

*Matthieu Charpe*

*Carl Chiarella*

*Peter Flaschel*

*Christian R. Proaño*

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# Business Confidence and Macroeconomic Dynamics in a Nonlinear Two-Country Framework with Aggregate Opinion Dynamics

Matthieu Charpe<sup>\*a</sup>, Carl Chiarella<sup>b</sup>, Peter Flaschel<sup>c</sup>, and Christian R. Proaño<sup>d</sup>

<sup>a</sup>International Labor Organization, Geneva, Switzerland

<sup>b</sup>University of Technology Sydney, Australia

<sup>c</sup>Bielefeld University, Bielefeld, Germany

<sup>d</sup>The New School for Social Research, New York, USA

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## Abstract

The main objective of the present paper is to investigate explicitly the role of the state of confidence for the macroeconomic dynamics of two interacting economies using the opinion dynamics approach by Weidlich and Haag (1983) and Lux (1995). Particularly, the overall state of confidence in the world (two-country) economy plays not only for the dynamics of the nominal exchange rate but also for the dynamics of the real economy through the determination of aggregate investment. This novel feature allows us to consider far richer international macroeconomic interactions than most standard models. Further, it features wage-price dynamics that interact with output and employment fluctuations – leading to a Goodwin (1967)-type of distributive cycle –, as well as debt dynamics due to a credit-financed investment behavior. The resulting framework is both advanced as well as flexible enough to generate various types of persistent fluctuations, and also complex dynamics.

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<sup>\*</sup>Corresponding author. E-mail: charpe@ilo.org

## 1 Introduction

As it is generally acknowledged, expectations and beliefs about future developments play a crucial role in the current decision making of economic agents. However, as already pointed out by Keynes (1936), such expectations are not only the reflection of economic fundamentals but they are also – and probably mainly – determined by “animal spirits”. In Keynes’s own words: “a large proportion of our positive activities depend on spontaneous optimism rather than mathematical expectations, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of animal spirits – a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities” (Keynes, 1936, p. 161-62).

Despite the potential importance of such “animal spirits” for the economic agents’ expectations formation and decision making, they played only a secondary role in the mainstream macroeconomics literature of the last forty years given the predominance of the rational expectations paradigm originally proposed by Muth (1961).<sup>1</sup> Nonetheless, since the outbreak of the 2007-08 global financial crisis, alternative approaches to the rational expectations paradigm based on bounded rationality and social norms have become increasingly interesting to the great majority of the economics profession (see, for example, the recent edited volume by Frydman and Phelps, 2013).

In this context, a particularly interesting field of research is the modelling of bounded-rationality based aggregate opinion dynamics. Indeed, given the social context in which economic agents interact, the beliefs and opinions by one agent are quite likely to affect other agent’s decision making, and contribute thus to formation of an aggregate opinion which is in turn likely to affect the agents’ decisions through a process of “social learning” (cf. Acemoglu and Ozdaglar, 2011). Lux (2009) provides for example empirical evidence which suggests that survey data on business confidence may be the result of a social process of opinion formation among the respondents, instead of being the representation of rational forecasts of future economic developments.

From a theoretical perspective, a particularly elegant approach for the modeling of such opinion dynamics was proposed by Weidlich and Haag (1983). In this framework – which has been widely applied in the behavioral finance following the work by Lux (1995) – agents’ beliefs are modelled through a binary choice problem where the stochastic transitions of agents between two types of beliefs alternatives due to exogenous factors and group pressure. The great majority of studies in the behavioral finance literature along these lines, however, restrict their analysis to the financial markets, and do not consider in an explicit manner how heterogeneous behavioral expectations at the microeconomic

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<sup>1</sup>In the DSGE framework animal spirits are related with sunspot equilibria, i.e. a situation where the equilibrium is indeterminate and endogenous persistent fluctuations are generated by “coordinated” (and thus self-fulfilling) revisions of expectations (since, technically, agents can be conceived).

level may interact with macroeconomic fundamentals in an international setup.<sup>2</sup> However, given the quite advanced integration of international goods and financial markets, the analysis of international macroeconomic interactions has become increasingly relevant both from the academic as well as from the policy perspective.

