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Growth, Cycles, Asset Prices, and Finance

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Abstract

Several Kaleckian models are set out, with illustrations from US macroeconomic data since around 1980. Harrod- and Domar-style investment functions are specified, and combined with distributive dynamics to generate Goodwin-style cycles. A counter-clockwise cycle in a two-dimensional phase diagram for capacity utilization and the wage share is compatible with US data. It is based on long-run profit-led demand and a profit squeeze at high activity levels, consistent with Domar. Harrod-style investment and short-run wage-led demand generate counter-factual clockwise cycles. Still in two dimensions, financial dynamics involving the equity valuation ratio are considered. Both the ratio and equity price inflation demonstrate positive own-feedback. The ratio can be stabilized cyclically by capital stock growth and/or rising household net worth or debt. Debt accumulation persists (or "overshoots") after equity price inflation switches to a negative rate, consistent with US data. A Minsky-style model with Harrodian investment can be stabilized by accumulation of business debt. Higher dimensional models combining the 2 x 2 specifications just described are briefly discussed.

Keywords: Distributive cycle, wage-led and profit-led demand regimes, Harrodian investment, household debt.

JEL CLASSIFICATION SYSTEM: E12, E22, E44

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Introduction

Since the 1980s many authors have worked on macroeconomic models incorporating ideas from Michal Kalecki, Nicholas Kaldor, Richard Goodwin, Hyman Minsky, and Charles Kindleberger. The premises are widely known. Relevant questions are whether the models provide insight into how economies behave in terms of cyclical growth, and how macroeconomic performance is affected by institutional and policy changes. Events leading up to the 2007-2009 crisis provide a natural tests and illustrations of the models. Summaries appear in Palma (2009) and Taylor (2010). This paper provides background for much of the verbal discussion in the latter reference. The presentation focuses on simple, partial models designed to highlight different aspects of macroeconomic adjustment and growth.

We begin on the real side of the economy by sketching an investment-driven theory of growth and then bringing in Goodwin-style distributive cycles. One key issue, implicit in the contrast between the pioneering growth models of Roy Harrod and Evsey Domar, is whether capital stock growth is intrinsically stable or not. If not, how can it be stabilized?

Changes in income distribution may play a role. The Goodwin extension describes how the level of economic activity and distribution interact over the cycle, and how the economy is influenced by distributive trends. Linearized 2 x 2 phase diagrams are used to present patterns of cyclical behavior that are generated under Harrod- and Domar-style assumptions about investment demand growth. Domar is broadly consistent with US data.

The emphasis then shifts to asset price inflation (using the price of equity as an illustration) and its effects on the real side of the system. Like investment in Harrodian theory, an asset price during a boom is subject to positive feedback of its level into its own rate of growth. As emphasized by Kindleberger and Minsky the bubble is supported by endogenous credit creation. Bubbles do not last forever. Using another phase diagram, two illustrative models are developed to show how they may deflate. Growth in debt may persist for a time after relevant asset prices start to fall, as demonstrated by US data.

Minsky's own crisis model can be viewed as an elaboration of Keynes's analysis of the trade cycle in Chapter 22 of the *General Theory* (1936). Using the asset price of capital as an argument in the liquidity preference function, the model shows how interest rate adjustments can stabilize a Harrod-style investment function.

Finally, a few ideas are suggested regarding higher dimensional Kaleckian models.

Capital stock growth

Kaleckian models on the whole do *not* treat the capital stock as a factor that limits production. Rather capital K sets the overall scale of the economy and may serve as a vehicle for new technology that increases labor productivity $\xi = X/L$ with X as the level of output and L as the employed labor force.

The principle of effective demand in a simple closed economy without a government sector states that output is determined by an injection of real investment demand I and a saving leakage at rate s,

$$X = \frac{I}{s} \tag{1}$$

or

$$u = \frac{g}{s} \tag{2}$$

with $u = X/K$ (u for "utilization") and $g = I/K$. The paradox of thrift arises because a fall in s increases X and u.

Let $\psi = \omega/\xi$ stand for the labor share of output with ω as the real product wage. The profit share is $\pi = 1 - \psi$. Households receive income from various sources – wages, dividends, interest (and in practice transfers net of direct taxes from the state). In advanced capitalist economies the gross household saving rate s_h from disposable income may range from zero to 15%. The profit share might be around 20-25%, with a profit rate $r = \pi u$ around 10%. Fairly general accounting worked out in connection with equation (20) below shows that the total saving rate from the value of output will be $s = s_h + (1 - s_h)\pi$ which will be an increasing function of π or a decreasing function of ψ.

π

Econometrically estimated investment functions usually don't fit the data well and are notoriously unstable – perhaps a tribute to Keynes's insistence on the fundamental instability of capitalism. Nevertheless, various macro level variables do appear to influence investment, including the profit rate r and/or the capital share π, the nominal (i) or real (j) interest rate, and the valuation ratio $q = P_e E/PK$ with P_e as a price index for equity, E an index of shares outstanding, and P a price index applicable to the capital stock (the idea that q can influence investment traces to Thorstein Veblen, Keynes, Richard Kahn and Kaldor who used the symbol v for the ratio, and James Tobin at least). An alternative expression for a valuation ratio (equivalent to q under strong assumptions along Modigliani-Miller lines) is $v = r/j$, i.e. the asset price of capital is its return r capitalized by the real interest rate j. This version figures in the discussion of Minsky's model below.

Some version of an accelerator still often does the best job of "explaining" investment demand. A broad swath of the Kaleckian literature treats investment as centered around a "desired" level $\bar{u}$of capital utilization which emerges from some (ill-defined) aggregation procedure across firms.

Broadly following Peter Flaschel (2009) if $D = u/\bar{u}$ then one variant for the *change* in the level of investment can be stated as

$$\dot{g} = \frac{dg}{dt} = \tau(1-D) = \tau\left(1-\frac{u}{\bar{u}}\right) = \tau\left(1-\frac{g}{s\bar{u}}\right) = \frac{\tau}{s\bar{u}}(s\bar{u}-g) \qquad . \tag{3}$$

When $u = \bar{u}$ and $g = s\bar{u}$ (a "Harrod-Domar equation") the economy arrives at steady state growth with $\dot{g} = \dot{u} = 0$. Because X/K is perhaps the only "magic ratio" that has been stable (across business cycles) in the US economy since WWII, this approach is consistent with the data. Moreover, as emphasized by Peter Skott (2010), observed capacity utilization tends to fluctuate within a fairly narrow range around a level similar to $\bar{u}$ see Figure 3 below for an illustration (using trend output instead of the capital stock to normalize u.)

The idea that it is appropriate to work with the change in investment as the relevant endogenous variable is often attributed to Josef Steindl (1976). In the elaboration of the Kaleckian model framework that Robert Rowthorn (1982) and

Amitava Dutt (1984) originally set up almost three decades ago a common alternative has been to make g (not $\dot{g}$) a function of u and perhaps the other variables mentioned above. This specification has been criticized by Skott and others and is not pursued here.

Apart from US data, the question about whether a construct such as (3) with its target $\bar{u}$ makes sense historically is fraught. Palma (2009) points out that in the nine decades since WWI there have been substantial shifts in political economy regimes in the USA: a liberal (in the European sense of the word) deregulated expansionary period 1920-1929 with rapidly rising income inequality; then the Great Crash, Great Depression, and WWII. Next came a broadly Keynesian and highly regulated Golden Age with rapid growth that ended around 1970; a decade of stagflation; another liberal, deregulated period with growing inequality, falling inflation, and relatively slow growth 1980-2007; then financial crash and deep recession. Why firms are going to plan around a long-run steady state level of capacity utilization under such shifting circumstances is a mystery (at least to me).

Its verisimilitude or lack of same notwithstanding, a steady state serves as a point of reference in almost all analyses of growth, and is interpreted in that way here. The steady state built into (3) may be unstable or stable. With τ assumed to be a negative constant, $\dot{g}$ becomes an increasing, elastic function of g. This case reflects Harrod's (1939) ideas about the instability of capitalist accumulation. If (3) with $\tau < 0$ and $\frac{\partial \dot{g}}{\partial g} > 0$ is the only dynamic relationship the economy will diverge from the steady state. Of course, as will be seen below the dynamics of other variables can stabilize the system.

The Kaleckian literature often draws attention to wage-led and profit-led macro responses to changes in the income distribution. Usages are ill-defined (I'm certainly guilty on that score). The labels have been applied to comparative static responses of output, growth, and other variables; slopes of nullclines in a phase diagram; and other aspects of the macro system. The distributive shifts may involve movements in the real wage or profit rate, wage and profit shares, etc.

