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Hysteresis in the Kaleckian growth model: a Bayesian analysis for the US manufacturing sector from 1984 to 2007*

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Abstract

Responding to the Classical criticism of the baseline Kaleckian growth model which is not fully adjusted in the long run, post-Kaleckians have proposed model variants that imply the economy to converge to a steady state in which the *realized* and the *normal* utilization rates as well as the *realized* and the *expected* secular rate of sales growth are congruent. Convergence is caused by endogenous adjustments of the conventional rates to their respective *realized* rates which is theoretically justified by hysteresis effects. Using a dynamic linear specification of the Kaleckian investment function in state-space form and by the aid of the Kalman filter, this paper studies the endogeneity of the *normal* utilization rate and the *expected* secular rate of sales growth empirically for the US manufacturing sector and its sub-sectors. We find evidence for an endogenous adjustment of both variables.

Keywords: Kaleckian growth model, hysteresis, investment, Bayesian econometrics, Kalman filter
JEL Classification: E12, E22

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1 Introduction

The baseline Kaleckian growth model pioneered by Rowthorn (1981), Dutt (1984) and Amadeo (1986) has been subject to severe criticism as it exhibits a long-run inconsistency between the *realized* rate of capacity utilization and the *normal* rate. If the firm's expected or target utilization rate is independent of realized rates, the question arises why it should settle even though its target is incongruent with the realization (cf. Committeri 1986, Auerbach and Skott 1988).[1]

Two responses to this criticism have been put forward by Kaleckians which aimed at preserving the core feature of the Kaleckian theory, i.e. the principle of effective demand, in long-run analysis: According to the first argument made by Dutt (2010), there may exist a continuum of feasible long-run equilibria. Motivated by Shackle's (1961) concept of *potential surprise* according to which a decision maker's behavior only alters in case of considerable changes in the environment, he argues that firms may not revise their investment behavior as long as the realized rates stay within a band of admissible target rates. While this is an appealing rationale for the reluctance of firms to adjust their behavior over a medium time horizon, the underlying assumption that the bands are constant as the economy converges to a long-run equilibrium may be contested.[2]

In contrast to the reasoning above, Lavoie (1995, 1996) and Dutt (1997, 2009) proposed model variants featuring fully adjusted steady states. Convergence is caused by endogenous adjustments of both the *normal* rate of utilization as well as the *expected* secular rate of sales growth to their respective *realized* rates via hysteresis effects. The endogeneity of this variables follows from their perception as conventions formed by firms facing fundamental uncertainty regarding future events. Firms tend to ground their assessment of *normal* rates and expectations in previously observed realizations which gives rise to a slowly and adaptively changing convention (Lavoie 1996). *Normal* capacity utilization may also be interpreted as a target of firms chosen according to the threat of competitive market entries (Dutt 1997).[3]

Endogenous adjustments of the *normal* utilization rate as well as *expected* secular rate of sales growth are crucial for making the Kaleckian growth model consistent with *steady-state* analysis while preserving the principle of effective demand (cf. Lavoie 1995).[4] Assuming

[1]For a survey of the debate from a Kaleckian perspective, see Hein et al. (2011a,b) and Lavoie et al. (2004).

[2]Similar to the initial criticism of the long-run inconsistency between the *realized* and the *normal* utilization rate put forward by Committeri (1986) and Auerbach and Skott (1988), one could argue that, in the long-run, firms become increasingly confident with an optimal utilization rate, and as a consequence, increasingly aware of a divergence to the *realized* rate and decreasingly willing to tolerate this deviation. This line of argument implies the bands around the optimal rate to become infinitely narrow in the long run.

[3]The terminology used in the literature for what we refer to as the *normal* rate of utilization depends on its interpretation. Using the attribute *standard* or *expected* suggests something defined by conventions whereas *target*, *planned* or *desired* indicate the belief that firms actively choose an optimal level of capacity utilization which they attempt to reach in the long run. As we will argue below, both interpretations are consistent with an endogenously adjusting long-run utilization rate. Hence, we use the attribute *normal* to indicate that the long-run rate my be a convention or a target.

[4]Note that the steady state requires no more than the consistency of expectations, targets and realized

that the continuum of admissible target rates becomes infinitely narrow in the long run and accepting the steady-state interpretation of the long run, a failure of the conventional rates to adjust endogenously causes the baseline Kaleckian model to be either valid only in the short and medium run or mis-specified as some crucial forces aligning the *realized* to the conventional rates in the long run are omitted.[5]

Despite their crucial roles for the long-run consistency of the Kaleckian growth model, the hypotheses of an endogenous *normal* utilization rate and of an endogenous *expected* secular rate of sales growth have not yet been studied empirically in a satisfying way. The endogeneity argument has been addressed econometrically by Lavoie et al. (2004) who compare the performance of different econometric investment models. Yet, they do not explicitly test the endogeneity hypotheses.[6] Skott (2008) is able to estimate the parameters of the adjustment of the conventional to the realized rates by OLS after transforming the Kaleckian model to exclude unobservable variables. However, in order to perform this transformation, a rather simple investment model has to be assumed. Further, the obtained estimates may be biased due to serial correlation in the residuals.[7]

To contribute to the empirical literature, this paper is an attempt to estimate the adjustment parameters and test the endogeneity hypotheses using quarterly data for the US manufacturing sector from 1984:1 to 2007:4. Since neither the *normal* rate of capacity utilization nor the *expected* secular rate of sales growth are observable, we specify the Kaleckian model as a dynamic linear model in the state space and utilize the Kalman filter which is an appealing method for estimating unobserved variables as well as their dynamics. It has been widely used in the orthodox literature to estimate unobservable variables such as the NAIRU (Gordon 1997) as well as the agency cost of external finance (Ogawa and Iiboshi 2008).

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values. Using the steady state as a theoretical point of reference does not imply that it is actually reached in reality. Most importantly and in contrast to Classical or Marxian long-run models, a steady state in the Kaleckian framework is not predetermined but endogenous and path-dependent.

[5]For instance, Marxians such as Duménil and Lévy (1999) and Shaikh (2009) accept the Kaleckian model for the short run but extend the model for the long run by introducing additional dynamics in the investment and/or savings function to ensure the adjustment of the *realized* utilization rate to a predetermined *normal* rate. Duménil and Lévy (1999) introduce a central bank performing anti-cyclical monetary policy. Shaikh (2009) assumes fixed investment to rise faster (slower) than output if the utilization differential is positive (negative).

[6]Lavoie et al. (2004) assesses the performance of various investment specifications based on different Marxian and Kaleckian adjustment processes. Using annual Canadian data on three sectors covering the period 1960-98, they find the Kaleckian model to outperform all the others based on information and encompassing tests. However, one may cast some doubt on the validity of these conclusions. First, the econometric specification of the Kaleckian model found to be superior gives rise to a severe endogeneity problem and, hence, to biased estimates (cf. Schoder 2011). Second, their interpretation of the results is questionable as their preferred specification does not in fact represent the hysteresis argument (cf. Skott 2008).

[7]Skott (2008) starts from a simple Kaleckian growth model with adaptive changes in the *normal* utilization rate and *expected* secular rate of sales growth. To eliminate these unobservable variables, he applies a Koyck (1954) transformation to the model. Based on his parameter estimates, Skott concludes that the long-run stability condition of the Kaleckian model is not met empirically. However, there is some reason to believe that his parameter estimates may be biased as discussed below.

As suggested by the Kaleckian literature, we suppose the dynamics of the *normal* utilization rate to follow a random walk process including an adjustment term which is a function of the lagged difference between the *realized* and the *normal* utilization rate. Similarly, the *expected* secular rate of sales growth is modeled as a random walk process including an adjustment term which is proportional to the difference between the lagged realized accumulation rate and the lagged *expected* secular rate of sales growth. Specifying the model in the state space allows us to directly estimate the adjustment parameters.