In this light, the main objective of the present paper is to fill in this gap in the behavioral (macro-)finance literature by explicitly investigating the role of the state of confidence for the macroeconomic dynamics of two interacting economies using the opinion dynamics approach by Weidlich and Haag (1983) and Lux (1995).<sup>3</sup> Particularly, the overall state of confidence in the world (two-country) economy plays not only for the dynamics of the nominal exchange rate but also for the dynamics of the real economy through the determination of aggregate investment. This novel feature allows us to consider far richer international macroeconomic interactions than most standard models. Further, it features wage-price dynamics that interact with output and employment fluctuations – leading to a Goodwin (1967)-type of distributive cycle –, as well as debt dynamics due to a credit-financed investment behavior. The resulting framework is both advanced as well as flexible enough to generate various types of persistent fluctuations, and also complex dynamics. This model also provides a platform for the analysis of economic policy as a necessary stabilization mechanism. The fluctuations shown in the paper preserve the insight of seminal partial contributions of the literature such as the Goodwin distributive cycle, the related inflation-stagflation cycle, debt cycles, and more.

The remainder of the paper is organized as follows. In section 2 we present the theoretical framework representing two large open economies interacting with each other through international trade of goods as well as through financial linkages. In section 3 we describe the model’s intensive form and its steady state. Various numerical simulations of the model focusing among other things on the role of the state of confidence for the macrodynamics of the two economies are discussed in section 4. Finally, we draw in section 5 some concluding remarks from this study.

## 2 The Model

In the following we consider two countries which interact with each other through the international trade of goods and services as well as financial assets. For the sake of simplicity, we assume however

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<sup>2</sup>Indeed, the analysis of such international interactions has been based over the last decades – and hand in hand with increasing predominance of the neoclassical rational expectations approach following the work by Lucas (1976) – on the presumption of rational agents, see e.g. Frenkel and Razin (1985) and Corden (1985), with the outcome that the resulting theoretical frameworks – being the New Open Economy Macroeconomics (NOEM) approach developed by Obstfeld and Rogoff (1995) the most recent incarnation – suggest an inherent and often misleading stability of such international interactions.

<sup>3</sup>See Taylor and O’Connell (1985) and Franke and Semmler (1989), for two early contributions thematizing the role of the state of confidence for the economy’s dynamics and stability, as well as Franke (2012) and Flaschel, Hartmann, Malikane and Proaño (2014) for other macroeconomic models along the lines of the present paper.

that only consumption goods are internationally traded, while investment and government consumption goods are country-specific, and that there is no labor mobility across countries. Further, we assume that while the behavioral equations and macroeconomic adjustments mechanisms of both economies are characterized by the same structure, they may differ in some specific numerical parameter values in order to allow for different types of economic asymmetries.<sup>4</sup>

## 2.1 The private sector

In a standard manner, both economies consist of a household sector, an entrepreneurial sector and a government sector. The household sector consists of asset holders – who own the means of production and obtain an income out of the firms’ profits – and workers who provide their skills and labor for a wage income on a continuous basis. Asset holders and workers are assumed to have different marginal consumption propensities out of their respective income streams: While workers are assumed to consume the totality of their labor income, the asset holders’ marginal consumption propensity out of their capital income  $c_c$  is assumed to be significantly smaller than one.

Let  $w$  denote the nominal wage,  $L^d$  the firms’ labor demand,  $T_w$  the taxes paid by the workers out of their labor income,  $T_c$  the taxes paid by the asset holders out of their gross profits, and  $\rho_g$  be the firms’ gross profit rate, defined as

$$\rho_g = \frac{pY - wL^d - r\Lambda - \delta pK}{pK} \quad (1)$$

where  $Y$  is aggregate output,  $r$  the nominal interest rate,  $\Lambda$  the firms’ debt level,  $K$  the domestic capital stock, and  $\delta$  the capital depreciation rate (analogous expressions for the foreign economy are denoted by the superscript  $*$  in the following). Aggregate domestic private consumption is then described by

$$pC = wL^d - pT_w + c_c(\rho_g pK + r\Lambda - pT_c), \quad c_c \leq 1 \quad (2)$$

where  $wL^d - pT_w$  represents the workers’ disposable labor income and  $\rho_g pK - pT_c$  the asset holders’ capital income after taxes. The distribution of income between workers and asset holders is thus non-trivial for aggregate demand, and by extension for the overall macroeconomic development in both domestic and foreign countries as aggregate private consumption is assumed to be a function of the workers’ and asset holders’ current income and asset holders feature a lower marginal propensity to consume than the workers.