In the present context with $\tau < 0$ it will be true that $\frac{\partial \dot{g}}{\partial \psi} > 0$ because an increase in the wage share reduces the saving rate. In other words (ignoring the possibility that there is a direct positive effect of the profit share on $\dot{g}$) the comparative static change of capital stock growth in a Harrodian economy can be called "wage-led".

The case in which $\tau > 0$ can be associated with Domar's (1946) analysis of the consistency of investment demand with balanced growth. When τ is positive, (3) becomes a stable differential equation for *g*. In the short run it is easy to see that $\frac{\partial \dot{g}}{\partial \psi} < 0$, or the change in capital stock growth is "profit-led".

Flirting with Say's Law

The distinction between Harrod-style investment instability and Domar-style stability is maintained throughout the following discussion. As a lead in, it makes sense to review quickly the contrasts and similarities between the models being developed here and mainstream growth analysis based on Say's Law. Two closures of the supply side accounts are considered – with and without pre-determined growth of employment.

In the Kaleckian models outlined below, "supply" is represented by differential equations for changes in the income distribution and labor productivity over time – they serve to constrain output and affect stability. There is a fixed stock of capital at any time, but as noted above it does not directly limit output through "decreasing returns". The levels of output and employment of inputs (especially labor) are determined by demand as influenced by the income distribution, in direct contrast to neoclassical models.

If capital is not a limiting factor on growth (at least within a "reasonable" range of values for the output/capital ratio *u*) but employment growth is pre-determined then the major source of per capita output expansion must be rising labor productivity. With a "hat" over a variable denoting its growth rate ($\hat{X} = \dot{X}/X$), the definition $\xi = X/L$ of productivity can be restated as

$$\hat{X} = \hat{\xi} + \hat{L} = \hat{\xi} + n \qquad (4)$$

with *n* as the growth rate of employment. Suppose for the moment that both $\hat{\xi}$ and *n* are exogenous, as in standard neoclassical models built around an aggregate production function and Say's Law. One implication of imposing (4) on the system is that an independent investment function such as (3) cannot exist.

If $\delta > 0$ is the depreciation rate then the growth of capital stock is given by

$$\hat{K} = g - \delta = su - \delta \,. \tag{5}$$

Plugging (4) and (5) into the definitional equation

$$\hat{u} = \hat{X} - \hat{K}$$

gives an expression for the evolution of *u* over time,

$$\dot{u} = u\left[-su + \left(n + \hat{\xi} + \delta\right)\right] = u[-g + \left(n + \hat{\xi} + \delta\right)] \,. \tag{6}$$

This differential equation will be stable around a steady state at which

$$su = g = n + \hat{\xi} + \delta \,. \tag{7}$$

With output growth set by (4), capacity utilization adjusts toward the steady state via shifting capital formation.

The stability of (6) demonstrates that one does not need to bring in an aggregate production function to derive a typical neoclassical result – basically all that is needed is an equation set like (4) and (5). In the version of the Harrod-Domar equation appearing in (7) the term $\left(n + \hat{\xi} + \delta\right)$ is the "natural" growth rate frequently invoked in the neoclassical literature on convergence. Taking a linear approximation to (6) around the steady state shows that

$$\frac{\partial \dot{u}}{\partial u} = -\left(n + \hat{\xi} + \delta\right) .$$

In the convergence literature based on the Solow-Swan model the warranted growth term on the right-hand side is multiplied by the labor share emerging from an aggregate production function. In comparison, convergence in the present model is "fast."

In practice, East Asian economies including Japan beginning in the 1950s and more recently China have had falling output/capital ratios with $su > n + \hat{\xi} + \delta$ consistently for decades. Their *lack* of convergence is due to high saving rates. In the US on the other hand, the output/capital ratio is stable across business cycles, signaling that convergence is rapid (with falling national saving since around 1980 offset by an increasing current account deficit). For more detail see Rada and Taylor (2006) and Figure 3 below.

Now drop Say's Law and reinstate the investment function. Employment growth n must be determined by demand. From equation (3) the change in capacity utilization can be expressed as

$$\dot{u} = \frac{\tau}{s\bar{u}}(\bar{u} - u) + u\sigma\hat{\psi}. \tag{8}$$

The first term after the equals sign captures the effect of investment demand on $\dot{u}$. The second shows the effect of a change in the multiplier in response to a shift in the labor share, with $-\sigma$ as the elasticity of the saving rate with respect to ψ (perhaps lying between zero and 0.5). For the moment, assume that the saving rate is constant (or) as $\sigma = 0$ in most standard growth models.

With the productivity growth rate $\hat{\xi}$ pre-determined (another assumption soon to be relaxed), equations (6) and (8) for $\dot{u}$ can be compatible if n is endogenous. The solution for n is

$$n = \frac{\tau}{su}\left(1 - \frac{u}{\bar{u}}\right) + su - (\hat{\xi} + \delta). \tag{9}$$

For u not "too close" to $\bar{u}$ along with plausible values of s and $\tau > 0$ $\partial n/\partial s$ can take either sign – the paradox of thrift may apply if $u > \bar{u}$. At the steady state we have $u = \bar{u}$, $\bar{g} = s\bar{u}$, and $\bar{n} = s\bar{u} - (\hat{\xi} + \delta)$. The term containing $\frac{\tau}{su}$ in (9) vanishes so that $\partial n/\partial s > 0$ and the paradox of thrift drops out.

Unless it is stabilized by other dynamic relationships a Harrodian economy with $\tau < 0$ will diverge from the steady state. In an expanding system with $u > \bar{u}$ the paradox will apply. In a contracting economy it ultimately will not.

Cyclical growth

The next step is to bring in dynamics for the wage share ψ and thereby cyclical growth along Goodwin's (1967) lines. Papers written during the 2000s have taken up this task. Here we use phase plane analysis for u and ψ .

A generic bargaining equation for the real wage can be written as

$$\hat{\omega} = \beta(u, \psi) \tag{10}$$

with positive and negative first and second partial derivatives respectively of the function $\beta(u, \psi)$ To hold $\hat{\omega}$ constant (equal to productivity growth in a steady state with $\hat{\psi} = 0$), u must increase along with ψ. This relationship pushes the dynamics toward a falling profit share or a "profit squeeze" when the level of economic activity rises.

A "story" about (10) is that a higher level of economic activity increases labor's bargaining power, while firms are more resistant to wage pressure when unit labor costs as measured by ψ are high. Flaschel (2009) and various co-authors suggest that an equation like (10) emerges from separate Phillips curves for nominal prices and wages. However, much of their work dispenses with the dynamics of labor productivity which figures centrally in the discussion to follow. Tavani, Flaschel, and Taylor (2010) extend the analysis to incorporate productivity changes through a version of Okun's law.

For dynamics of productivity growth, the "Kaldor-Verdoorn" equation (Kaldor, 1978) tying $\hat{\xi}$ to output growth is used in many demand-driven models. Ignoring depreciation for simplicity so that $\hat{X} = \hat{u} + g$ and assuming that $\hat{u} = \hat{g} + \sigma\hat{\psi}$ as in (8) we get

$$\hat{\xi} = \xi_0 + \gamma\hat{X} = \xi_0 + \gamma(\hat{g} + \sigma\hat{\psi} + g) \tag{11}$$

where γ is the Kaldor-Verdoorn elasticity (usually somewhere around 0.5). One could also throw in a term for induced innovation in response to a rising real wage but I'll leave that out for simplicity (it would tend to amplify the effect on saving of ψ). Both g and $\hat{g}$ positively affect productivity growth, consistent with a role for new capital formation as a vehicle for technological improvement.

Using $\hat{\psi} = \hat{\omega} - \hat{\xi}$, plugging in (10) and (11), and simplifying gives

$$\dot{\psi} = \psi(1+\gamma\sigma)^{-1}[\beta(u,\psi) - \xi_0 - \gamma(\hat{g} + g)]. \tag{12}$$

As in (3) with $\bar{u}$, dynamics of the wage share could be centered around a "natural" level $\bar{\psi}$ (related to target levels in the wage and price Phillips curves and the trend in productivity growth). But because ψ has been trending downward in the US for the past several decades (Figure 3) that extension seems cosmetic at best.