The model is fitted to both aggregate data and sectoral panel data. In both cases, we are able to reject the null hypothesis of no endogenous adjustment. For both the *normal* rate of capacity utilization and the *expected* secular rate of sales growth, we find positive and robust adjustment parameters as predicted by the long-run Kaleckian growth model.

To qualify our contribution, we do not seek to assess the overall empirical performance of the Kaleckian growth model which has been critized, among others, by Skott (2008).[8] Rather, we focus on estimating investment dynamics to study the endogenity of the *normal* rate of capacity utilization and the *expected* secular rate of sales growth which is a precondition for the validity of the hysteresis argument. Note, however, that the endogeneity of the conventional rates *per se* is only a necessary but not a sufficient condition for the validity of the Kaleckian growth model. As argued by (cf. Skott 2008), for instance, the *normal* utilization rate could follow the actual one within a predetermined narrow band, which would be equipollent to the assumption of an exogenous *normal* utilization rate. Finding answers to this question is delegated to future research.

The remainder of the paper proceeds as follows: Section 2 briefly outlines the baseline Kaleckian growth and distribution model which has been subject to criticism by various economic strands of Classical inspiration. Then, the mechanics by which the economy converges to a fully adjusted position through the adjustment of the *normal* utilization rate as well as the *expected* secular rate of sales growth are discussed. Section 3 briefly summarizes the econometric techniques applied, i.e. the state-space modeling approach and the Kalman filter. Section 4 presents the empirical model, discusses the estimation strategy and reports the estimation and test results. Section 5 concludes the paper.

2 Long-run adjustment in a Kaleckian growth model

There seems to exist a consensus among many economists on the behavior of the economy in the short run. The literature emphasizes the principle of effective demand stating that investment demand being, *ex ante*, independent of savings drives aggregate demand which determines output and employment. Regarding the long-run behavior of the economy, most economists agree that the *realized* utilization rate cannot deviate from the *normal* rate persistently. Yet, the adjustment processes are highly disputed as we will discuss in the

[8] The simple Kaleckian model predicts the *realized* utilization rate to fluctuate enormously if the economy is hit by shocks, given the linear form of the model and its constant parameters. With reasonable parameter values, shocks may even cause the equilibrium rate to leave its boundaries of 0 and 1. However, u seems to be relatively stable empirically fluctuating moderately between 70% and 90%.

following.

2.1 The baseline Kaleckian growth model

For simplicity, we consider a closed economy with a fixed-coefficient technology and without government activity. There is only one homogeneous good produced which is used for both consumption and investment. We assume away fixed costs. The economy comprises two classes: a representative capitalist who owns the means of production and hires labor-power; a representative worker who sells labor-power for a wage which is spent on consumption entirely. In the short-run, our economy is represented by the following set of aggregate relations:

$$g = \gamma + \beta_r r + \beta_u (u - u^N) \tag{1}$$

$$\sigma = sr \tag{2}$$

$$g = \sigma \tag{3}$$

$$r \equiv \frac{\pi u}{v} \tag{4}$$

$$\pi = \bar{\pi} \tag{5}$$

Equation (1) is one variant of the post-Kaleckian investment functions. The accumulation rate, g, is assumed to be determined by the following: the *expected* secular rate of sales growth, γ, the profit rate, r, and the difference between the *realized* and the *normal* rate of capacity utilization, $u - u^N$, which, for convenience, we will refer to as *utilization differential*.[9]

The *normal* rate of utilization as defined by post-Keynesians implies that each plant in use has to be operated to an optimal extent consistent with cost minimization. Firms, however, operating under fundamental uncertainty and confronting unexpected demand shifts attempt to defend or raise their market share by maintaining idle plants in order to stay flexible, a behavior often compared to Keynes' liquidity preference (Lavoie 1992, pp. 124-6). Hence, the *normal* utilization rate is perceived as a convention which may change over time. In fact, the adjustment of this convention will be crucial for endogenizing the *normal* utilization rate in the long run.

Equation (2) tells us that savings normalized by the capital stock, σ, is equal to the product of the capitalists' propensity to save, s, and the profit rate. (3) is the macro balance condition stating that income equals expenses, i.e. aggregate demand equals aggregate supply. (4) is the profit function as seen from the cost side. The profit rate can be decomposed into the profit share, π, times the rate of capacity utilization, u, over the capital-capacity output ratio, v. (5) states that distribution is determined exogenously as firms set prices according to a fixed mark-up on unit labor costs.

[9] To be precise, γ is interpreted as the expected *secular* rate of sales growth as it is independent of r and $u - u^N$. The expected steady-state rate of sales growth is $\gamma + \beta_r r^N$ where r^N is the profit rate consistent with u^N given $\bar{\pi}$ and v.

Simple algebra leads us to the the short-run equilibrium position of the economy which is given by

$$u^* = \frac{\gamma - \beta_u u^N}{(s - \beta_r)\frac{\bar{\pi}}{v} - \beta_u} \tag{6}$$

which, in general, will be different from the u^N as there is no adjustment built into the short-run model considered here. The short-run equilibrium of the accumulation rate is given by

$$g^* = s\frac{\gamma - \beta_u u^N}{(s - \beta_r)\frac{\bar{\pi}}{v} - \beta_u}.$$

Keynesian stability requires that savings respond more sensitively to changes in utilization than investment does, i.e.

$$(s - \beta_r)\frac{\bar{\pi}}{v} > \beta_u. \tag{7}$$

By taking the appropriate derivatives with respect to s and π one can easily show that, given plausible parameters, the *paradox of thrift*—an increase in the propensity to save slows down growth—and the *paradox of cost*—a higher real wage is consistent with faster growth—hold, respectively.

2.2 Introducing long-run adjustment

The baseline Kaleckian model has been accepted by many macro-economists as a sensible description of the economic forces at work in the short run (cf. Shaikh 2007 and Duménil and Lévy 1999). For long-run analysis, the validity of the Kaleckian framework has been questioned on theoretical grounds. Among others, Kurz (1986), Auerbach and Skott (1988) and Skott (2008) have argued that, as the *normal* rates are targets of the firms, the capitalists and, hence, the economy will not settle as long as u^* deviates from u^N. Today, the view that a valid theory of the long run requires the congruency of expectations, targets and realizations in the steady state seems to prevail among economists. Hence, in terms of our Kaleckian model, there seems to be consensus that

$$u^{**} = u^N$$

in the long run. In the steady state, the *realized* utilization rate needs to be equal to the target or *normal* rate. In this case, $r^{**} = r^N$ also holds at a given distribution and technology. Given (1) the economy will grow at rate $g^{**} = \gamma + \beta_r r^N$ and the growth expectations are in line with growth realizations.

Despite the consensus on the conceptual importance of a fully adjusted steady state, there is no consensus on how this steady state is reached. Several traverses towards the fully adjusted position have been proposed. Classical and Marxian authors alike have proposed mechanisms by which the *realized* rate adjusts to an exogenously determined *normal* rate in the long run. In Duménil and Lévy (1995, 1999) a positive (negative) utilization differential, $u - u^N$, induces the investment function to shift downwards (upwards) as the monetary authority fighting inflationary (deflationary) pressures is argued to raise (lower) interest rates as long as the disequilibrium persists. For Shaikh (1991, 2009) investment is also adjusting in the long run so as to equalize *realized* and *normal* utilization. Firms are argued to respond to a positive (negative) utilization differential by raising (lowering) fixed investment faster than output. This will cause the share of material investment to decline (rise) and output growth and utilization to fall (rise) eventually.

These classical long-run traverses reverse the implications of the initial model fundamentally. The *paradox of thrift* and the *paradox of cost* disappear in the long run and equilibrium output is predetermined, i.e. not caused by aggregate demand: higher economic growth is now associated with a higher saving rate; a higher profit rate implying lower real wages is beneficiary to growth. Moreover, many post-Keynesian authors stressing the role of uncertainty and conventions as the guidance for economic behavior as well as the path dependence of economic development have not been convinced by the proposed trajectories towards an exogenously given position independent of short-run demand dynamics as suggested by Classical economists.