Concerning the activities of the entrepreneurial sector in both countries (which is own by the respective asset holders, as previously mentioned), we assume as in previous related work a production technology with fixed coefficients and that labor demand by firms is determined by

$$L^d = Y/x, \quad (3)$$

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<sup>4</sup>For a more elaborate framework along the same modeling lines where the interconnectedness of a high-income and a low-income labor markets is explicitly modeled, see Charpe, Flaschel, Krolzig, Proaño, Semmler and Tavani (2014).

with  $Y$  denoting aggregate output,  $K$  the capital stock and  $x$  the average labor productivity.

The growth rate of capital stock (i.e. aggregate investment by the entrepreneurial sector  $I$  in terms of domestic capital units  $K$ )  $\iota$  is in turn assumed to depend on the firms' expected gross profit rate  $\rho^{ge}$ , on the real interest rate  $r - \hat{p}$  (where  $r$  represents the nominal interest rate and  $\hat{p}$  the price inflation rate), on the utilization rate of the workforce of firms  $u = Y/Y^p$ , where  $Y_p$  represents the firms' potential output which in turn is determined by the given output-capital ratio  $y_p = \text{const.}$ , i.e.  $Y^p = y^p K$ ),<sup>5</sup> on the firms' debt to capital ratio  $\lambda$ , (all these variables as deviations from their corresponding steady state levels), and in addition on a given trend term  $\gamma$  which determines all other trend terms considered in this model, i.e.

$$\iota = I/K = i_\rho(\rho^{ge} - \rho_o^g) - i_r(r - \hat{p} - \bar{r}) + i_u(u - u_o) - i_\lambda(\lambda - \lambda_o) + \gamma \quad (4)$$

where  $\dot{K} = I - \delta$ , where  $I$  represents net aggregate investment,  $\dot{K}$  the change in the firms' capital stock, and  $\rho^{ge} = \rho^g + \psi$ , where  $\psi$  is the state of confidence in the economy. Accordingly, the growth rate of capital is assumed to be driven by expectations of profitability, financing costs, the rate of capacity utilization relative to its normal level, the firms' indebtedness (also relative to its "normal" level), and by an exogenous term  $\gamma$ .

The state of confidence of the economy – which determines the expected profit rate, since  $\rho^{ge} = \rho^g + \psi$  – is determined by

$$\psi = \psi_a(u - u_o) + \psi_b(u^* - u_o^*) \quad (5)$$

The state of confidence  $\psi$  is assumed to depend positively on the current state of the business cycle in both countries represented by the respective capacity utilization rates  $u$  and  $u^*$  and negatively on an overvalued real exchange rate, and viceversa (cf. Charpe, Flaschel, Hartmann and Proaño, 2014). The state of confidence in the economy is thus assumed to affect not only the dynamics of the exchange rate, as it will be discussed below, but also the real economy through its impact on the expected profit rate and by extension on aggregate investment.

We assume that firms distribute all profits to asset owners and that firms are not confronted with any type of credit rationing. It follows that new credit is given by the level of aggregate investment according to the following rule:

$$\dot{\Lambda} = (\alpha_f^o + \alpha_f^1(\lambda_o - \lambda))pI \quad (6)$$

The wage-price inflation dynamics is of the kind considered e.g. by Chiarella and Flaschel (2000), Flaschel and Krolzig (2006) and are described by

$$\hat{w} = \beta_{we}(e - e_o) + \kappa_w(\sigma\hat{p} + (1 - \sigma)(\hat{p}^* + \hat{s})) + (1 - \kappa_w)\pi^c + \hat{x} \quad (7)$$

$$\hat{p} = \beta_{pu}(u - u_o) + \kappa_p(\hat{w} - \hat{x}) + (1 - \kappa_p)\pi^c \quad (8)$$

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<sup>5</sup>Note that by denoting with  $y$  the actual output-capital ratio, it holds for the utilization rate of capital that  $u = Y/Y^p = y/y^p$ .

where  $\hat{w}$  and  $\hat{p}$  represent the wage- and price inflation rates,  $\hat{x} = \text{const.}$  the growth rate of labor productivity,  $v = \omega/x$  the wage share and  $\pi^c$  the economy's long-run inflation rate, which for the sake of simplicity will be assumed to be equal to the central bank's target inflation rate in the following. Accordingly labor- and goods market disequilibria have a positive direct impact on wage and price inflation, respectively, as well as an indirect impact through the cross-over inflationary expectational terms, with the respective weights  $\kappa_{wp}$  and  $\kappa_{pw}$ . In this context, following the assumption that only consumption goods are traded internationally, the firms's expectations are given by a weighted of the the growth rate of domestic unit labor costs  $\hat{w} - \hat{x}$ , and long-term inflation  $\pi^c$ . By contrast, nominal wage inflation takes into account not only the domestic price inflation, but also the imported inflation given by  $\hat{p}^* + \hat{s}$  by the extent  $(1 - \sigma)$ .