We can analyze the system (8) and (12) when it is linearized around a steady state. First, in (8) if $\tau < 0$ in the term for investment demand (Harrodian dynamics) it is easy to see that $\partial\dot{u}/\partial u > 0$ and $\partial\dot{u}/\partial\psi > 0$. As noted above the latter response reflects the negative effect of an increase in the wage share on the saving rate. This wage-led behavior might reverse if there is a strong direct positive influence of the profit share on the change in investment demand.

When $\tau > 0$ in a Domar-style investment regime we get stable local adjustment of u ($\partial\dot{u}/\partial u < 0$) and a profit-led response of $\dot{u}$ to ψ ($\partial\dot{u}/\partial\psi < 0$).

In both the Harrod and Domar cases, these broad conclusions will be affected by the $+u\sigma\hat{\psi}$ term in (8) but by considering how $\dot{\psi}$ responds to u and ψ in (12) we can see that the unstable/wage-led and stable/profit-led dynamics in (8) may well carry over.

$\partial\dot{\psi}/\partial u$: There is a positive real wage effect of u on $\hat{\omega}$ and $\dot{\psi}$ from $\beta(u,\psi)$ in (10). In a Harrod-style investment regime with $\tau < 0$ there is a negative effect from $\frac{\partial\hat{g}}{\partial u} > 0$. The sign reverses if $\tau > 0$. There is a direct negative effect from $g = su$. A profit squeeze with $\frac{\partial\dot{\psi}}{\partial u} > 0$ is more likely in a Domar-style regime. Under Harrodian conditions a strong bargaining response $\frac{\partial\beta}{\partial u} \gg 0$ would be needed to make $\frac{\partial\dot{\psi}}{\partial u} > 0$.

$\partial\dot{\psi}/\partial\psi$: There is a negative direct effect of ψ on $\dot{\psi}$ via β; a positive effect β from because u jumps upward in response to an increase in ψ; and depending on the investment function an ambiguous effect from $\hat{g}$ via u. Overall it is probably safe to assume a negative own-partial derivative.

For two potentially stable cases of interest Table 1 shows the signs of responses of $\dot{u}$ and $\dot{\psi}$ in Jacobian matrixes for (8) and (12) under Harrod- and Domar-style assumptions about investment demand. Because of the opposite signs in the off-diagonal entries the steady state in both variants can easily be a focus. In the Harrod case potential instability shows up with the positive sign for $\partial\dot{u}/\partial u$. (If a strong direct profitability effect of the profit share on $\dot{g}$ makes $\partial\dot{u}/\partial\psi < 0$ the system would demonstrate saddlepoint instability.)

Table 1 about here

Figure 1 is a phase diagram for stable dynamics in the Harrodian case. Redeploying the ambiguous distributive terminology mentioned above, the negative slope of the nullcline for $\dot{u} = 0$suggests that -- as opposed to the positive impact effect of ψ on $\dot{u}$ discussed above and shown in Table 1 -- "in the long run" the level of capacity utilization appears to be "profit-led." This is a typical instability result -- because of positive feedback to keep the system on the nullcline the direct boost to $\dot{u}$ from a higher level of ψ in Table 1 must be offset by a *decrease* in *u*. The nullcline for $\dot{\psi} = 0$ shows a positive long run relationship between output and the profit share, something like the traditional forced saving mechanism in Keynes's *Treatise on Money* and Kaldor's growth models from the 1950s.

Figure 1 about here

The trajectory sketched in the diagram suggests that economic recovery from a low level of *u* relies on a rising wage share which boosts output due to the paradox of thrift. It leads Harrodian investment demand to rise. There are clockwise oscillations with the profit share increasing in the latter part of the upswing as Kaldor-Verdoorn productivity growth cuts into unit labor costs. As drawn, the distribution vs. demand cycle is a stable converging spiral – the typical econometric result.

In the Domar case illustrated in Figure 2, long-run demand is also profit-led because a reduction in ψ would have to be met by an increase in *u* to hold $\dot{u} = 0$. An increase in *u* squeezes the profit share (increases the wage share) along the $\dot{\psi} = 0$

nullcline. Economic recovery now features a falling wage share, consistent with the stylized fact that productivity growth increases as the economy emerges from recession. With high unemployment the real wage is falling or stable so that $\psi = \omega/\xi$ goes down. Ultimately the real wage starts to rise while productivity growth slows, the labor share recovers, and demand growth decelerates as the economy approaches its cyclical peak level of u. There is a counter-clockwise oscillation in the phase plot, again drawn to be dynamically stable.

Figure 2 about here

Broadly speaking, the second case is a better description of US cyclical dynamics. Figure 3 shows the behavior since the immediate post-WWII period of indexes of the wage share and utilization measured by output relative to trend. The shaded areas represent recessions as defined by the NBER.

Figure 3 about here

On the whole capacity utilization leads the wage share, as in the trajectory shown in Figure 2. The share typically drops during the initial recovery, and then recovers as utilization approaches its cyclical peak. This same pattern shows up in econometric model estimates for the US by Barbosa-Filho and Taylor (2006) and for the US and "Europe" by Flaschel (2009) and colleagues.

The other striking aspect of the diagram is a visible downward trend in the wage share over cycles after 1980 (which was accompanied by a dramatic increase in the concentration of the size distribution of income). This pattern is consistent with the resurgence of liberalism and rising inequality since around 1980 that was mentioned above. In Figure 2, a downward shift of the $\dot{\psi} = 0$ nullcline due to loss of labor's bargaining power and/or more rapid productivity growth would be associated with a lower long-run level of ψ. In Figure 1 it would lead to a perverse increase in the labor share.

Overall, Domar-style investment and a profit squeeze appear to fit the data better than Harrod and short-run wage-led aggregate demand.

Non-steady state behavior

The wage share is by no means the only strongly trended variable in the US economy since 1980. Figure 4 shows that the consumption share of household income rose dramatically as the wage share declined. One immediate interpretation is that consumers were attempting to maintain living standards at the same time as the main source of income for many of them stagnated or fell (Veblenesque imitation of conspicuous consumption at the top of the income distribution may also have been involved). More consumption was made possible by a rising household debt/income ratio – at least until the 2007 recession struck.

Figure 4 about here

Collateral for the debt was provided in part by rising share prices until the end of the 1990s. Thereafter, as shown in Figure 5, the housing price boom after the early 1980s permitted real household debt to increase. Debt began to fall in 2007-08, overshooting the break in housing prices that took place a year or two earlier (a point further discussed below). Borrowing was abetted by a strong downward trend in interest rates, especially after the mid-1990s.

Figure 5 about here

On the other hand households did not "over-borrow", at least insofar as their expenditure relative to net worth trended downward until the late 1990s (Figure 6). Then the ratio spiked upward when net worth dropped in response to the collapses of the equity and residential housing price booms. The ratio of debt to net worth rose gradually through the 1990s, until its level was also shocked upward by the asset price collapses.

Figure 6 about here

This glance at the data suggests that distributive and financial characteristics of the US economy were nowhere near steady states for three decades after 1980 (or four after the end of the Golden Age). The saving and wage shares of income trended downward at the same time as asset prices and real household debt went up. Some trends began to reverse after the financial crash. The next step is an attempt to apply Kaleckian modeling to analyze these changes, in particular mechanisms under which financial trends might reverse.

Asset prices and investment demand

The framework is highly simplified, relying on macro accounting laid out in Lavoie and Godley (2001-02) and Taylor and Rada (2007). It omits the high drama underlying the crash, for example the huge increase in leverage in the financial sector built around derivatives based on mortgages and financed by an explosion of repo lending among investment banks and other Wall Street actors (Adrian and Shin, 2008).

To keep the number of symbols under control, we focus on one asset price – for equity in productive firms – and associated debt. The modeling could be extended to bring in housing prices, household borrowing, and mortgage securitization, but that would take another paper, at least. Table 2 presents balance sheets for households, the corporate business sector, and banking system. All three sets of actors are assumed to have non-zero levels of net worth -- Ω_h, Ω_b, and Ω_c respectively.

Endogenous net worth is the background for the models to be developed here because it rules out restrictions on asset returns such as the Modigliani-Miller theorem asserting that net worth is always equal to zero. In Table 2, endogenous credit creation can be absorbed by changes in net worth (or by declining net foreign assets of the economy as a whole, as in the US since around 2000!). Other symbols are explained as we proceed.

