:

The main sources for outstanding debt and corporate equities at market value for the United States are the *Flow of Funds Accounts* published by the Federal Reserve Bank. Corporate equities are the time series FL103164003 and outstanding debt is the time series LA104104005. (http://www.federalreserve.gov/releases/Z1/Current/data.htm). Capital stock is taken from BEA, NIPA Table 6.1. Current-Cost Net Stock of Private Fixed Assets by Industry Group and Legal

Form of Organization, and represents the capital stock of the corporate non-financial sector. The same pattern can be observed running the plot with both gross and net capital stock. In Figure 1 we used gross capital stock.

The data for the prewar debt-equity cycle for the US were obtained from Goldsmith and Lipsey (1963). They appear in the national balance sheet, Table 1a (pp. 72-85, Volume II). We used the S&P index from Shiller (2000) for the benchmark years in the upper diagram of Figure 4.

2) United Kingdom:

The London Stock Exchange (http://www.londonstockexchange.com/market/historic.asp) publishes the market value of equities for UK and Irish companies used in Figure 2b. In Figure 2a we used the FTSE All Shares stock price index, provided by Global Financial Data (http://www.globalfindata.com). The Cambridge Endowment for Research in Finance, at Cambridge University, provided data for capital stock and outstanding debt for the non-financial corporate sector.

3) Japan:

Data for capital stock and outstanding debt (or financial liabilities) are taken from *Financial Statements Statistics of Corporations by Industries*, Quarterly, Japanese Ministry of Finance, http://www.mof.go.jp/english/ssc/historical.htm. For capital stock we used the time series for fixed assets after removing land in order to avoid the problem of real estate speculation. In order to obtain the value of outstanding debt we added liquid financial liabilities - time series no. 23 (short-term financial liabilities) - to the value of fixed liabilities - time series no. 29 (long-term financial liabilities).

For estimates on the value of equity we used data released by the Bank of Japan for 1970-1999, supplemented for 2000-2001 with data from the Flow of Funds Accounts for 2000-2001 http://www2.boj.or.jp/en/dlong/dlong.htm. The Nikkei 225 obtained from Global Financial Data was another estimate for the value of equity that we used to calculate *q* (upper diagram of figure 5).

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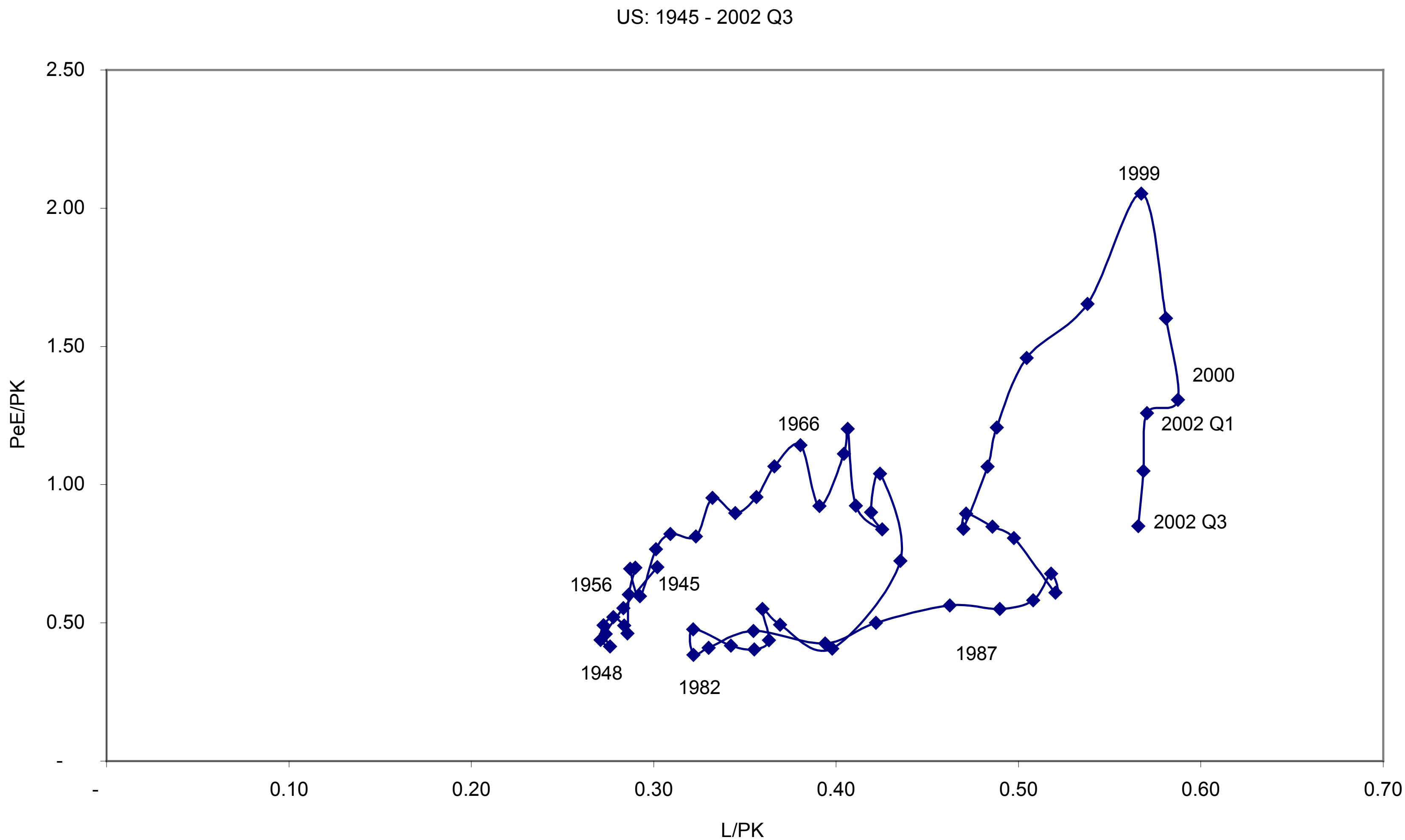