As an alternative to the Classical adjustment stories, some Kaleckians, most notably Lavoie (1995, 1996) and Dutt (1997, 2009), put forward channels by which the economy converges to the fully adjusted position without necessarily loosing the short-run implications of the Kaleckian model.

Two channels have been proposed: One follows from the perception of u^N as a convention which changes slowly over time through hysteresis effects. If firms face an inconsistency between the *realized* and *normal* rates of utilization or profit, they may adjust their perception of what is *normal* (cf. Robinson 1956, 186-90). As argued by Lavoie (1995, 1996) and Cassetti (2006), hysteresis in the adjustment of u^N and r^N may affect the investment decision, the pricing decision or both. Here, we want to focus on a Kaleckian traverse based on hysteresis through the investment function only.[10]

Going beyond a mere conventional determination of the u^N, Dutt (1997) attempts to derive an endogenous adjustment of u^N from strategic considerations of the firm. Expecting a rate of market entries higher than it is today, so he argues, may induce firms to reduce their target rate allowing them to respond more spontaneously to demand fluctuations. Dutt specifies the change in u^N as

$$\dot{u}^N = -\lambda((\gamma + \beta_r r) - g) \quad \text{with } \lambda > 0 \tag{8}$$

[10] Apart from the long-run adjustment through changes in the firms' investment behavior, Lavoie (1996) additionally outlines a possible mechanism through the price setting behavior of firms. Forming their profit and utilization targets adaptively and endogenizing distribution, firms are argued to increase the mark-up in response to a higher profit rate target. However, it is questionable that firms have control over the mark-up to the extent required for this adjustment process to work. Pricing decisions are contested decisions which depend on the relative power of different social groups such as owners, managers and workers (cf. Dallery and van Treeck 2011).

where $\dot{u}^N$ is the derivative of u^N with respect to time. (8) is justified by assuming a proportional relationship between the accumulation rate and planned excess capacity. Here, $\gamma + \beta_r r$ is interpreted as the expected rate of market entries which can be expected to be highly correlated with the steady-state rate of demand growth whereas the realized rate of market entries is reflected by g.[11]

Both channels imply the change of u^N to be a function of the divergence of the *realized* from the *normal* utilization rate (Amadeo 1986, p. 155). Substituting (1) into (8) yields

$$\dot{u}^N = \phi_{u^N}(u^* - u^N) \quad \text{with } \phi_{u^N} > 0 \tag{9}$$

where $\phi_{u^N} = -\lambda\beta_u$ captures the speed of adjustment. One can now substitute the short-run equilibrium value u^* from (6) into (9) and easily derive the fully adjusted position.

The endogeneity of u^N is crucial for the Kaleckian traverse to work. If it was exogenous, i.e. independent of past values of *realized* utilization, then the economy would not converge towards a steady state. Hence, the endogenous adjustment of the *normal* rate of utilization is of great importance for the validity of the Kaleckian model as a tool for long-run analysis. Endogenizing the dynamics of u^N is sufficient to maintain the principle of effective demand in the long run but not to have the model economy feature the *paradox of thrift* and the *paradox of cost* in the steady state. As can be verified easily, the steady-state growth rate is solely determined by the autonomous part of the accumulation function in (1) which is interpreted as the secular rate of sales growth (Lavoie 1996, p. 138).[12] Change in the saving propensities or distribution do not alter the steady-state growth rate. To ensure the economy to exhibit Keynesian features in the long run, Lavoie (1995) motivates hysteresis in the secular rate of sales growth. If u^* exceeds u^N, the realized growth rate, g^*, will exceed the secular rate of sales growth, γ. Hence, equivalently to the normal utilization rate, also the secular growth rate may adjust adaptively to past realizations. In particular it is argued that

$$\dot{\gamma} = \phi_\gamma(g^* - (\gamma + \beta_r r)) \quad \text{with } \phi_\gamma > 0. \tag{10}$$

The Kaleckian model comprising (1)-(5) and (9)-(10) is fully adjusted in the long-run and sustains the role of effective demand as well as the paradoxes of thrift and cost.

Following Skott (2008), one can show that convergence of the dynamic system specified in (1)-(5), (9) and (10) to a stationary solution requires that

$$(s - \beta_r)\frac{\bar{\pi}}{v} > \frac{\phi_\gamma}{\phi_{u^N}}\beta_u. \tag{11}$$

3 Bayesian inference and state-space models

The main difficulty in studying the *normal* utilization rate is the fact that it is not observable. Typically, attempts to estimate investment specifications which take into account *normal*

[11] Skott (2008) criticizes this line of argument as a slight deviation of the expected from the realized rate causes a permanent change in the utilization target.

[12] In case of additional determinants of investment such as the profit rate or the profit share, the steady-state growth rate would still be equal to the secular rate of sales growth which, however, now includes all terms of the accumulation function independent of the utilization differential.

utilization apply a smoothing method such as a Hodrick-Prescott filter to the time series of *realized* utilization in order to obtain an approximation of the normal rate (cf. Lavoie et al. 2004, Hein and Schoder 2011). To analyze the possible endogeneity of u^N, however, this procedure involves a major problem: The filtered series does not necessarily describe an endogenous u^N better than an exogenous u^N as the causality between u and the filtered series might go in both directions.[13] In fact, the opposite might be the case since the filtered series is inconsistent with the suggested adaptive behavior of the endogenously changing *normal* utilization rate (Skott 2008).

Another method to handle the unobservability of u^N and γ is to eliminate them by applying a Koyck (1954) transformation to the system as demonstrated by Skott (2008). A Koyck transformation removes an unobservable auto-regressive variable from an equation by subtracting from the latter an appropriately manipulated and appropriately lagged variant of that equation. However, for the Koyck transformation to work, the system of equations has to be sufficiently simple. Even if an equation without unobservables can be obtained, it is not always possible to recover the parameters of the initial equation.

For our purposes, Bayesian estimation techniques such as the Kalman filter are an appealing alternative to traditional filter methods and the Koyck transform. It allows us to estimate the *normal* utilization rate and the *expected* secular growth rate of sales as well as their respective dynamic behavior over time as implied by the representative firm's investment behavior.

In the orthodox empirical literature, the Kalman filter is widely used for estimating unobserved variables. One particular field of application is the NAIRU theory. Equivalent to our analysis of investment and the *normal* rate of capacity utilization, Gordon (1997), for instance, applies the Kalman filter to a dynamic Phillips curve specification to estimate the unobserved NAIRU and study its changes over time. Ogawa and Iiboshi (2008) uses the Kalman filter to estimate the agency cost of external finance within the framework of a dynamic investment and borrowing model.

Since the econometric techniques employed in this study do not yet belong to the standard repertoire of most heterodox economists, the remainder of the section seeks to provide the intuition of estimating dynamic linear models using Bayesian econometrics.[14]

3.1 Some Bayesian concepts

Traditional statistical analysis makes use of point estimators and hypothesis testing to make inference on an unknown characteristic, θ, of the data generating process which is typically a parameter vector. In contrast to that, Bayesian approaches seek to compute $\pi(\theta|y)$, i.e.

[13] For instance, Lavoie et al. (2004) use an HP-filtered series of u as a proxy for an endogenous u^N and the sample mean of u as a proxy for an exogenous u^N. However, it is not clear why the HP-filtered series should not be interpreted as an exogenous *normal* utilization rate.