Like investment demand in a Harrod-style growth model, asset prices display positive feedback which underlies their growth in a boom, but must ultimately be reversed by other forces. There is no good descriptive theory of asset prices – there aren't that many stock market billionaires around. Nevertheless, rationales for growth followed by decline of an asset price such as P_e for equity E can be provided. Two examples focusing on the valuation ratio $q = P_e E/PK$ will be presented. The one in this section shows how growth in the capital stock can stabilize q. Then comes a discussion of how shifting financial positions may drive asset price inflation rate into negative territory. Finally Minsky's (1975) version of the business cycle which can be set up in terms of the alternative valuation ratio $v = r/j$ is quickly reviewed. Trajectories from all three scenarios take the same shape in a 2 x 2 phase diagram.

The first two models presuppose that $\dot{P}_e$, the change in the asset price, can be observed correctly. In other words, economic actors share myopic perfect foresight about asset price changes. This assumption stretches credibility, but is far more transparent than some sort of adaptive expectations about $\dot{P}_e$. Meanwhile the price P of the capital stock is assumed to stay constant.

The off-the-shelf Gordon (1962) equation states that the return to equity ρ is generated by dividends and capital gains according to the formula

$$\rho P_e E = \dot{P}_e E + D \tag{13}$$

with D as the level of dividend payments. In the medium term it is convenient to set $D = \phi PK$ (with ϕ taking a value in the vicinity of 0.02).

Theories about asset price growth can be set up around the determination of ρ. For the moment, hold it constant as a "required" return. Then (13) becomes

$$\rho = \hat{P}_e + \frac{\phi}{P_e E / PK} = \hat{P}_e + \frac{\phi}{q}$$

which immediately provides an equation for the growth rate of the equity price,

$$\hat{P}_e = \rho - \frac{\phi}{q} \tag{14}$$

Because $q = P_e E / PK$, we have $\partial \hat{P}_e / \partial P_e > 0$ in (14). The Gordon equation automatically incorporates positive feedback of the equity price into its own growth rate.

Suppose that new issues of equity are proportional to investment demand $P_e\dot{E}/PK = \chi g$ (For the business sector, due to share buybacks the parameter χ is in fact negative in the US. Households, in contrast, assumed higher debt burdens during the boom.) Evidently,

$$\hat{E} = \frac{\chi g}{q}. \tag{15}$$

Ignoring depreciation, differentiating the definition of q gives $\dot{q} = q[\hat{P}_e + \hat{E} - (\hat{P} + g)]$. Plugging in (14) and (15) with $\hat{P} = 0$ shows that

$$\dot{q} = (\rho - g)q + \chi g - \phi \tag{16}$$

The effect of q on $\dot{q}$ is $\partial\dot{q}/\partial q = \rho - g$. The real long-run equity return ρ in the US is famously in the range of 7%, higher than any reasonable capital stock growth rate so that q demonstrates positive feedback with $\partial\dot{q}/\partial q > 0$. Also $\partial\dot{q}/\partial g = -q + \chi < 0$ especially if $\chi < 0$.

Suppose that as in many models the change in investment $\dot{g}$ depends positively on q ($\partial\dot{g}/\partial q > 0$). Under a Domar-style investment regime ($\partial\dot{g}/\partial g < 0$) dynamics of g can stabilize the macro system in the (g, q) phase plane. These sign patterns are summarized in Table 3.

Table 3 about here

A trajectory based on (16) and these investment responses is sketched in Figure 7. When a boom gets underway, P_e starts to rise and q increases. If the higher valuation ratio spurs investment demand the *PK* term in the denominator of q will grow, ultimately making $\dot{q} < 0$. Investment demand overshoots the fall in q but ultimately $\dot{g}$ turns negative as well. As in Figure 5, overshooting is not uncommon in financial cycles.

Figure 7 about here

Asset prices and household borrowing

As pointed out above, heavy household mortgage borrowing in response to rising residential prices was a key factor leading into the financial crash. Restricting the accounting to equity, a similar cycle is analyzed in this section under the assumptions that the growth rate g and wage share ψ stay constant. To begin, national income and flows of funds accounts for consumers have to be set out, scaled to the value of the capital stock *PK*.

From their balance sheet in Table 2, households hold money M and borrow H from the banking system, receiving interest on the former and paying on the latter (for simplicity assume that the same real rate j applies to both transactions). Money/capital and debt/capital ratios can be expressed as $\mu = M/PK$ and $\eta = H/PK$ respectively. They also receive income from dividends (ϕ) and wages. Wage income relative to capital stock is $(1-\pi)u = u - r$. Net household income becomes

$$y_h = u - r + j(\mu - \eta) + \phi \ .$$

In the US, household debt exceeds money holdings or $\eta > \mu$.

Sources of funds for household financial transactions are saving, equity buybacks with $\chi < 0$, and new borrowing. In the present model without residential investment, the only use is to increase holdings of money for transactions purposes, growing (say) at the rate g. The flows of funds balance becomes

$$s_h y_h - \chi g + \eta \hat{H} = \mu g$$

so that the increase in household debt is

$$\dot{H} = \frac{H}{\eta}[(\mu + \chi)g - s_h y_h] \ . \qquad (17)$$

with $\mu > -\chi$. Figures 4 and 5 suggest that since around 1980 in the USA there was a downward trend in the gross household saving rate s_h, accompanied by substantial accumulation of debt. This linkage shows up in (17) -- s_h goes down and $\dot{H}$ has to rise.

Again, the model could be extended to deal with investment in residential housing as another use of funds but that is outside the scope of this paper.

The change in household net worth is the sum of saving and capital gains. With $\Delta = \Omega_h/PK$ the growth rate of wealth is

$$\widehat{\Omega}_h = \frac{1}{\Delta}(s_h y_h + q\widehat{P}_e) .$$

Besides $q\widehat{P}_e$, or capital gains on equity, an extended model would include asset price changes for residential housing as well.

Using $\widehat{\Delta} = \widehat{\Omega}_h - g$ and $q\widehat{P}_e = \rho q - \phi$ from (14) gives an equation for the increase in Δ,

$$\dot{\Delta} = s_h y_h + \rho q - \phi - \Delta g \qquad (18)$$

which in principle allows a steady state with $\dot{\Delta} = 0$ and $\widehat{\Omega}_h = g$.

Equations (16) and (18) make up a dynamical system in q and Δ. Under a couple of plausible behavioral assumptions it demonstrates the same cyclical pattern as q and g in Figure 7 (with Δ taking on the role of g in Table 3 and the diagram). One assumption is that the household saving rate $s_h$declines (or at most increases weakly) as the net worth ratio Δ increases. The implication is that $\partial\dot{\Delta}/\partial\Delta < 0$, with the saving effect augmenting (or not offsetting) the impact of the term $-\Delta g$. From (18) it is also clear that $\partial\dot{\Delta}/\partial q > 0$.

The other assumption is that an increase in Δ reduces the required return to equity ρ. As capital gains accumulate and Ω_h builds up, pressure for high returns to equity declines. Similar responses are mentioned in the political economy literature on financial cycles. For example, Kindleberger and Aliber (2005) point out that at some point during a mania more sellers than buyers begin to emerge in the market in question (in Wall Street parlance, near the top of an upswing "fear" overwhelms "greed" and on average players exit the market).

In the financial cycle described in Keynes's *Treatise on Money* (1930) at some point during an upswing asset prices have risen "more than sufficiently" for bear speculators to start building up money holdings and begin short selling in anticipation of a crash (a similar idea is built into Minsky's model of investment determination below). In the present framework, the implication is that $\partial \dot{q}/\partial \Delta < 0$ in (16). With $\partial \dot{q}/\partial q > 0$ the Figure 7 cycle recurs.

A further variant can be set up with the household debt/capital ratio $\eta = H/PK$. Using (17) one can show that

$$\dot{\eta} = (\mu + \chi)g - s_h y_h - \eta g \ . \tag{19}$$

Post-crisis, the US household saving rate has risen and there have been large withdrawals from the stock market, both directly and via mutual funds. In terms of the present model, the implication is that a higher value of η induces households to save more ($\partial \dot{\eta}/\partial \eta < 0$). They also become more cautious in pursuing capital gains ($\partial \dot{q}/\partial \eta < 0$). On the other hand rising equity prices may reduce saving and lead to more accumulation of debt ($\partial \dot{\eta}/\partial q > 0$). The same sign pattern as in Table 3 reappears (with η assuming the role of *g*), broadly consistent with observed household borrowing over time as shown in the empirical "phase diagram" in Figure 8.

Figure 8 about here

Using the algebra developed above one can also write out an equation for the growth of debt/net worth ratio H/Ω_h. It increases with a falling saving rate s_h and decreases with the valuation ratio *q*, consistent with the data shown in Figure 6.