Figure1: Postwar debt-equity cycles in the US.

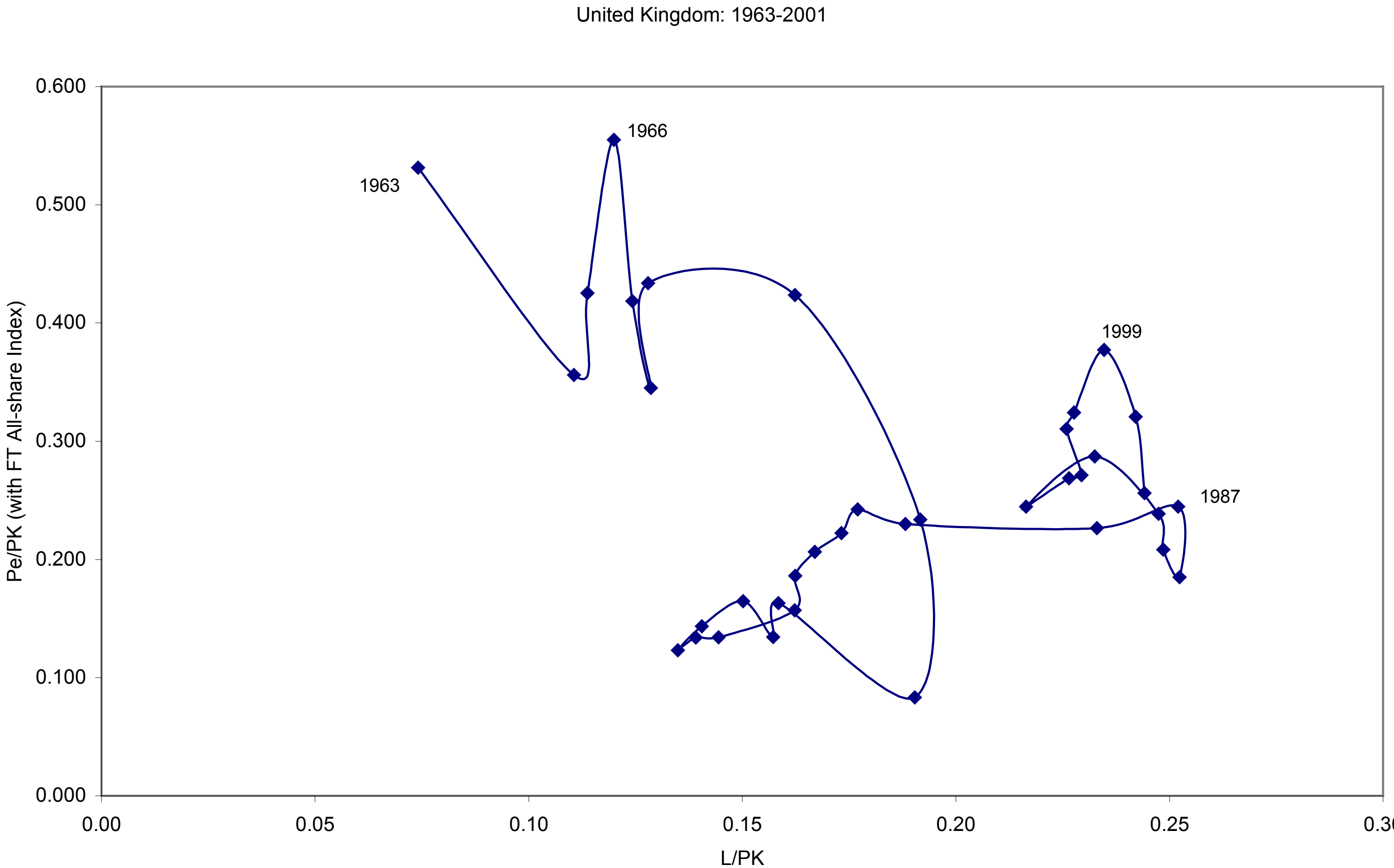


Figure 2a: Postwar debt-equity cycles in the UK.