[14] An excellent treatment of Bayesian analysis is provided by Bernardo and Smith (1994). Dynamic linear models and the Kalman filter are concisely presented by Hamilton (1994). For a more practical guide how to perform Bayesian analysis with the software package *R*, see Petris et al. (2009). The following subsections draw from Petris et al. (2009) and Hamilton (1994).

the conditional distribution (or posterior distribution) of θ, which may also include variables such as the *normal* rate of capacity utilization or the *expected* secular growth rate of sales, given the sampling result described by a random vector Y with a possible realization y. In particular, knowledge of the conditional distribution $\pi(y|\theta)$ taken from economic theory as well as of the prior distribution $\pi(\theta)$ reflecting the uncertainty attached to θ are required. Then $\pi(\theta|y)$ can be computed using the well-known Bayes' formula,

$$\pi(\theta|y) = \frac{\pi(y|\theta)\pi(\theta)}{\pi(y)}$$

with $\pi(y) = \int \pi(y|\theta)\pi(\theta)\mathrm{d}\theta$ being the marginal distribution of Y.

In time series applications the assumed dependence structure of a vector of time series Y_t (with $t = 1, \ldots, n$) is crucial. A common assumption made is that $Y_1, \ldots, Y_n$ are conditionally independent and identically distributed (i.i.d.) given a characteristic θ. In practice, this usually means that the disturbance terms in parametric time series models are assumed to be i.i.d. Note, however, that Y_{n+1} still depends on the previous observations $Y_1, \ldots, Y_n$ through θ. The main implication of this assumption for the Kalman filter is that the posterior distribution of the characteristic θ, i.e the distribution $\pi(\theta|y_{1:n})$ of θ given the previous observations $y_1, \ldots, y_n$, can be computed recursively. To see this, note first that

$$\pi(\theta|y_{1:n}) = \frac{\pi(y_{1:n}|\theta)\pi(\theta)}{\pi(y_{1:n})} \propto \prod_{t=1}^{n} \pi(y_t|\theta)\pi(\theta) \tag{12}$$

where the equality follows from Bayes' formula and the proportionality (indicated by the sign $\propto$) follows from the fact that $\pi y_{1:n}$ is independent from θ and the assumption of conditional independence which implies that $\pi(y_{1:n}|\theta) = \prod_{i=1}^{n} \pi(y_i|\theta)$. Hence, using Bayesian terms, the posterior distribution of θ is proportional to the product of the likelihood and the prior distribution of θ. Note that the the posterior in period $n-1$, $\pi(\theta|y_{1:n-1}) \propto \prod_{t=1}^{n-1} \pi(y_t|\theta)\pi(\theta)$, can be interpreted as the prior in period n. Combining it with the likelihood, which is $\pi(y_n|\theta, y_{1:n-1}) = \pi(y_n|\theta)$ due to the assumption of conditional independence, we can use the Bayes formula to update the prior in period n, i.e.

$$\pi(\theta|y_{1:n-1}, y_n) \propto \pi(\theta|y_{1:n-1})\pi(y_n|\theta) \propto \prod_{t=1}^{n-1} \pi(y_t|\theta)\pi(\theta)\pi(y_n|\theta)$$

which is equal to (12).

3.2 Dynamic linear models

State-space models perceive time series as dynamic systems perturbed by random shocks. For most time series, the assumption of independence is unjustified. Yet, building on the dependence structure of a Markov chain state-space models can represent complex dynamic systems in a way that allows for Bayesian estimation and inference as outlined above. A process Y_t with $t \geq 1$ is a Markov chain if the observations prior to $t-1$ do not contain any information about Y_t which is not already contained in y_t, i.e. if, for any $t > 1$, $\pi(y_t|y_{1:t-1} = \pi(y_t|y_{t-1})$. State-space models assume an underlying possibly unobservable Markov chain, θ_t, describing the evolution of the state of affairs as well as Y_t to be an imprecise measure of the state θ_t. The Y_t's are independent conditionally on θ_t and Y_t depends only on θ_t. If this assumptions are meet, the state-space model is fully specified by the initial distribution $\pi(\theta_0)$ as well as the conditional distributions $\pi(\theta_t|\theta_{t-1})$ and $\pi(y_t|\theta_t)$. For $t > 0$, we have $\pi(\theta_{0:t}, y_{1:t}) = \pi(\theta_0)\prod_{j=1}^{t}\pi(\theta_j|\theta_{j-1})\pi(y_j|\theta_j)$ (Petris et al. 2009, pp. 39-41).

The dynamic linear model is a special case of a state-space model which is linear and features Gaussian disturbances and a Gaussian prior distribution of the state vector. The model comprises the prior for the p-dimensional state vector

$$\theta_0 \sim \mathcal{N}_p(m_0, C_0) \tag{13}$$

as well as the *observation equation* and *state equation*

$$y_t = F_t\theta_t + v_t, \quad v_t \sim \mathcal{N}_m(0, V_t), \tag{14}$$

$$\theta_t = G_t\theta_{t-1} + w_t, \quad w_t \sim \mathcal{N}_p(0, W_t). \tag{15}$$

y_t is an $(m \times 1)$ vector including variables observed at t which is described by a $(p \times 1)$ vector comprising both observable predetermined variables as well as non-observable variables which determine the state of the system at t. The $(m \times p)$ dimensional parameter matrix F_t specifies how the elements of the state vector at t affect the observations at t. The dynamics of the state vector are described in the state equation which models the evolution of the state vector over time as a first order autoregressive process with the $(m \times p)$ dimensional transition matrix G_t together with the $(p \times 1)$ vector of evolution errors w_t specifying the transition of the state vector from one period to the other. Note that F_t and G_t may be time-variant (but not in our model). The vectors of disturbance terms v_t and w_t are assumed to be Gaussian with zero mean and variance-covariance matrices V_t and W_t, respectively, which both may also be time-variant. If it is assumed that θ_0 is independent of v_t and w_t, one can show that (13)-(15) describes a fully specified state-space model.

3.3 The Kalman filter

In many time series applications such as our analysis of the unobserved *normal* utilization rate and the *expected* secular growth rate of sales, one is interested in making inference on the state vector θ_t. The aim is to employ the state-space model assumed to accurately describe the data generating process to extract the information on the states from the observations y_t. To achieve this, filtering (such as the Kalman filter for linear models) and smoothing algorithms may be applied which differ in the set of information used to make inference on θ_t. Here, we focus on filtering.[15] Filtering methods such as the Kalman filter allow us to make

[15] Conceptually the difference is that the filter uses the information in $y_{1:t}$ to make inference on θ_t, whereas the smoother uses $y_{1:t}$ to infer on $\theta_{0:t}$. Hence, at the end of the iterative process the filter produced a series whose element θ_t uses only information up to time t whereas the smoother yields a series whose element θ_t

inference on the posterior distribution of the state vector $\pi(\theta_t|y_{1:t})$ which can be updated recursively as we have sketched out above.

To illustrate the filtering recursions for a general space state model which the Kalman filter is a special case of, let us reproduce here Petris et al.'s (2009) Proposition 2.1 which provides a concise insight into the filtering algorithm:

Proposition 1. *For a general state-space model defined by the assumptions that (a) θ_t is a Markov chain and (b) the Y_t's are independent conditionally on θ_t and Y_t depends on θ_t only, the following statements hold.*

(i) The one-step-ahead predictive distribution for the states can be computed from the filtered distribution $\pi(\theta_{t-1}|y_{t-1})$ according to

$$\pi(\theta_t|y_{1:t-1}) = \int \pi(\theta_t|\theta_{t-1})\pi(\theta_{t-1}|y_{1:t-1})\,d\theta_{t-1}. \tag{16}$$

(ii) The one-step-ahead predictive distribution for the observations can be computed from the predictive distribution for the states as

$$\pi(y_t|y_{1:t-1}) = \int \pi(y_t|\theta_t)\pi(\theta_t|y_{1:t-1})\,d\theta_t. \tag{17}$$

(iii) The filtering distribution can be computed form the above distributions as

$$\pi(\theta_t|y_{1:t}) = \frac{\pi(y_t|\theta_t)\pi(\theta_t|y_{1:t-1})}{\pi(y_t|y_{1:t-1})}. \tag{18}$$