A Minsky cycle

Minsky's (1975) analysis focused on how investment demand responds to changes in profit and interest rates. To set up a formal version we can use the income and flow of funds statements for corporate business and banks. In its balance sheet in Table 2, business liabilities are loans *L* from banks and outstanding equity $P_e E$. Let $\lambda = L/PK$. We want to set up a differential equation for the evolution of λ over time.

Relative to the value of capital stock, corporate retained earnings (and saving) are $y_c = r - j\lambda - \phi$ with $r = \pi u$. Saving by banks is equal to their net interest income

$j(\lambda + \eta - \mu)$. Setting investment equal to overall saving from households, business, and banks gives

$$g = s_h[(1-\pi)u + j(\mu - \eta) + \phi] + (\pi u - j\lambda - \phi) + j(\lambda + \eta - \mu)$$

Solving for *u* produces a multiplier equation

$$u = \frac{g + (1 - s_h)[\phi + j(\mu - \eta)]}{s_h + (1 - s_h)\pi} \tag{20}$$

which is a direct extension of (2) to the more complicated accounting of the present model. There is a demand injection term $(1 - s_h)[\phi + j(\mu - \eta)]$ in the numerator corresponding to consumption from financial transfers (dividends and net interest) to households and an overall saving rate from income $s = s_h + (1 - s_h)\pi$ in the denominator.

Business flows of funds are

$$(r - j\lambda - \phi) + \lambda\hat{L} + \chi g = g \tag{21}$$

with sources of funds being retained earnings, new borrowing $\dot{L}/PK = \lambda\hat{L}$, and issuance of new equity (or buybacks when $\chi < 0$). The use is investment demand *g*. The profit rate *r* and growth rate *g* are determined on the real side of the model, so in (21) the change in the supply of bank loans $\lambda\hat{L}$ has to be endogenous to allow firms to carry through their investment plans. Endogenous banking system net worth Ω_b permits money demand *M* and loan demands *H* and *L* to be determined elsewhere in the system.

Rearranging (21) produces an equation for $\dot{\lambda}$,

$$\dot{\lambda} = (j - g)\lambda + \phi + (1 - \chi)g - \pi u\ . \tag{22}$$

If the solvency condition $g > j$ applies then $\partial\dot{\lambda}/\partial\lambda < 0$. For plausible levels of the variables and parameters in (22) it is likely that $\partial\dot{\lambda}/\partial g > 0$, i.e. more capital accumulation increases business debt.

Minsky can be interpreted as working with an investment demand function

$$\dot{g} = \phi(g, v, \lambda) \qquad (23)$$

in which $\partial \dot{g}/\partial g > 0$ as with Harrod, and $\partial \dot{g}/\partial v > 0$ with $v = r/j$ as discussed above. Firms may well cut back on investment when their debt burden increases, $\partial \dot{g}/\partial \lambda < 0$. He has a fairly complicated theory regarding the role of the valuation ratio v. The key assumptions are that speculative demand for money is a decreasing function of the interest rate j *and* an increasing function of v. The first assumption is standard. As noted above, the second depends on expectations. It broadly follows Keynes's idea in the *Treatise* that when asset prices are high, speculators will withdraw from the market in anticipation of a crash.

The analysis also follows Keynes (1936) and most monetary economists into the 1980s in assuming that the money supply is controlled by the authorities with an endogenous interest rate. Contemporary discussion has switched from Keynes toward Wicksell in assuming that the authorities seek to adjust the rate to meet shifts in economic activity and the inflation rate according to some sort of Taylor (1993) rule (which of course should be called a Wicksell rule). Minsky could be restated along these lines but it is simpler to stick with his original approach.

The money supply must equal the sum of transactions and speculative demands. If it increases at a given level of transactions demand, then j can decrease and v can rise. Higher output means that transactions demand increases. With a fixed money supply speculative demand would have to be forced downward by a higher interest rate and/or lower asset prices.

But Minsky also brings in a shift in liquidity preference. If a boom is underway "... increasing the surety of income from capital-asset ownership, then the liquidity preference function will shift..."(p.73), presumably downward. The interest rate does not have to increase very much as output rises. In effect, Keynes's liquidity trap has been relocated from a low level of economic activity to a high one. During an upswing the profit rate will rise, feeding into an increase in v and stimulating still higher investment and output. The braking factors are an increasing debt burden λ and gradually rising interest rates.

Figure 9 presents the local dynamics of (22) and (23), similar to the financial cycles already discussed. Both the growth rate and debt/capital ratio would stay constant at the steady state at point A where the nullclines cross. Suppose that firms suddenly lose confidence at the steady state so that the growth rate jumps down along the dashed line to B. After the fall, firms will start to pay back debt and further reduce investment until the solid trajectory crosses the $\dot{g} = 0$ nullcline at C. Enough debt has been repaid to give firms an incentive to increase the capital stock growth rate. When the trajectory crosses the $\dot{\lambda} = 0$ nullcline at D they start to run up debt again.

Because of the rise in the profit rate relative to the interest rate sketched above this phase of expansion may last for a considerable time. Meanwhile the financial system becomes increasingly fragile. With increasing asset prices Minsky's version of speculative demand suggests that interest rates will have to rise. With higher debt/capital ratios and interest rates the increase in the capital stock growth rate will slow down. Sooner or later the trajectory will cross the growth rate nullcline at E. Investment will fall but there will be typical overshooting of debt. As in Figure 1, Harrod-style investment demand is stabilized by other factors in a clockwise spiral. In practice, both distributive and interest rate changes could play roles in reversing a cyclical investment upswing.

Figure 9 about here

The whole shebang

Simple little 2 x 2 phase diagrams go only so far in analyzing macroeconomic systems. The equations set out above extend to at least a 7 x 7 system (for a *closed* economy) with ample possibilities for instability, cycles, and potentially chaotic behavior.

In detail, dynamics for capacity utilization u are needed, from an equation such as (8) incorporating the principle of effective demand and an investment function for $\dot{g}$ with $g = I/K$. The investment function in principle could incorporate either positive (Harrod) or negative (Domar) feedback of g into $\dot{g}$. It presumably will be influenced by some subset of the variables listed below.

The wage share can be written as $\psi = (w/P)/\xi$ with w as the money wage and P as the price level. Dynamic Phillips curve equations are needed for $\dot{w}$ and $\dot{P}$ along with a model such as (11) for $\dot{\xi}$.

The valuation ratio q is presumably determined from an equation such as (16), with positive feedback. If two or more assets are included (say equity and residential housing) each would have its own valuation dynamics.

Equations such as (19) and (23) are needed for changes in household and business debt ratios.

This system incorporates positive feedbacks and several likely examples of cross dynamics from one variable to another with opposite signs. If it has a steady state (or states) it could be unstable. If it is stable, convergence will almost certainly be cyclical. Chaotic attractors could certainly emerge. A lot of Kaleckian macrodynamics remains to be explored.

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Table 1: Signs of responses in differential equations for a Goodwin model involving capacity utilization u and the wage share ψ

Harrod-style investment regime

	u	ψ
$\dot{u}$	+	+
$\dot{\psi}$	-	-

Domar-style investment regime

	u	ψ
$\dot{u}$	-	-
$\dot{\psi}$	+	-

Table 2: Balance sheets for households, corporate business, and banks

Households		Corporate		Banks	
M	H	PK	L	L	M
$P_e E$	Ω_h		$P_e E$	H	Ω_b
			Ω_c		

Table 3: Signs of responses in differential equations for an asset price models based on the investment capital ratio g and the valuation ratio q