It can be shown that the structural wage- price inflation equations given by eqs. (7) and (8) can be reduced to one equation which describes the law of motion of the labor share  $v$  (under the assumption that  $\hat{x} = 0$ )

$$\dot{v} = \kappa[(1 - \kappa_{pw})\beta_{we}(e - e_o) - (1 - \kappa_{wp})\beta_{pu}(u - u_o) + \kappa_{pw}(1 - \kappa_{pw})(1 - \sigma)(-\hat{\eta})] \quad (9)$$

where  $\kappa = 1/(1 - \kappa_w\kappa_p)$  and  $\hat{\eta} = \hat{p} - \hat{s} - \hat{p}^*$ .

Finally, aggregate output is assumed to be determined in a delayed manner in reaction to the aggregate demand

$$Y^d = C + I + G + \delta K + NX \quad (10)$$

where  $G$  represents the government expenditures (to be described below), and  $NX$  the net exports, given by

$$NX = (1 - \sigma^*)C^*/\eta - (1 - \sigma)C, \quad (11)$$

with  $s$  being the nominal exchange rate,  $\eta = p/(sp^*)$  the real bilateral exchange rate and  $(1 - \sigma)$  representing the share of foreign goods in the domestic households's aggregate consumption  $C$ , and viceversa.

Finally, aggregate output is assumed to be determined by

$$\dot{Y} = \beta_y(Y^d - Y) + nY \quad (12)$$

with  $\rho^g$  given by eq.(1),  $I = \iota K$ , with  $\iota$  given by eq.(4),  $NX$  by eq.(11) and  $G$  by eq.(20).

## 2.2 FX Market Dynamics

Concerning the determination of the nominal exchange rate, we assume in the following that its growth rate is determined by two sources: real trade-linked demand and supply of foreign currency, as well as by financial trading. Specifically, we assume that the dynamics of the nominal exchange rate is driven

$$\hat{s} = \beta_s(r^* - r + \pi_s^e) - \beta_x nx. \quad (13)$$

where the first term represents arbitrage possibilities results from deviations of the expected rates of return of foreign and domestic bonds from the Uncovered Interest Rate Parity (UIP) to an extent  $\beta_s$ , and by  $nx$ , the net exports to domestic capital ratio, which represent the excess supply for foreign currency and its appreciative effect on the domestic currency.<sup>67</sup>

The expectations driving financial trading are modeled through an aggregate opinion dynamics approach along the lines of Lux (1995), and more recently, Franke (2012). Accordingly, financial trading in the FX market can be assumed to be characterized by two main forecasting strategies: *fundamentalism* (denoted by a superscript  $f$ ), and *chartism* (denoted by a superscript  $c$ ). Fundamentalist traders expect the nominal exchange rate to stabilize in the long run, or equivalently, that the growth rate of nominal exchange rate becomes zero – its steady state position in the present model – while chartists traders use a simple adaptive rule to forecast the depreciation rate of the nominal exchange rate  $\hat{s}$ .

Given that agents have heterogeneous expectations, it is not obvious a priori what *market* expectations should be. Indeed, while in standard equilibrium models with efficient markets heterogeneous information and beliefs are spontaneously aggregated and made uniform under the pressure of market forces, this is not the case in our framework given the implicit bounded rationality underlying our modeling approach.