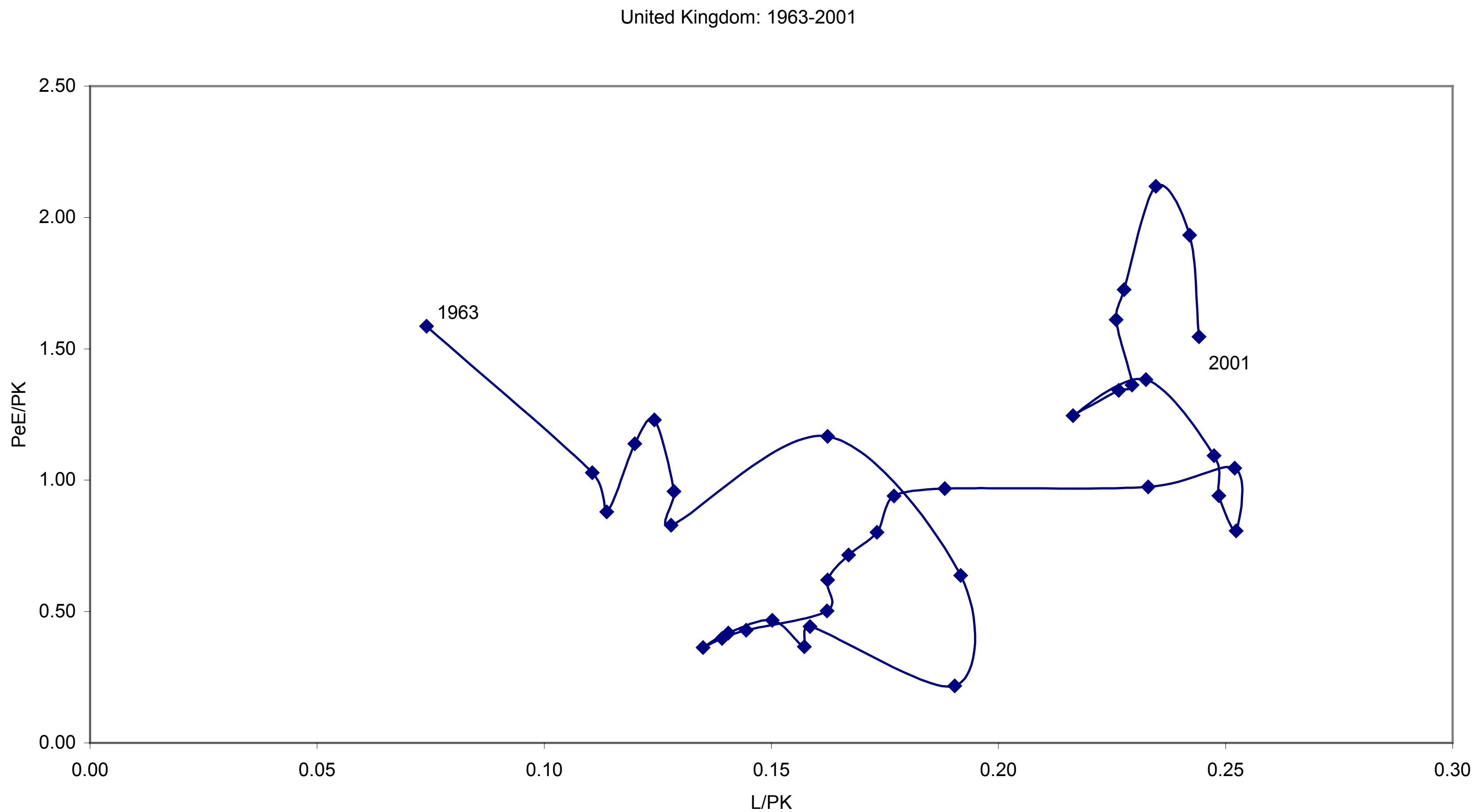


Figure 2b: Postwar debt-equity cycles in the UK.

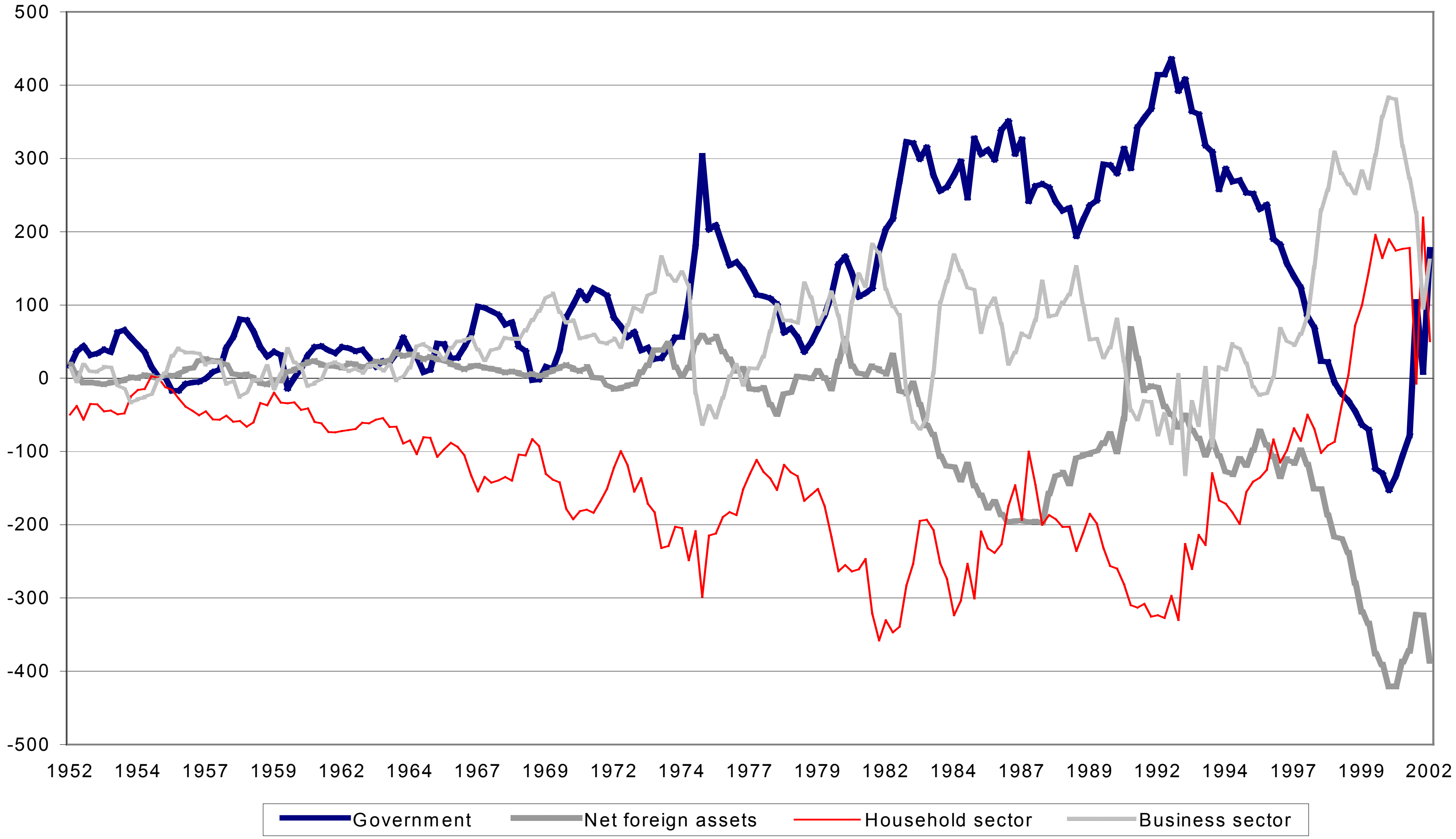


Figure 3: NIPA-based financial needs of the government, foreign sector, household and rest of the private sectors - real dollars of 1996.