	g	q
$\dot{g}$	+	-
$\dot{q}$	+	-

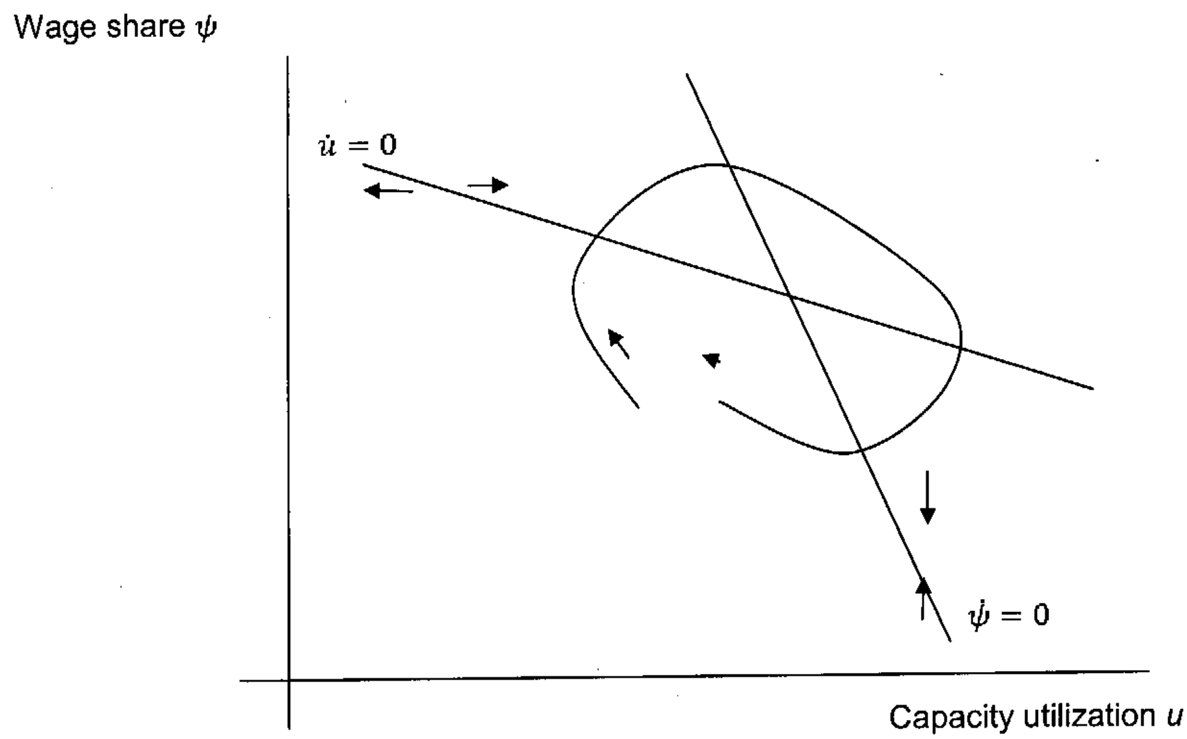

Figure 1: Distribution and demand dynamics with a Harrod-style investment function

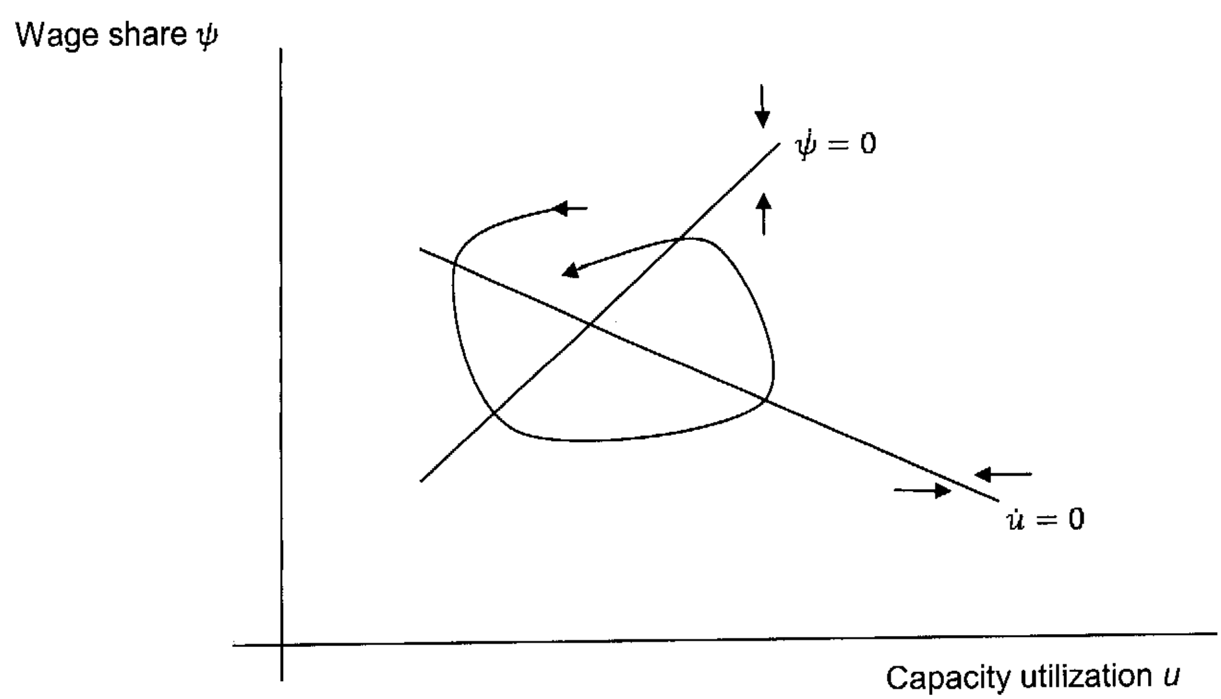

Figure 2: Distribution and demand dynamics with a Domar-style investment function