Proof. See Petris et al. (2009, p. 52) □

Starting from an assumed prior distribution of the state $\pi(\theta_0)$, the algorithm first uses the conditional distribution $\pi(\theta_1|\theta_0)$ implied by the state transition equation to compute the predictive distribution $\pi(\theta_1)$ (which is unconditional as y_0 does not exist) in equation (16). In the second step, the distribution of the next observation, i.e. $\pi(y_1)$ as there is no y_0 available, is computed using the previously estimated filtered distribution of θ_1 as the prior and the conditional distribution of y_1 given θ_1 as specified in the observation equation as the likelihood (equation 17). In the last step, the latter conditional distribution, $\pi(y_1|\theta_1)$, the predictive distribution $\pi(\theta_1)$ and the predictive distribution $\pi(y_t|y_{1:t-1})$ are used to compute the filtered distribution $\pi(\theta_1|y_1)$ which is the result of the filter for period $t = 1$ (equation 18). The estimate of the distribution of θ_1 has been updated as the observation y_1 has become available. In period $t = 2$ the recursion starts over with the filtered distribution $\pi(\theta_1|y_1)$ serving as the prior for computing the predictive distribution $\pi(\theta_2|y_1)$.

uses all information present in the period covered, i.e. up to time T. Hence, in our context the use of the smoother seems inappropriate as u_t^N and γ_t cannot be plausibly argued to be affected by all its leads until T.

This recursive algorithm for sequentially updating the projection of a system of equations is called Kalman filter, if the underlying state-space model is a dynamic linear model, i.e. consists of linear equations and features Gaussian disturbances and a Gaussian prior of the initial state vector, such as the one given in (13)-(15). These additional assumptions facilitate the computation of the relevant distributions $\pi(\cdot)$ enormously which are, then, all Gaussian, i.e. fully described by an expected value and a variance.

3.4 Estimating unknown parameters

Some parameters in the dynamic linear model described in (13)-(15) may be unknown. In fact, the adjustment parameters of the *normal* utilization rate and of the *expected* secular growth rate of sales in the Kaleckian model specified below are not known and of interest to us. Fortunately, a vector of unknown parameters, ψ, and the variance-covariance matrix can be estimated through Maximum Likelihood Estimation (MLE). Let $\pi(y_{1:T};\psi)$ denote the joint probability distribution of the observations for a given parameter vector ψ. Recall that the assumption of conditional independence implies for dynamic linear models that $\pi(y_{1:T};\psi) = \prod_{t=1}^{T} \pi(y_t|y_{t-1};\psi)$ where $\pi(y_t|y_{t-1})$ is the predictive distribution for the observations as specified in (17). One can show that under the assumption of Gaussian disturbances and a Gaussian prior on the initial state vector the conditional distribution $\pi(y_t|y_{t-1})$ is also Gaussian with mean $f_t = F_t G_t m_{t-1}$ and variance $Q_t = F_t(G_t C_{t-1} G_t' + W_t)F_t' + V_t$ which both depend on ψ. The resulting log-likelihood function is

$$\log L(\phi) = -\frac{1}{2}\sum_{t=1}^{T} \log|Q_t| - \frac{1}{2}\sum_{t-1}^{T}(y_t - f_t)' Q_t^{-1}(y_t - f_t)$$

which is numerically maximized with respect to ψ. This yields a point estimate $\hat{\psi}$ and an estimate of the variance, $\mathrm{Var}(\hat{\psi})$.

Note that especially $\mathrm{Var}(\hat{\psi})$ has to be interpreted with care, as it is obtained numerically and not analytically. In the empirical analysis, we therefore provide additional Likelihood-Ratio tests of the significance of those variables which are crucial for our research question.

4 Empirical analysis

4.1 Data

We use quarterly data covering the US manufacturing sector as well as 18 sub-sectors from 1984:1 to 2007:4.[16] The former limit is due to the availability of investment data for the sectors under study. The latter has been chosen in order to avoid distortions caused by recent financial crisis. Data on the rate of capacity utilization for the manufacturing sectors considered in our study have been taken from the Fed's G.17 Release–Industrial Production and

[16] The sub-sector of manufacturing are the sectors identified by the NAICS codes 311-2, 313-4, 315-6, 321, 322, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335, 336, 337, 339.

Capacity Utilization. Note that these data are based on surveys.[17] The accumulation rate and the profit rate have been constructed using Compustat data on market-listed manufacturing firms.[18] The respective items in the firms balance sheets and income statements have been aggregated over the manufacturing sector and the sub-sectors considered. The series have been seasonally adjusted for the total manufacturing sector, but not for the sub-sectors as they are used in a panel estimation after removing fixed group and time effects.

We pursue our analysis using both aggregated data and panel data because of the following reason. We want our results to be comparable to previous empirical attempts to estimate investment in a general post-Keynesian framework such as Lavoie et al. (2004) and Skott (2008). These attempts usually employed aggregate data of the manufacturing or business sector. However, as is well-known in the empirical macroeconomic literature, estimations of investment functions on an aggregate level are inherently prone to endogeneity issues and thus to biased parameter estimates. Recalling the Kaleckian short-run model outlined above, this is because the equilibrium utilization rate, u^*, is not an exogenous but an equilibrating variable. Hence, investment g^* may affect u^* (and its lags), for instance, through cyclical changes in the saving propensities. These potential endogeneity problems can be overcome either by using an instrumental variable estimator or by estimating the investment dynamics on a lower level of aggregation. Here, we follow the second approach and estimate the investment model on a three-digit NAICS level by pooling the data after removing fixed group and time effects by applying appropriate within transforms to the data.[19]

4.2 The econometric model

This section discusses how the Hysteresis Kaleckian model outlined above can be transformed into state-space form which allows us to estimate unknown parameters such as the speed of adjustment of the *normal* utilization rate, ϕ_{u^N}, and the *expected* secular growth rate of sales, ϕ_γ, by MLE as well as the state vector including u_t^N and γ_t by the Kalman filter.

The dynamics of the underlying investment model are represented by an econometric version of the investment function in (1) as well as the adjustments specified in (9) and (10). In the benchmark specification the accumulation rate is assumed to be a linear function of the profit rate as well as the utilization rate differential both accompanied by some lags as investment expanses may respond delayed to the profit and utilization impulse. Contempo-

[17] The documentation reads the following: "For a given industry, the capacity utilization rate is equal to an output index (seasonally adjusted) divided by a capacity index. The Federal Reserve Board's capacity indexes attempt to capture the concept of *sustainable maximum output*—the greatest level of output a plant can maintain within the framework of a realistic work schedule, after factoring in normal downtime and assuming sufficient availability of inputs to operate the capital in place." Hence the capacity utilization data suits our purpose well, as it is derived from the firms' perceptions.

[18] The accumulation rate is the ratio of quarterly capital expenses for property, plant and equipment minus quarterly depreciation and the net stock of capital. The profit rate is computed as the ratio of quarterly net income before dividends and the net stock of capital.

[19] Note that there is an additional source of endogeneity of u which disaggregation cannot solve. Obviously, investment creates additional capacity. In the empirical analysis, this is taken care of by excluding the contemporaneous values of the covariates.

raneous values of the regressors are excluded as the estimation procedure applied requires the covariates to be predetermined.[20] In order to account for serial correlation we model the disturbance terms in the investment function as an AR(1) process with parameter ρ. We have

$$g_t = \gamma_t + \sum_{i=1}^{L} \beta_{r,i} r_{t-i} + \sum_{i=1}^{L} \beta_{u,i}(u_{t-i} - u_{t-i}^N) + \mu_t \tag{19}$$

$$\mu_t = \rho \mu_{t-1} + v_t, \quad v_t \sim \mathcal{N}(0, \sigma_v^2) \tag{20}$$

where g_t, r_t and $u_t - u_t^N$ are the accumulation rate, the profit rate and the utilization differential, respectively. $\beta_{j,i}$ with $j = r, u$ are parameters. The disturbance term in the investment function μ_t is assumed to follow a first order autoregressive process with normally distributed disturbances with mean zero and variance σ_v^2.