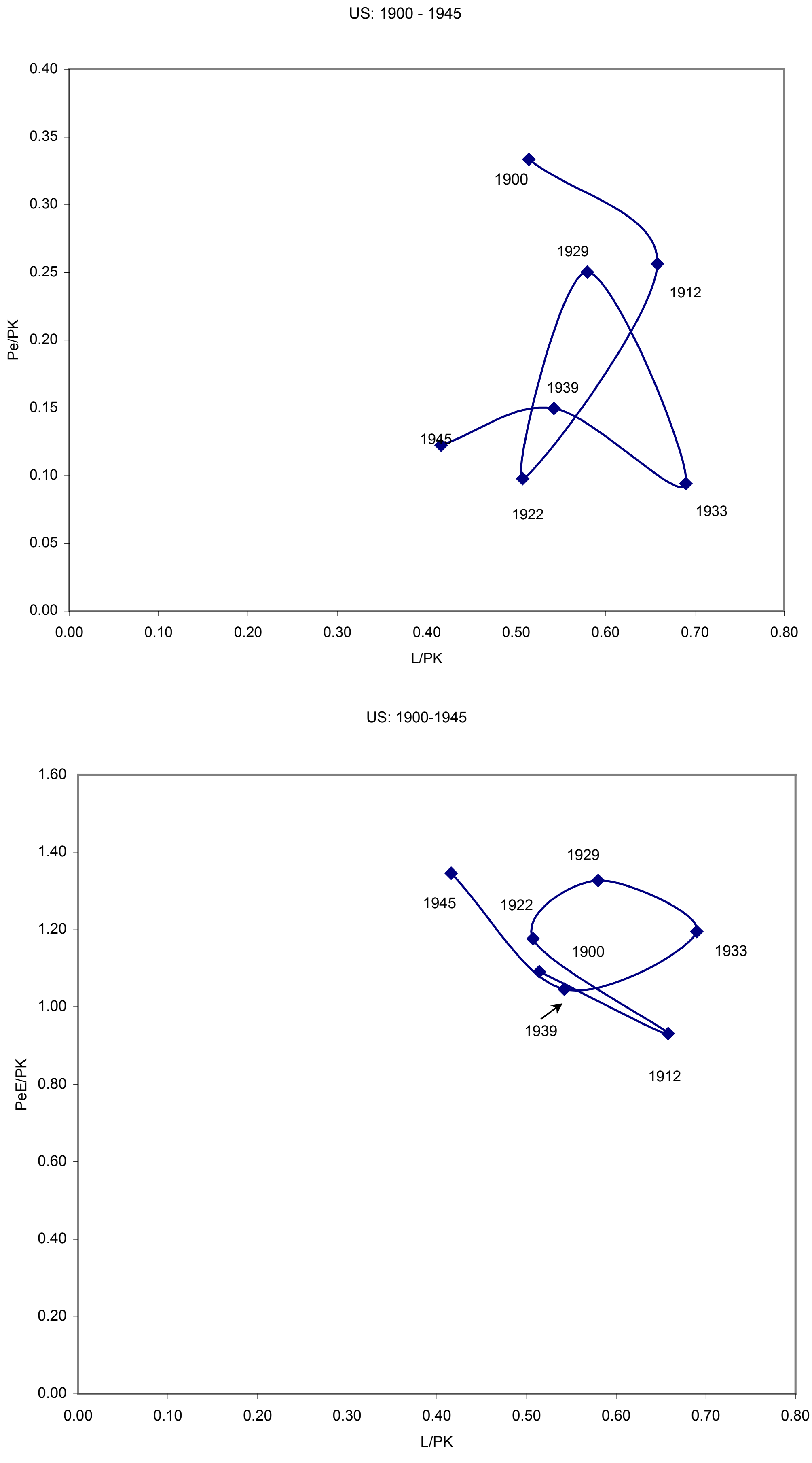


Figure 4: Prewar debt-equity cycles in the US.