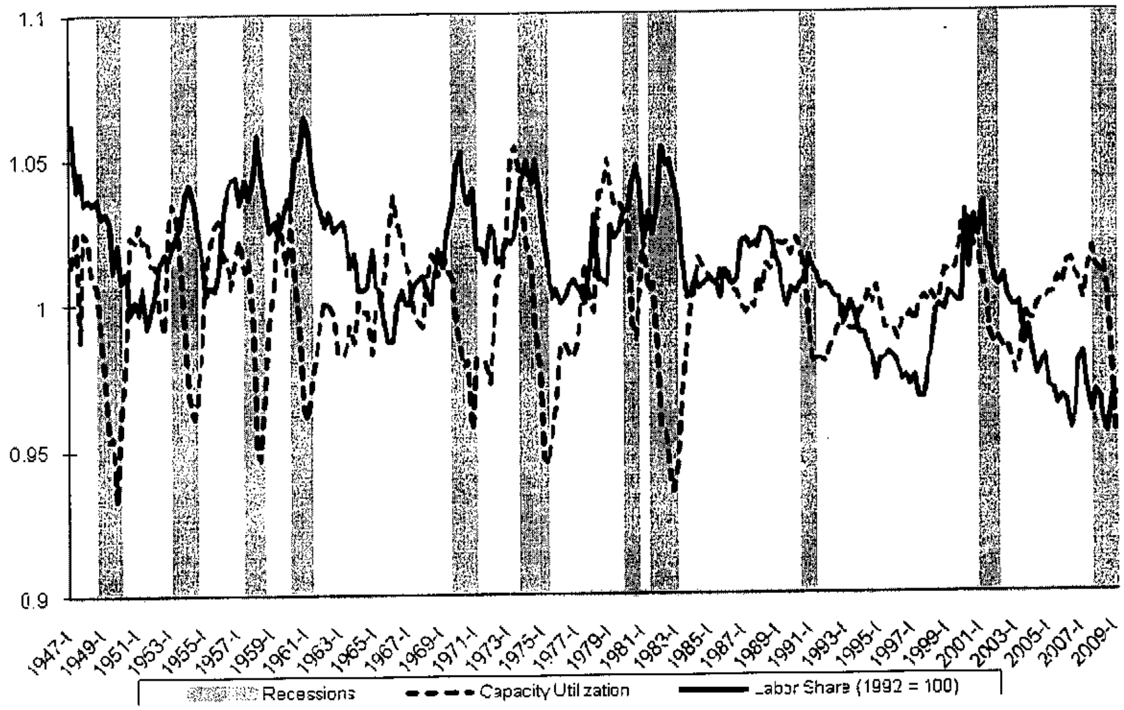

Figure 3: Time series for capacity utilization and wage share

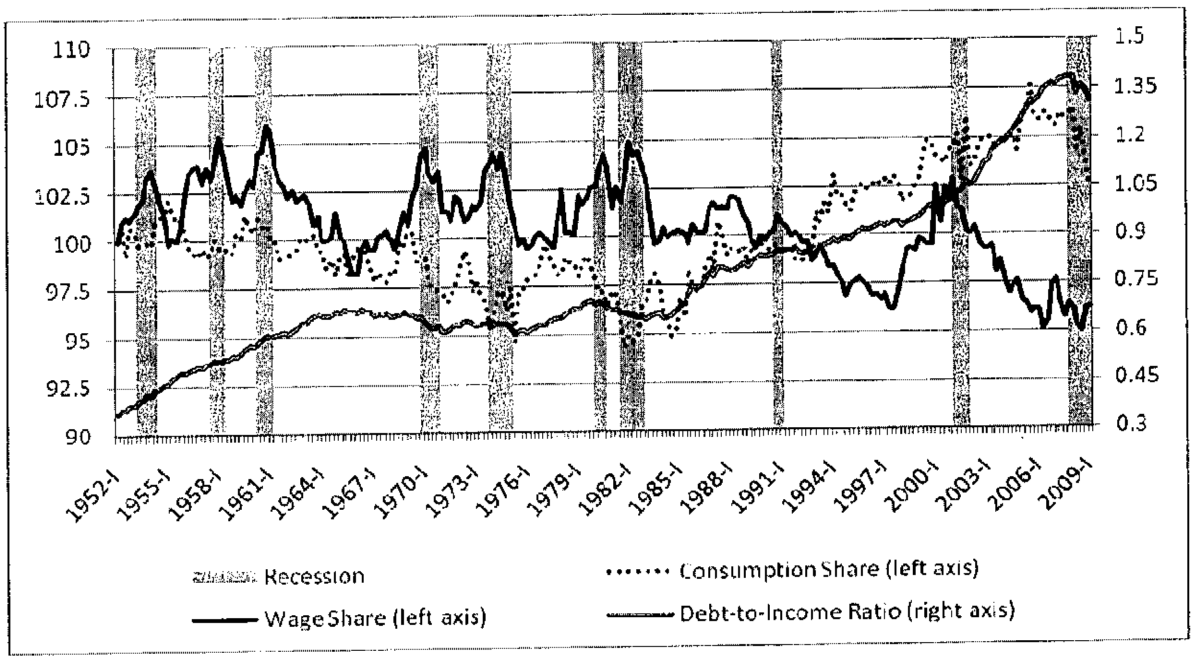

Figure 4: Wage share of value-added, consumption share of disposable income, and household debt to income ratio with NBER reference recessions

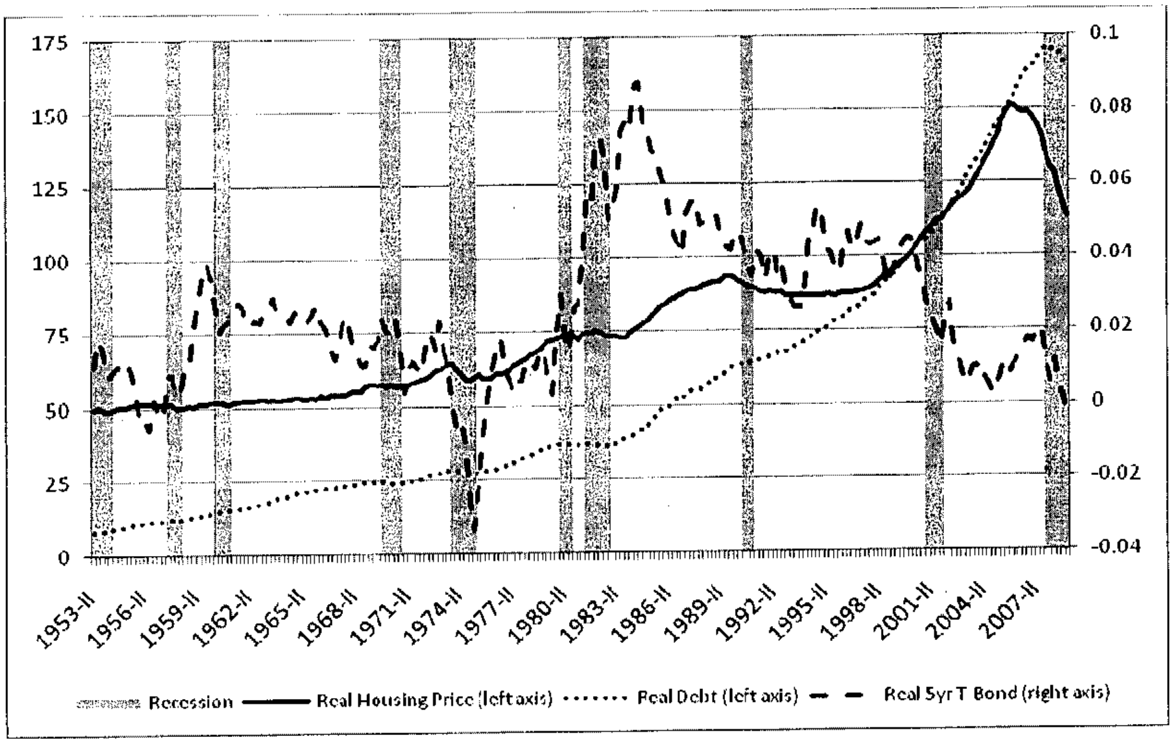

Figure 5: Real medium-term interest rate, housing prices, and real household debt with NBER reference recessions