The econometric discrete-time version of the dynamics of u_t^N given in (9) is

$$\begin{aligned} u_t^N &= u_{t-1}^N + \phi_{u^N}(u_{t-1} - u_{t-1}^N) + w_{u^N,t} \\ &= \phi_{u^N} u_{t-1} + (1 - \phi_{u^N}) u_{t-1}^N + w_{u^N,t} \quad w_{u^N,t} \sim \mathcal{N}(0, \sigma_{u^N}^2) \end{aligned} \tag{21}$$

where ϕ_{u^N} measures how fast the *normal* rate adjusts to a change in the utilization differential. The econometric equivalent to (10) which specifies the dynamics of γ_t is

$$\begin{aligned} \gamma_t &= \gamma_{t-1} + \phi_\gamma (g_{t-1} - (\gamma_{t-1} + \sum_{i=1}^{L} \beta_{r,i+2} r_{t-(i+2)})) + w_{\gamma,t} \\ &= \gamma_{t-1} + \phi_\gamma \sum_{i=1}^{L} \beta_{u,i+1}(u_{t-(i+1)} - u_{t-(i+1)}^N) + \phi_\gamma \mu_{t-1} + w_{\gamma,t} \quad w_{\gamma,t} \sim \mathcal{N}(0, \sigma_\gamma^2) \end{aligned} \tag{22}$$

where the second line is obtained by substituting (19) into the first line and rearranging.

Let us now transform (19), (20), (21) and (22) into the linear state-space form of (13) to (15) in order to estimate the the unknown model parameters and to make inference on the *normal* utilization rate as well as the *expected* secular growth rate of sales. Although various specifications with different lag structures have been estimated with the results reported below, we here assume a lag structure of $L = 2$ for illustrative purposes. In this case, the variable and parameter vectors read

$$y_t = \begin{bmatrix} g_t & r_{t-1} & r_{t-2} & u_{t-1} & u_{t-2} & u_{t-3} \end{bmatrix}' \tag{23}$$

$$\theta_t = \begin{bmatrix} \gamma_t & r_{t-1} & r_{t-2} & u_{t-1} & u_{t-1}^N & u_{t-2} & u_{t-2}^N & u_{t-3} & u_{t-3}^N & \mu_t \end{bmatrix}' \tag{24}$$

$$F = \begin{bmatrix} 1 & \beta_{r,1} & \beta_{r,2} & \beta_{u,1} & -\beta_{u,1} & \beta_{u,2} & -\beta_{u,2} & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{25}$$

[20] However, our results are robust to including the contemporaneous values of the covariates.

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \phi_\gamma \beta_{u,2} & -\phi_\gamma \beta_{u,2} & \phi_\gamma \beta_{u,3} & -\phi_\gamma \beta_{u,3} & \phi_\gamma \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_{u^N} & 1-\phi_{u^N} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho \end{bmatrix} \tag{26}$$

$$W = \mathrm{diag}(\sigma_\gamma^2, \sigma_r^2, 0, \sigma_u^2, \sigma_{u^N}^2, 0, 0, 0, 0, \sigma_v^2) \tag{27}$$

$$V = 0. \tag{28}$$

diag(x) is a $(p \times p)$ dimensional diagonal matrix with the vector x as the diagonal elements. σ_r^2 and σ_u^2 denote the variances of the disturbances in the models of the profit rate and the utilization rate, respectively.

Substituting the variables and parameters matrices defined in (23) to (28) into equations (14) and (15) yields a rather ostentatious representation of the simple system of equation given by (19) to (22). They can be easily recovered by appropriate matrix manipulations. Specifying the model in state-space form allows us not only to fit the model through Maximum Likelihood Estimation but also to apply the Kalman filter to estimate the state vector.

Identification is a serious issue for estimating dynamic linear models by MLE. To ensure identification of unknown parameters at least one parameter has to be restricted. In practice, however, the model should be as simple as possible with as many parameter restrictions as economic theory can provide given the problem of small samples in most economic applications. The Kaleckian model imposes most of the restrictions on the parameter matrices F, G, W and V. However, some additional restrictions have been imposed which do not necessarily arise from Kaleckian theory but are required for estimation and shall be identified here.

First, given the set-up of the dynamic linear model, the observed covariates which enter the state vector θ_t in contemporaneous values, i.e. r_t and u_t, need to be assumed to follow random walk processes. The variances of the respective disturbances, σ_r^2 and σ_u^2, are unrestricted. Hence, since the variables are observed the estimated variances will be such that the disturbances perfectly describe the data given the random walk hypothesis. This assumption is innocent because it does not affect the estimation of u_t^N or γ_t as all relevant cross-correlations are restricted to zero. It raises the degree of freedom and allows for a better fit.[21]

Second, for the econometric analysis we restrict the variances of the *normal* utilization rate and of the *expected* secular growth rate of sales to zero, i.e $\sigma_{u^N}^2 = \sigma_\gamma^2 = 0$. The resulting series for $\hat{u}_t^N$ and $\hat{\gamma}_t$ can then be understood as filtered series abstracting from the

[21] Assuming the observed variables in the state vector to follow a random walk process is common practice in the literature (cf. Gordon 1997).

influence of random disturbances. These restrictions are required in order to obtain results which are robust to changes in the lag structure and the specification of the investment function. Without these additional assumptions, the MLE is only weakly identified since, given the limited size of the sample, it cannot easily differentiate between disturbances in the investment function, in the *expected* secular growth rate of sales or *normal* utilization rate. In this case, the MLE assigns too much noise from the disturbances of the investment function to $\hat{u}_t^N$ and $\hat{\gamma}_t$ which then overshoot/undershoot the actual rates by a multiple. Restricting the variances of the unobserved processes is thus very common in the literature (Gordon 1997, Semmler et al. 2005). Moreover, our results are robust to any choice of $\sigma^2_{u^N}$ and σ^2_γ as long as they are sufficiently small.

4.3 Estimation results

As the Kalman filter needs information on the prior distribution of the state vector in the initial period, we initialize the system in the following manner: For the profit rate and the utilization rate and their respective lags, we use the first observation in the respective series as these are the best guesses given the persistence in the data. The *normal* rate of utilization and its lags are initialized by the average of the ten first observations of the *realized* utilization rate series.[22] The initial value for *expected* secular rate of sales growth is zero. For the MLE, we initialize the unrestricted parameters to zero.

The results of fitting the model given in (19) to (26) and (23) to (28) to data on the aggregate manufacturing sector as well as to a panel of sub-sectors are reported in Table 1. Estimation results for three different lag structures, i.e. $L = 3, 4, 5$, are presented. Since quarterly data is being used, these lag structures seem to be a reasonable choice. Including longer lags is considerably more computationally intensive, but does not affect the results substantially.

Notice that the estimated t-statistics have to be interpreted with caution. The t-test is likely to have low power as the variance-covariance matrix is not computed analytically but approximated numerically. To check the robustness of the t-tests, we additionally apply conventional Likelihood-Ratio (LR)-tests on the estimated coefficients and report the test results for the estimates of the adjustment parameters as well as for those coefficients for which a negative variance has been computed. We also apply LR-tests to check the joint significance of a variable's lags in explaining investment.

For the models fitted to aggregate data, we find the following: The estimates for the parameter ϕ_{u^N} which measures the adjustment of the *normal* utilization rate are in the range of 0.05 and 0.06. The estimates for ϕ_γ which is the parameter measuring the adjustment of the *expected* secular growth rate of sales reach from 0.09 to 0.13. These results are fairly robust to the lag-length chosen. The t-statistics which follow a t-distribution indicate, however, that we cannot reject the null hypothesis of the adjustment parameters being significantly different from zero at any reasonable level of confidence. The statistical significance of the estimated adjustment parameters is higher according to the computed likelihood ratios which

[22] Experiments with changing the time range for calculating u_0^N did not alter the results.