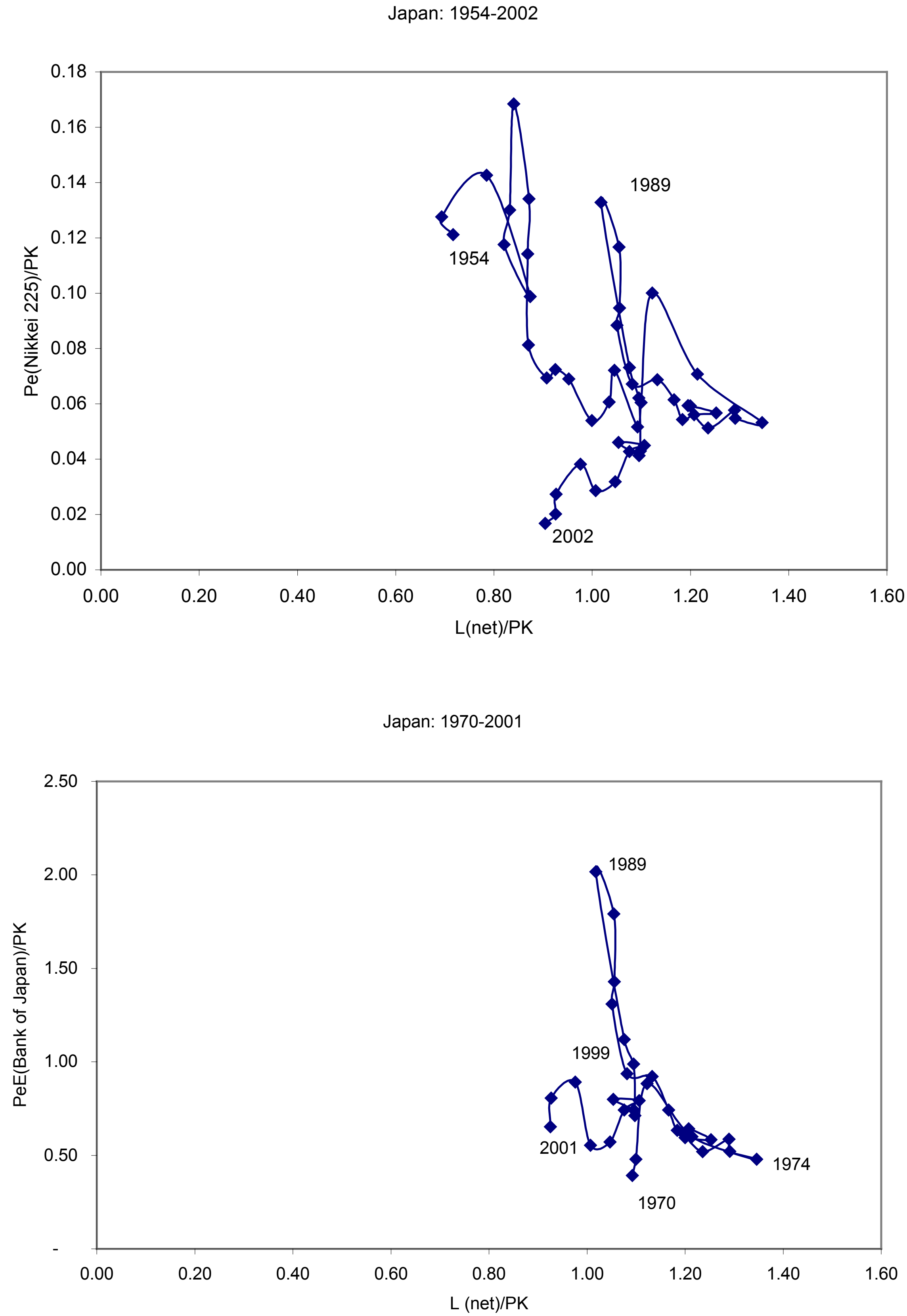


Figure 5: A postwar debt-equity cycle in Japan.