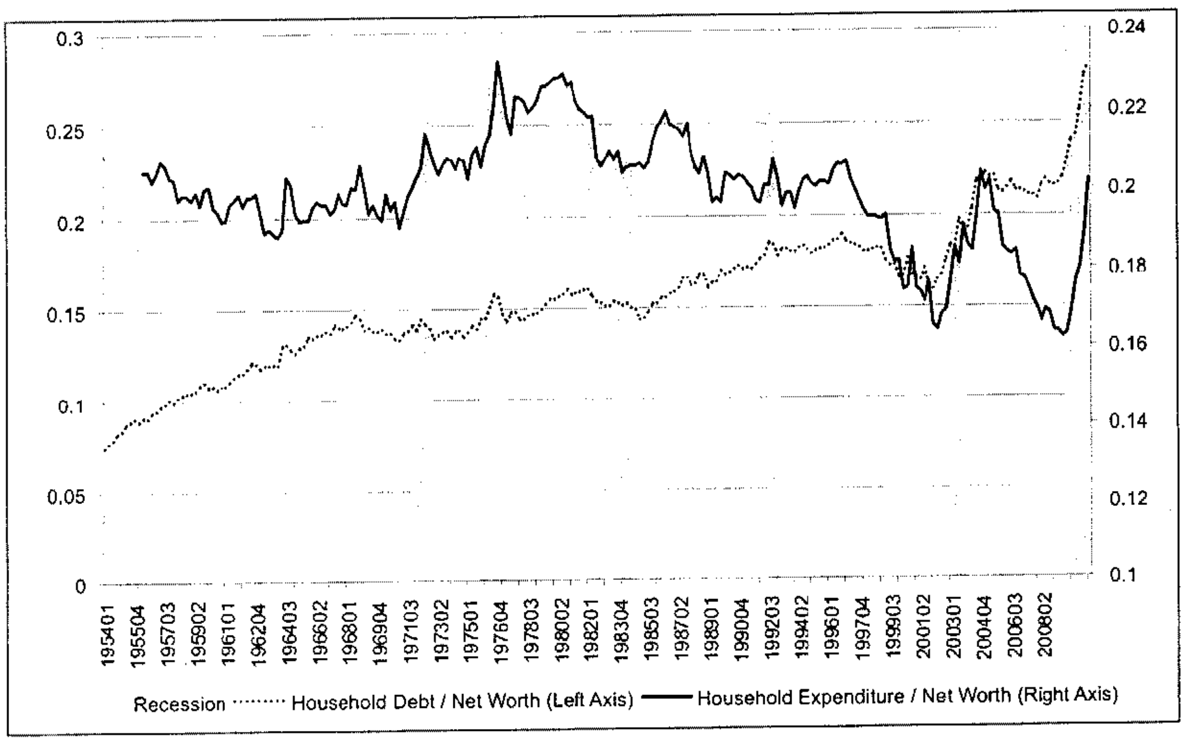

Figure 6: Household debt and expenditure relative to net worth

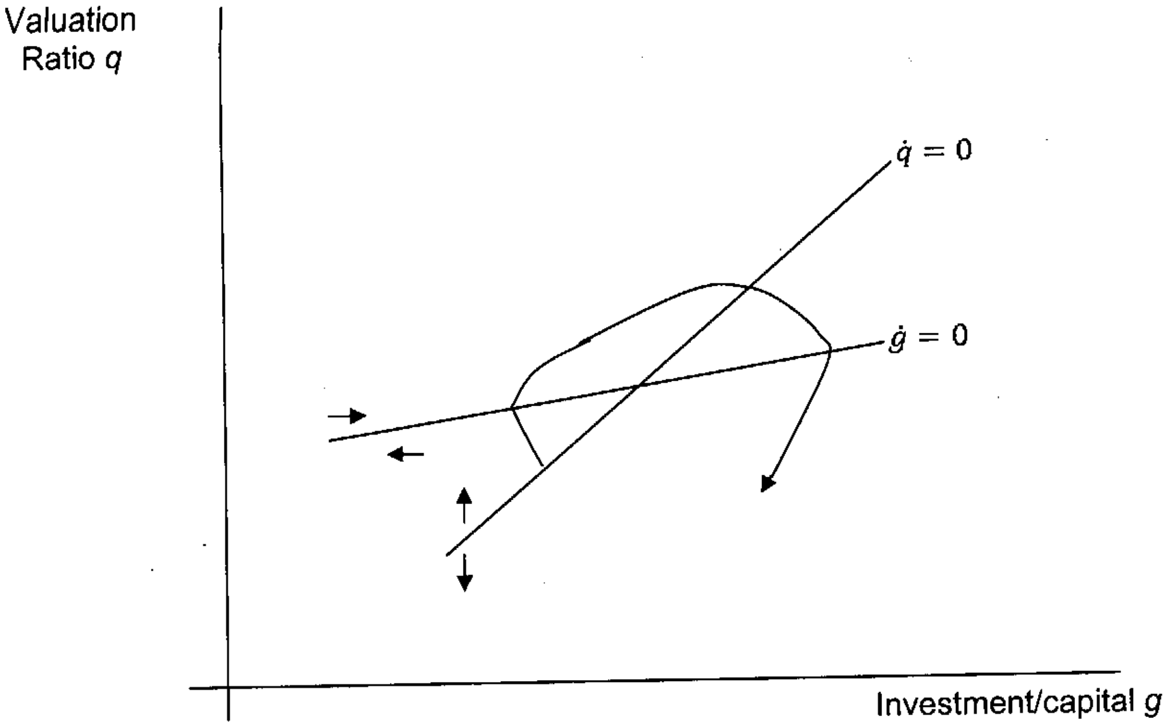

Figure 7: Dynamics of investment and the valuation ratio.

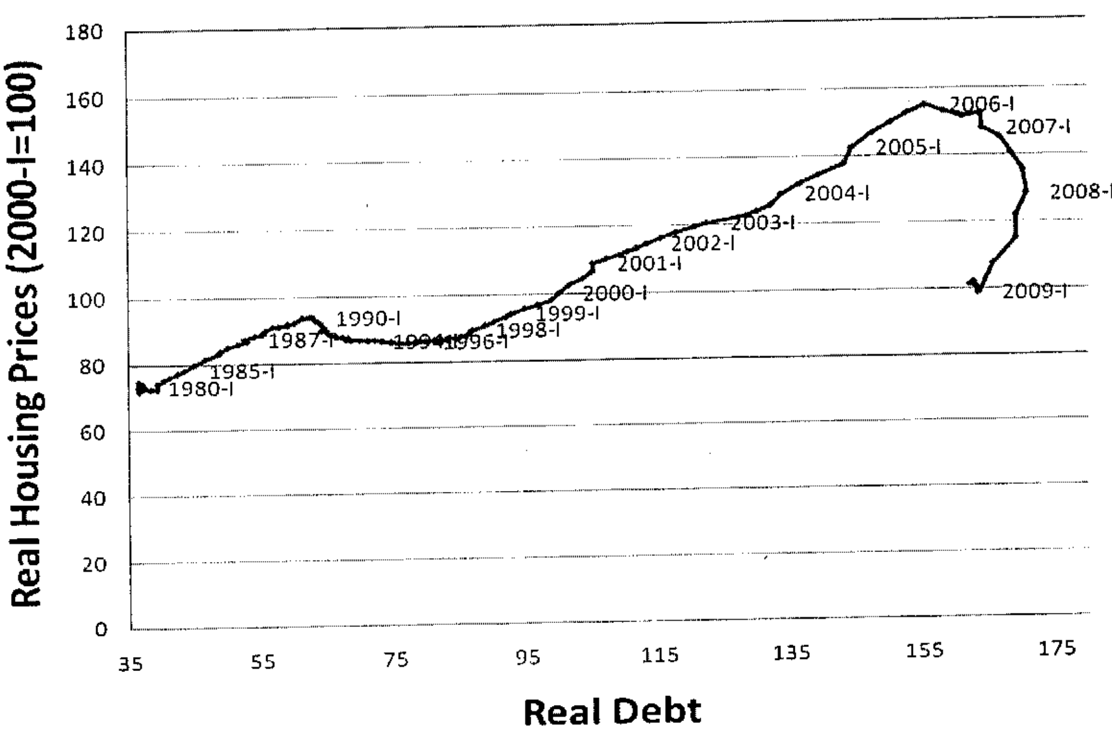

Figure 8: Real US housing price index vs. real consumer debt

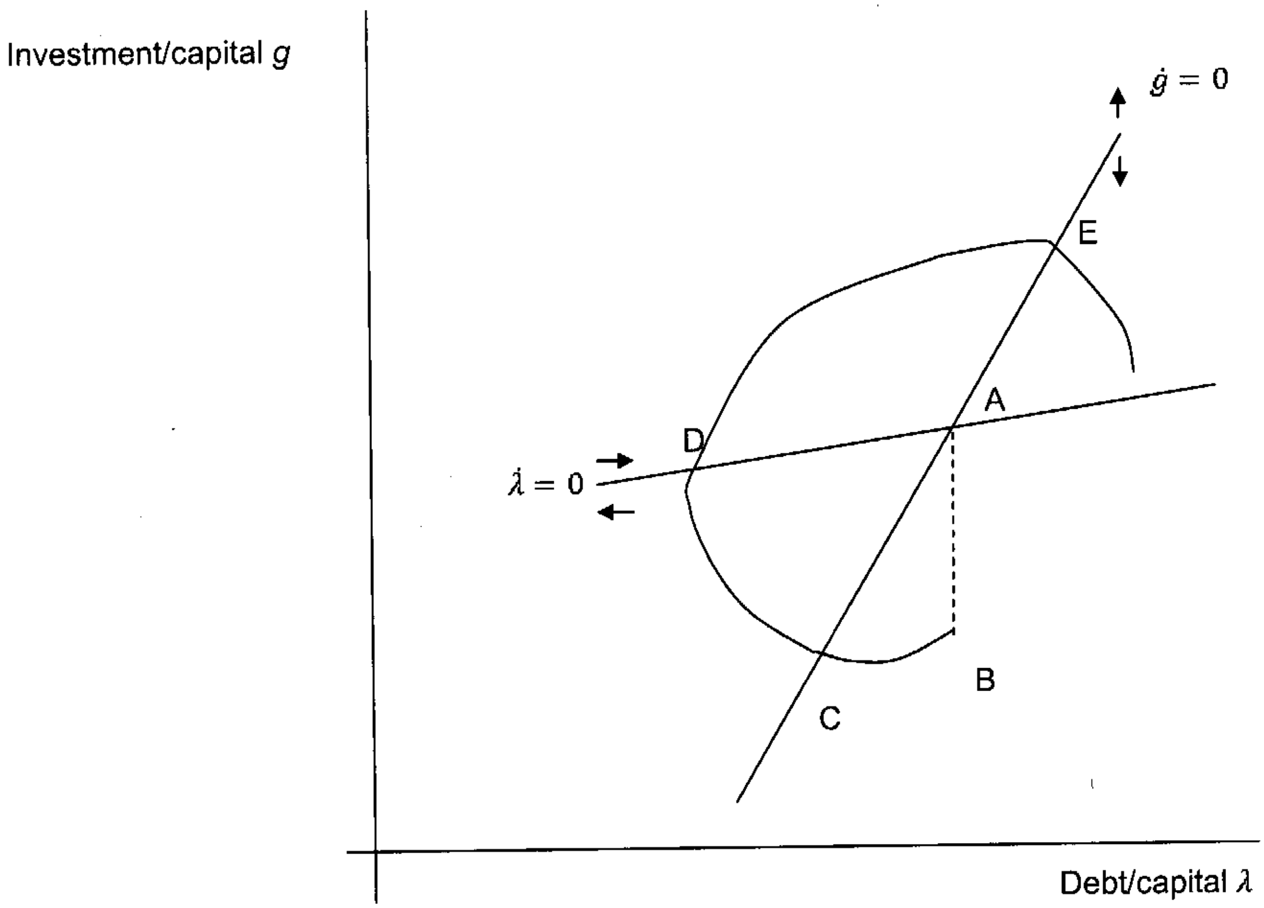

Figure 9: Dynamics of a Minsky investment cycle.