For the sake of simplicity, we assume as is Flaschel, Hartmann, Malikane, & Proaño (2014), the following simple law of motion for the expected depreciation rate:<sup>8</sup>

$$\begin{aligned}\dot{\pi}_s^e &= \beta_{\pi_s^e} \left[ \left(1 - \frac{1+z}{2}\right) (0 - \pi_s^e) + \frac{1+z}{2} (\hat{s} - \pi_s^e) \right] \\ &= \beta_{\pi_s^e} \left[ \frac{1+z}{2} \hat{s} - \pi_s^e \right],\end{aligned}\tag{14}$$

where  $\beta_{\pi_s^e} > 0$  represents an adjustment speed parameter,  $\pi_s^e$  the market expectation of growth rate of the nominal exchange rate, and  $z$  is the relative share of chartists in the whole population of chartists and fundamentalists, defined as

$$z = \frac{n}{N} \in [-1, 1], \quad 1 - z = \frac{N_f}{N}, \quad 1 + z = \frac{N_c}{N},\tag{15}$$

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<sup>6</sup>The economic rationale for this exchange rate specification can be related with the economy's balance of payments – which comprises the trade as well as the capital accounts –, namely

$$NX + (r^* - \hat{\eta})B_b - (r - \hat{p})B_a^* = \eta\dot{B}_b - \dot{B}_a^*,$$

where  $B_b$  and  $B_a$  the domestic economy's holdings of foreign and domestic bonds, respectively. As it should be clear, trading recorded in the trade and the capital accounts does not have to follow the same rationale or rules, and may therefore be modelled independently.

<sup>7</sup>Note that capital gain expectations are zero in the inflation-free steady state of the whole model.

<sup>8</sup>Obviously, this is only one possible formalization of the dynamics of aggregate expectations in markets with heterogeneous agents, and alternative approaches can be proposed (see, for example, the approach adopted by De Grauwe and Grimaldi (2006) in their analysis of the behaviour of agents on foreign exchange markets). Yet, we regard eq.(14) as a very parsimonious way of capturing *both* the influence of aggregate observed variables *and* the role of heterogeneity and self-driving forces in expectation formation.



where,  $N_c$  is the number of speculative traders using chartism and  $N_f$  those with fundamentalist expectations, with  $N_c + N_f = 2N$  as in Franke (2012), and  $n = \frac{N_c - N_f}{2}$ . The distribution of chartists and fundamentalists in the population is thus described by the difference in the size of the two groups normalised by  $N$ .<sup>9</sup>

We denote by  $p^{f \rightarrow c}$  the transition probability of a fundamentalist becoming a chartist, and viceversa for  $p^{c \rightarrow f}$ , and we assume that these transition probabilities are determined by

$$p^{f \rightarrow c} = \exp(\psi), \quad (16)$$

$$p^{c \rightarrow f} = \exp(-\psi). \quad (17)$$

where  $\psi$  represents the state of confidence as described by eq.(5). Accordingly, an increase in  $\psi$  is assumed to increase the probability that a fundamentalist becomes a chartist, and to decrease the probability that a chartist becomes a fundamentalist. Further, the evolution of  $z$  is assumed to be determined by

$$\dot{z} = \beta_z((1 - z)p^{f \rightarrow c} - (1 + z)p^{c \rightarrow f}) \quad (18)$$

Inserting this expressions in eq.(18) delivers<sup>10</sup>

$$\dot{z} = \beta_z((1 - z)\exp(\psi) - (1 + z)\exp(-\psi)). \quad (19)$$

We thus assume that exchange rate expectations are driven in a complex manner through the relative importance of chartists and fundamentalists in the FX market.<sup>11</sup>

Given the strong nonlinearity of the opinion part and also in the rate of return function of the 4D dynamics (despite the simple linear behavioral rules we have adopted), we shall address these questions by means of numerical simulations in the next section. They will show that interesting irregular and persistent fluctuations in the real and financial variables of the model can be generated, quite in contrast to what is possible in such a model type under the assumption of the homogeneous rational expectations of the mainstream literature.

### 2.3 The government sector

The government's behavior is described by

$$G = T_w + T_c - g_c(u - u_o)K \quad (20)$$

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<sup>9</sup>As in Franke (2012), we assume that  $N$  is large enough so that the intrinsic noise from different realisations when individual agents apply their random mechanism can be neglected.

<sup>10</sup>This law of motion is derived in detail in Franke (2012) and its use of the exponential terms guarantees that neither the chartist nor the fundamentalist group can be totally eliminated in the FX-market.

<sup>11</sup>As fundamentalist form their expectations in a simple regressive way, these expectations do not really show up in the formation of averages and the average expectation of their formation.