Households	
M P_eE	Ω_h

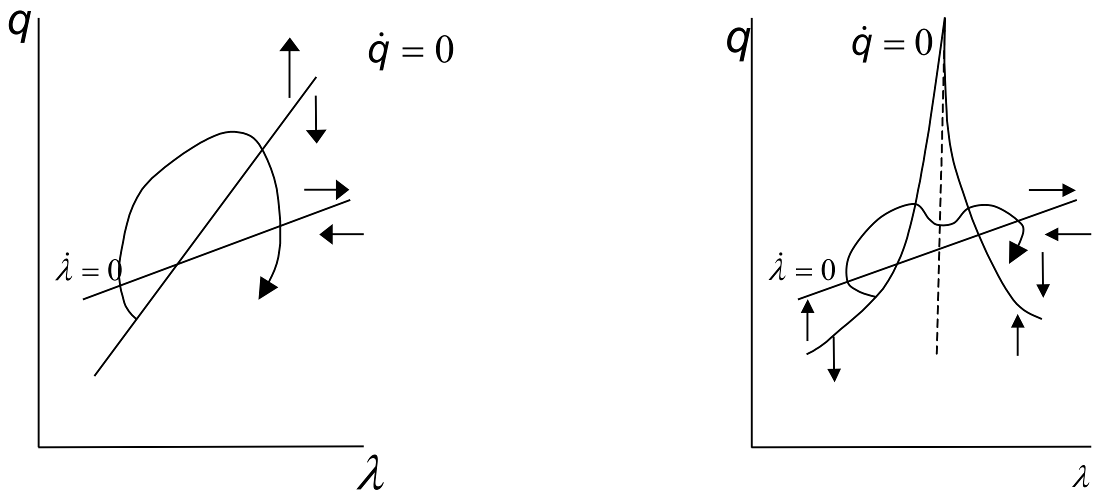
Corporations	
PK	L P_eE Ω_f

Banks	
L	M

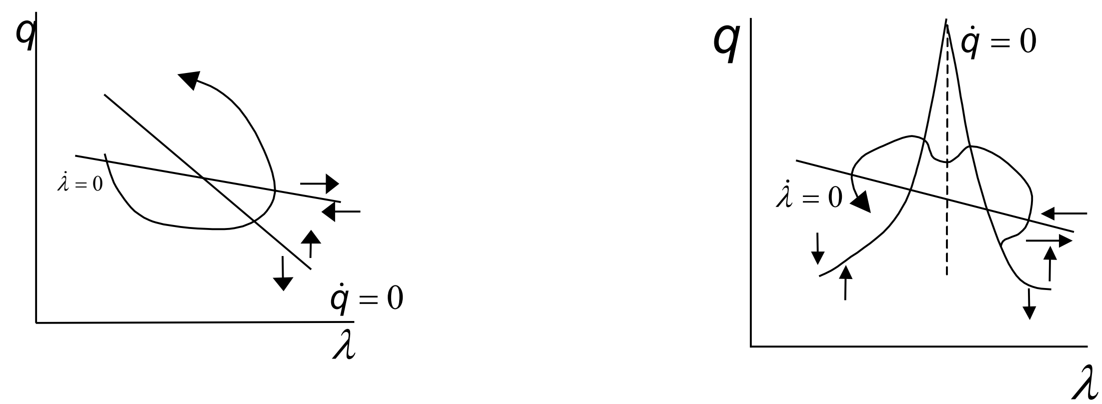
Table 1: Balance Sheets underlying the debt-equity model

	Debt-Led Growth	Debt-Burdened Growth
Equity decelerated debt ratio	λ, q $\dot{\lambda}$: $-$, $-$ $\dot{q}$: $-$, $\pm$ Saddlepoint if $\partial \dot{q} / \partial q > 0$	λ, q $\dot{\lambda}$: $-$, $-$ $\dot{q}$: $+$, $\pm$ Counter-clockwise cycle if $\partial \dot{q} / \partial q > 0$
Equity accelerated debt ratio	λ, q $\dot{\lambda}$: $-$, $+$ $\dot{q}$: $-$, $\pm$ Clockwise cycle if $\partial \dot{q} / \partial q > 0$	λ, q $\dot{\lambda}$: $-$, $+$ $\dot{q}$: $+$, $\pm$ Saddlepoint if $\partial \dot{q} / \partial q > 0$