Table 1: Estimation results for different lag structures

AGGREGATE DATA

	$\beta_{r,1}$	$\beta_{r,2}$	$\beta_{r,3}$	$\beta_{r,4}$	$\beta_{r,5}$	$\beta_{u,1}$	$\beta_{u,2}$	$\beta_{u,3}$	$\beta_{u,4}$	$\beta_{u,5}$	ρ	ϕ_{u^N}	ϕ_γ	$\sum_{i=1}^{L}\beta_{r,i}$	$\sum_{i=1}^{L}\beta_{u,i}$	# of obs.
L=3	0.084 (1.57)	0.138 (2.38)	−0.001 (−0.01)	—	—	−0.025 (−0.49)	−0.019 (−0.30)	0.100 (1.94)	—	—	0.548 (3.55)	0.051 (0.84) [1.75]	0.091 (0.93) [2.51]	0.221 [12.14***]	0.054 [6.12]	92
L=4	0.099 (1.87)	0.095 (1.69)	−0.097 (−1.63)	0.196 (3.70)	—	−0.028 (−0.59)	0.016 (0.26)	−0.007 (−0.11)	0.060 (1.26)	—	0.453 (2.94)	0.063 (0.98) [2.98*]	0.113 (1.22) [2.72*]	0.293 [27.19***]	0.041 [4.58]	91
L=5	0.090 (1.70)	0.094 (1.72)	−0.121 (−2.07)	0.132 (2.31)	0.130 (2.50)	−0.011 (−0.23)	0.006 (0.09)	0.018 (0.29)	0.017 (0.28)	0.001 (0.03)	0.395 (2.48)	0.060 (0.93) [1.96]	0.126 (1.33) [2.93*]	0.325 [29.76***]	0.032 [2.55]	90

PANEL DATA

	$\beta_{r,1}$	$\beta_{r,2}$	$\beta_{r,3}$	$\beta_{r,4}$	$\beta_{r,5}$	$\beta_{u,1}$	$\beta_{u,2}$	$\beta_{u,3}$	$\beta_{u,4}$	$\beta_{u,5}$	ρ	ϕ_{u^N}	ϕ_γ	$\sum_{i=0}^{L}\beta_{r,i}$	$\sum_{i=0}^{L}\beta_{u,i}$	# of obs.
L=3	0.039 (5.08)	0.039 (4.96)	0.027 (3.57)	—	—	0.012 (0.75)	0.034 (1.70)	0.019 (1.13)	—	—	0.315 (7.67)	0.070 (1.66) [30.06***]	0.099 (3.61) [22.18***]	0.107 [49.32***]	0.067 [51.50***]	1,656
L=4	0.038 (4.86)	0.042 (5.35)	0.031 (4.00)	0.027 (3.54)	—	−0.008 (−0.49)	0.027 (1.32)	0.017 (0.81)	0.020 (1.10)	—	0.303 (7.83)	0.091 (1.27) [28.04***]	0.093 (3.81) [32.69***]	0.141 [156.47***]	0.057 [37.44***]	1,638
L=5	0.036 (4.67)	0.036 (4.65)	0.026 (3.40)	0.032 (4.08)	0.002 (0.29)	0.016 (0.96)	0.031 (1.54)	0.020 (0.97)	0.001 (0.07)	0.000 (0.03)	0.266 (6.55)	0.074 (1.49) [51.11***]	0.124 (4.23) [33.12***]	0.134 [179.76***]	0.071 [41.06***]	1,620

Notes: t-statistics are in parenthesis, the log-likelihood ratio statistics in brackets. Significance levels are only reported for the log-likelihood ratio statistics. *, **, and *** denote the significance levels at 10%, 5%, and 1%, respectively.

follow a χ^2-distribution. ϕ_{u^N} and ϕ_γ are significantly different from zero at the 10%-level for $L = 4$ and $L = 4, 5$, respectively. The total effects of the profit rate, $\hat{\beta}_r = \sum_{i=1}^{L} \hat{\beta}_{r,i}$, and the utilization differential, $\hat{\beta}_u = \sum_{i=1}^{L} \hat{\beta}_{u,i}$, on investment range from 0.22 to 0.32 and from 0.03 to 0.05, respectively. While the former estimates seem to be statistically significant the latter do not.

Given the tendency of the endogeneity of the lags of the utilization rate to rise with the level of aggregation, one should put more weight on the panel estimation. Fitting the models to panel data yields similar coefficients than before but typically at higher levels of significance. The estimates for ϕ_{u^N} and ϕ_γ are in the range of 0.07 to 0.09 and 0.09 to 0.10, respectively, and are significant at the 1%-level. The point estimates of the cumulative short-run effects of the profit rate and the utilization differential are in the range of 0.11 to 0.14 and 0.06 and 0.07, respectively. The likelihood ratios indicate high levels of significance.

Our results suggest a fairly strong adaptive behavior of US firms in revising their notion of both the *normal* rate of capacity utilization as well as the *expected* secular rate of sales growth. A parameter of $\phi_{u^N} = 0.08$ implies that, in response to a 1%-point change in the utilization differential, it takes the firms 8.31 quarters to adjust their perception of the *normal* rate by 0.50%-points. Equivalently, a parameter of $\phi_\gamma = 0.10$ implies a half-life of 6.57 quarters. As we find evidence for endogenous adjustments of the two conventions considered, the precondition for the Kaleckian long-run closure based on hysteresis seems to be met empirically.[23]

Our results partly contradict the findings by Skott (2008) for the Canadian manufacturing sector. He estimates $\tilde{\phi}_\gamma = 1.40$ and $\tilde{\phi}_{u^N} = 0.11$ as well as $\tilde{\beta}_u = 0.11$ for the effect of the accelerator term on investment. Although our results are not directly comparable as Skott uses a different investment specification applied to a different country and annual instead of quarterly data, some crucial divergences seem to prevail. Taking the average of our adjustment-parameter estimates over the three specifications, the resulting annualized adjustment rates for the *normal* utilization rate and *expected* secular growth rate of sales are roughly $\tilde{\phi}_{u^N} = 0.32$ and $\tilde{\phi}_\gamma = 0.38$, respectively, which are quite different to Skott's findings. The annualized cumulative utilization-differential effect on investment is roughly $\tilde{\beta}_u = 0.25$.[24] Skott qualifies his results as preliminary which may be due to two reasons: First, the assumption of no random disturbances in the adjustments of u^N and γ ensures absence of serial correlation in the regression equation. Since the validity of this assumption is questionable, some serial correlation can be expected to prevail. However, serial correlation may likely imply downward biased estimates for ϕ_{u^N} and β_u and an upward biased estimate

[23] These results are robust to different values of the lag-length, L, as reported in Table 1. Also, dropping the profit rate in the investment specification, i.e. restricting $\beta_{r,i} = 0$ for $i = 1, \ldots, L$ in (19), does not alter the results substantially. Further the estimates for ϕ_{u^N} are robust to restricting $\phi_\gamma = 0$. We also estimated specifications including the lagged dependent variable as a regressor which yield similar estimates for ϕ_{u^N}. Yet, as we are interested here in the Kaleckian growth model, we do not report these estimation results. They can be obtained from the author upon request.

[24] Note that the estimated coefficient for the utilization differential has to be quadrupled in order to be comparable to an estimate based on annual data, as this variable is a ratio between flows whereas the dependent variable is a ratio between a flow and a stock.

for ϕ_γ.[25] Second, Skott includes only contemporaneous regressors in the investment function which may cause the accelerator effect to be understated.