Government expenditures (which include unemployment benefits that are totally consumed) follow an anti-cyclical fiscal policy, which is financed through the issuance of new government bonds. This anti-cyclical behavior fluctuates around a balanced-budget level determined by the tax revenues  $T_w + T_c$ . With respect to monetary policy, the following Taylor rule is assumed

$$r = \bar{r} + r_p(y/y^p - u_o) + r_p(\hat{p} - \bar{\pi}). \quad (21)$$

Accordingly, the monetary authorities simply raise the interest rate above a given level  $\bar{r}$  if domestic prices are higher than their steady state value and vice versa. This fixing of the rate of interest  $r$  at each moment of time only requires that money supply  $M$  (not explicitly modeled here) is endogenous and adjusted to the money demand of asset holders via open market operations in the form of short-term government bonds  $B$ .

### 3 The model's intensive form and steady state

We now reformulate the two-country framework just outline in terms of domestic capital units, so that  $y = Y/K$ ,  $y^d = Y^d/K$ ,  $t_w = T_w/K$ ,  $t_c = T_c/K$ ,  $\iota = I/K$ ,  $g = G/K$ , and  $k = K^*/K$ . The dynamic laws of motion which describe the evolution of the two-country system over time read on the intensive form level:

$$\begin{aligned} \dot{y} &= \beta_y(y^d - y) - y(\iota - \delta - n) \\ \dot{\omega} &= \kappa[\beta_w(e - \bar{e})(1 - \kappa_p) - \beta_p(u - \bar{u})(1 - \kappa_w\gamma_w) + \kappa_w(1 - \gamma_w)(\hat{p}^* + \hat{s})(1 - \kappa_p) - \pi(1 - \gamma_w)\kappa_w(1 - \kappa_p)] \\ \dot{\lambda} &= (\alpha_f^o + \alpha_f^1(\lambda_o - \lambda))\iota - [\iota + \hat{p}]\lambda \\ \dot{y}^* &= \beta_y(y^{d,*} - y^*) - y^*(\iota^* - \delta^* - n^*) \\ \dot{\omega}^* &= \kappa^*[\beta_w^*(e^* - \bar{e}^*)(1 - \kappa_p^*) - \beta_p^*(u^* - \bar{u}^*)(1 - \kappa_w^*\gamma_w^*) + \kappa_w^*(1 - \gamma_w^*)(\hat{p} - \hat{s})(1 - \kappa_p^*) - \pi^*(1 - \gamma_w^*)\kappa_w^*(1 - \kappa_p^*)] \\ \dot{\lambda}^* &= (\alpha_f^o + \alpha_f^1(\lambda_o - \lambda^*))\iota^* - [\iota^* + \hat{p}^*]\lambda^* \\ \hat{\eta} &= \hat{p} - \hat{s} - \hat{p}^*, \\ \dot{z} &= \beta_z((1 - z)\exp(\psi) - (1 + z)\exp(-\psi)) \\ \dot{\pi}_s^e &= \beta_{\pi_s^e}\left[\frac{1+z}{2}\hat{s} - \pi_s^e\right] \\ \hat{k} &= \iota^* - \delta^* - (\iota - \delta), \end{aligned}$$

with

$$\begin{aligned}
y^d &= \sigma(vy - t_w + c_c(\rho^g + r\lambda - t_c)) + \iota + g + \delta + k(1 - \sigma)(v^*y^* - t_w^* + c_c(\rho^{g*} + r^*\lambda^* - t_c^*))/\eta \\
\rho^g &= y - vy - r\lambda - \delta \\
\iota &= i_\rho(\rho^g + \psi - \rho_o^g) - i_r(r - \hat{p} - \bar{r}) + i_u(y/y^p - u_o) - i_\lambda(\lambda - \lambda_o) + \gamma \\
g &= t_w + t_c - g_c(y/y^p - u_o) \\
r &= \bar{r} + r_y(y/y^p - u_o) + r_p\hat{p} \\
nx &= (1 - \sigma^*)c^*k/\eta - (1 - \sigma)c \\
y^{d*} &= \sigma(v^*y^* - t_w^* + c_c(\rho^{g*} + r^*\lambda^* - t_c^*)) + \iota^* + g^* + \delta + (1 - \sigma)\eta(vy - t_w + c_c(\rho^g + r\lambda - t_c))/k, \\
\rho^{g*} &= y^* - v^*y^* - r^*\lambda^* - \delta \\
\iota^* &= i_\rho(\rho^{g*} + \psi - \rho_o^{g*}) - i_r(r^* - \hat{p}^* - \bar{r}^*) + i_u(u^* - u_o^*) - i_\lambda(\lambda^* - \lambda_o^*) + \gamma \\
g^* &= t_w^* + t_c^* - g_c(y^*/y^{p*} - u_o^*) \\
r^* &= \bar{r}^* + r_y(u^* - u_o^*) + r_p\hat{p}^* \\
\psi &= \psi_a(y^*/y^{p*} - u_o) + \psi_b(y^*/y^{p*} - u_o^*) - \psi_\eta(\eta - 1)
\end{aligned}$$