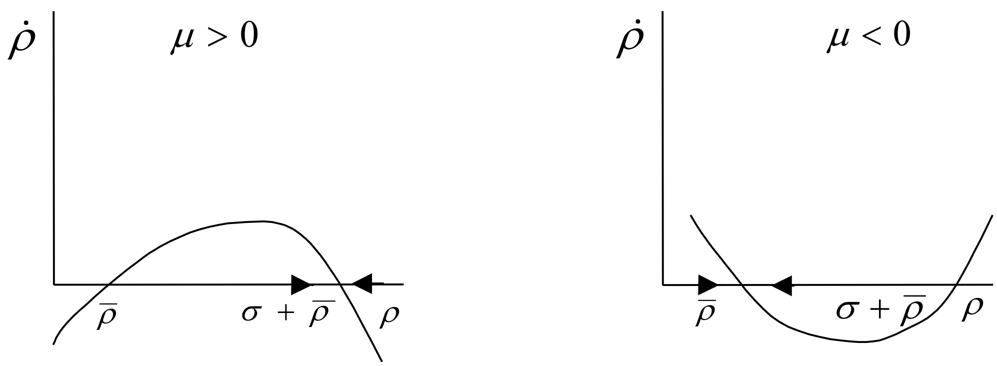
Table 2: Four Possible Scenarios for the two differential equation system



Figures 6a and 6b: Dynamics with an equity-accelerated debt ratio and debt-led growth



Figures 7a and 7b: Dynamics with an equity-deccelerated debt ratio and debt-burden growth



Figures 8a and 8b: 'Fast dynamics' and bifurcations in the bull-bear cycles.

Notes

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[1] Data sources are described in the Appendix. In Figure 2a, we use the FTSE all-shares index as the equity valuation indicator for the UK. A similar pattern (but with a higher peak value for q in the second cycle) shows up in Figure 2b where a measure of total corporate valuation in the UK is used instead. The latter diagram is somewhat problematical because available data on outstanding equity include not only UK corporations but Irish corporations as well for 1970-1994.

[2] In round numbers, non-financial corporate net worth at the end of 1999 in the US was -$9.6 trillion with the sector's stock market valuation at $19.8 trillion. Tangible assets were valued at $12 trillion, and other net financial liabilities were $1.8 trillion. Non-financial business had -$3.7 trillion in "holding losses" on financial paper (basically equity) during that year, while households enjoyed $4.3 trillion of "holding gains" (mostly on shares issued by domestic non-financial and financial business and the rest of the word).

[3] It can be shown that a constant value of π in the present model is equivalent to stating that the price level P is set by a constant mark-up over wage costs. For more on the modeling framework being used, see Taylor (2003).

[4] For the last two or three decades of the 20th century in the US, χ was negative due to stock buybacks.

[5] This assumption will be relaxed when we take up short-term dynamics in section 6. Alternatively, ρ could be interpreted as the observed return averaged over several decades.

[6] The turning point at the top of the last cycle in the US was driven by a fall in the growth rate of equity prices (using the S&P price index) from 30% observed in 1999 to 14% in 2000 and further to an absolute decrease in prices of –6% in 2001. The growth rate of capital followed a similar pattern but with a one year lag. It started to decrease only in 2001, when it grew at 4% relative to the peak of

7% observed in 2000. At the bottom of the previous cycle at the end of 1970s for the US, equity prices were extremely volatile but then began to outstrip capital stock growth in 1983. There was a pause after the 1987 stock market decline and then a strong price surge in the 1990s.

[7] Although our differential equation (13) for *q* incorporates positive feedback of the equity price P_e into its own increase $\dot{P}_e$, it does not generate a rational expectations bubble of the sort investigated by many people in the 1970s and 1980s. Bubbles involving saddlepoints don't show up in the SW and NE corners of Table 2 because of *q*'s cross-dynamics with λ. Blanchard and Fischer (1989) and Rosser (2000) summarize the now largely forgotten bubble literature.

[8] With the exception of Minsky and other Post Keynesians, most business cycle modelers did not take Keynes's emphasis on interactions between the real and financial sectors on board. The *General Theory*'s shifts in the marginal efficiency of capital perhaps influenced multiplier-accelerator formulations but they vanished along with that class of models. On the financial side, bulls, bears, and liquidity preference do not figure in contemporary formulations of financial dynamics which feature rational bubbles, feedback, and models with different types of actors such as "fundamentalists" and "chartists." Again, see Rosser (2000) for a review.

[9] In the jargon, (18) generates a "transcritical bifurcation" for ρ. A slightly fancier variant could be based on a "cusp catastrophe" as in Zeeman (1974) and Varian (1979).