Since we estimated only the investment dynamics and not the system of investment and saving functions, our estimates do not allow for drawing strong conclusions on short and long-run stability. Yet, using conventionally assumed parameter values for s, $\bar{\pi}$ and v and given our cumulative parameter estimates, one can verify that neither condition tends to hold.[26]

Having estimated the parameter values, the *expected* secular growth rate of sales and the *normal* utilization rate in the manufacturing sector can be estimated employing the Kalman filter. Doing so, we use the respective parameter estimates of the panel estimation. The resulting time series are plotted in Figure 1. The initial value for the *normal* utilization rate is around 0.78 which is the average of the utilization rate of the first ten quarters considered. For the *expected* secular rate of sales growth, the initial value is zero. Since for both unobserved variables, the adjustment parameters do not differ much across different specifications, the respective estimated time series are very similar. Both sets of plots indicate a strong adaptive behavior of the respective estimated series.

5 Concluding remarks

Classical and Marxian authors criticized the canonical Kaleckian growth model as it features a sustained divergence of the *realized* rate of capacity utilization from its *normal* level. Post-Kaleckians such as Lavoie (1996) and Dutt (1997) responded to this criticism by proposing fully adjusted Kaleckian models featuring an endogenous adjustment of the *normal* utilization rate as well as the *expected* secular rate of sales growth through hysteresis. The *normal* and *expected* rates are conceived as conventions which may slightly change over time driven by past realizations.

Although the Kaleckian approach is now widely used in models of distribution and growth, the question whether the *normal* rate of utilization adjusts endogenously over time has not yet been tackled empirically in a satisfying manner. To contribute to the empirical literature, we set out to empirically assess whether the *normal* utilization rate as well as

[25] Skott (2008) derives from the Hysteresis-Kaleckian growth model an equation of the form

$$g_t = c + (1 - \phi_{u^N})g_{t-1} + \beta_u \phi_\gamma u_{t-1} + \beta_u u_t - u_{t-1} + \mu_t$$

where c is a constant and $\mu_t = \rho\mu_{t-1} + \varepsilon_t$ are potentially serially correlated disturbances. Skott assumes $\rho = 0$. Yet, if $\rho > 0$ which follows from random disturbances in the adjustments of u_t^N and γ_t, it is straightforward to see that $E[g_{t-1}\mu_t] > 0$ causing the estimate for $1 - \phi_{u^N}$ to be upward biased. Since an acceleration of accumulation raises utilization in the aggregate, and therefore $E[g_{t-1}u_{t-1}] > 0$, the estimates for $\beta_u\phi_\gamma$ and β_u will be upward and downward biased, respectively. However, the question how relevant these biases are for the results obtained by Skott is beyond the scope of this paper.

[26] For instance, using $s = 0.3$, $\pi = 0.4$, $v = 4$, $\beta_r = 0.13$, $\beta_u = 0.06$, $\phi_\gamma = 0.1$ and $\phi_{u^N} = 0.08$, neither (7) nor (11) hold. Assuming that $s\frac{\pi}{v} \approx 0.1$, Skott's (2008) estimates imply both the short-run stability condition in (7) and the long-run stability condition in (11)—both with the restriction that $\beta_r = 0$—to be violated. For the long-run case, this conclusion is driven by a large $\tilde{\phi}_\gamma$ relative to $\tilde{\phi}_{u^N}$.

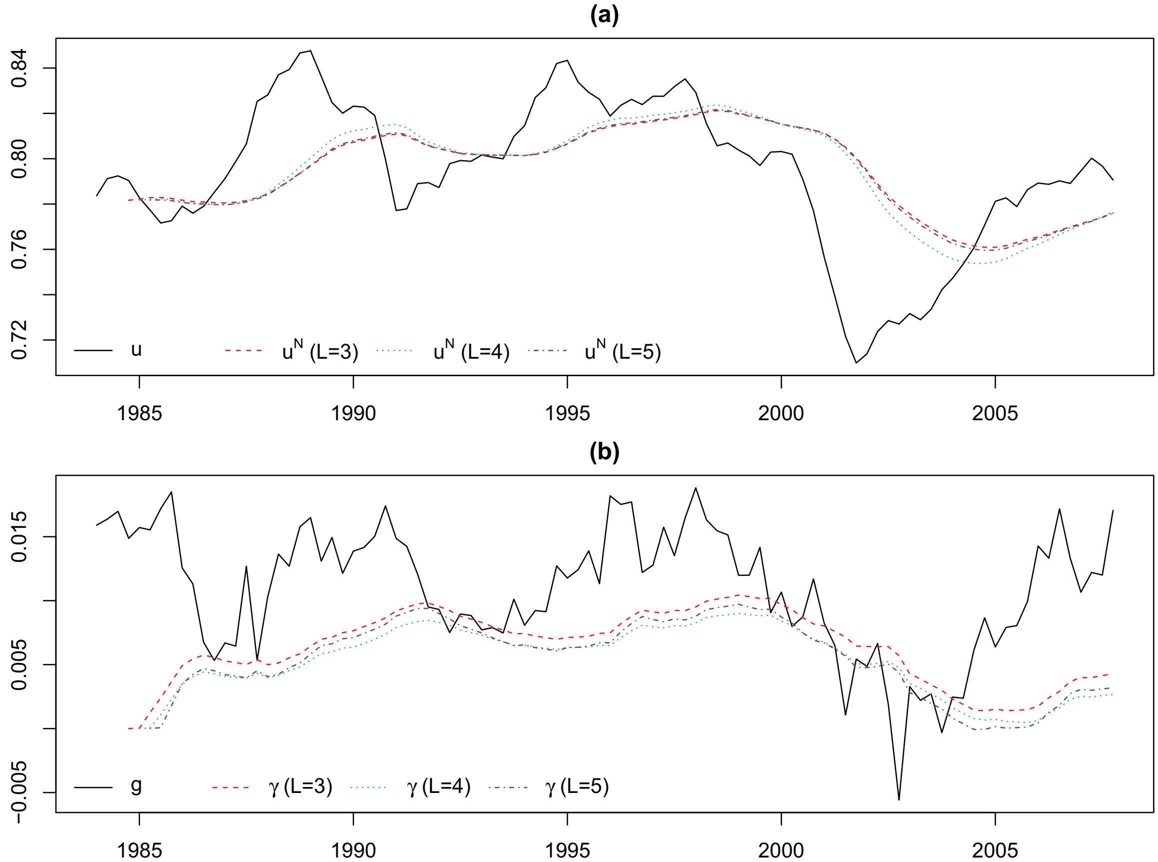

Figure 1: The *realized* and the estimated *normal* rate of capacity utilization (panel a) and the accumulation rate and the *expected* secular rate of sales growth (panel b) for the US manufacturing sector from 1984:1 to 2007:4

the *expected* secular rate of sales growth are endogenous employing quarterly data on the US manufacturing sector and its sub-sectors covering the period 1984:1-2007:4. We set up a dynamic econometric model of investment consistent with the hysteresis argument in state-space representation which allows for parameter estimation even though some variables are unobservable. Unknown parameters are estimated using Maximum Likelihood. The *normal* utilization rate as well as the *expected* secular rate of sales growth are estimated using the Kalman filter.

The model is fitted to both aggregate data and sectoral panel data. In both cases, we are able to reject the null hypotheses of no endogenous adjustments. For both the *normal* rate of capacity utilization and the *expected* secular rate of sales growth, we find positive and robust adjustment parameters as predicted by the long-run Kaleckian growth model. The estimated series of the conventional rates reflect a strong pattern of adaptive adjustment.

While we find empirical support for the theoretical prerequisite for applying the Kaleckian model in long-run analysis, note that our results cannot be interpreted as evidence against models built on the hypothesis of an exogenous *normal* utilization rate. Classical, Marxian and Harrodian models are typically not inconsistent with variables being influenced by conventions to some extent.

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