and

$$\begin{aligned}
\hat{s} &= \beta_s(r^* - r + \pi_s^e) - \beta_x nx \\
\hat{p} &= \kappa[\beta_p(u - \bar{u}) + \kappa_p\beta_w(e - \bar{e}) + \kappa_p\kappa_w(1 - \gamma_w)(\hat{p}^* + \hat{s}) + \pi\kappa_p\kappa_w(\gamma_w - 1)] + \pi \\
\hat{p}^* &= \kappa^*[\beta_p^*(u^* - \bar{u}^*) + \kappa_p^*\beta_w^*(e^* - \bar{e}^*) + \kappa_p^*\kappa_w^*(1 - \gamma_w^*)(\hat{p} - \hat{s}) + \pi^*\kappa_p^*\kappa_w^*(\gamma_w^* - 1)] + \pi^*.
\end{aligned}$$

The model's steady state expressions representing the balanced growth path of the two economies can be calculated in a recursive manner using this intensive form representation. Setting  $\hat{v} = 0$ , we get the steady state for output  $y = \bar{u}y^p$ , and the steady state for the employment rate  $e = y/x$ . Using the equation for the debt dynamic and setting  $\iota = \gamma$  we get the steady state for the debt to capital ratio  $\lambda = \alpha_f^o$ . Setting  $\dot{y} = 0$ , we get the steady state for aggregate demand  $y^d = y$ . The steady state for public spending is equal to the sum of the tax revenues:  $g = \tau_w + \tau_c$ . Setting  $\bar{r}$  exogenously and  $\eta = 1$ , we get the steady state for the labour share  $\omega$  and the steady state for the rate of profits using the following two equations:

$$\begin{aligned}
y^d &= vy - t_w + c_c(\rho^g + r\lambda - t_c) + \iota + g + \delta \\
\rho^g &= y - vy - r\lambda - \delta.
\end{aligned}$$

Solving for the labour share, we get:

$$v = \frac{x}{y(1 - c_c)} (y^d - c_c(y - \delta - t_c) + t_w - \iota - g - \delta).$$

Assuming that the interest rate are equal across the two country at the steady state, setting  $nx = 0$ , gives the steady state of the relative size between the two countries  $k = \frac{\eta(1-\sigma)c}{(1-\sigma^*)c^*}$ . The definition for  $k$  implies that the two country have the same size when the parameters are identical.

As most of the equilibrium conditions determining the economy's balanced growth path are self-explaining, there is no need to discuss them explicitly. It is however noteworthy that while both higher tax rates on asset-holders and on workers reduce  $v_o$ , the effect of an increase in the propensity to consume of asset-holders is ambiguous, because it depends ultimately on a complex combination of various parameters.<sup>12</sup>

Further, it should be pointed out that we have neglected in the presentation of the model all stock-flow relationships or budget equations which do not have an impact on its dynamics, most importantly the government budget constraint which is still set aside by way of the assumed taxation rules. Also, the stocks of assets – up to the capital stock and the debt to capital ratio of firms do not yet influence the laws of motion of the model, since investment only depends on the loan rate and not on Tobin's  $q$ .

## 4 Numerical Simulations

It should be clear that due to the high dimensional nature of the model, a huge range of outcomes can be expected in theory, ranging in particular from simple Hopf-bifurcation-generated limit cycles or persistent business fluctuations to truly complex attractors and trajectories converging towards them. Therefore, we restrict the following analysis to three scenarios: i) a baseline scenario where the economy behaves in a relatively tranquil manner, returning to its steady state in an oscillatory manner after an initial exogenous monetary policy shock in the domestic economy, ii) a scenario where the state of confidence reacts more strongly to business cycle developments, and iii) the exchange rate depreciation expectations have a higher speed of adjustment.<sup>13</sup>

When not stated elsewhere we assume symmetric economies using the parameter values summarized in Table 1. Our choice of parameters, while not based entirely on econometric estimations, reflects nonetheless the modeling philosophy we pursue here. So for example is  $c_c$ , the asset holders' marginal propensity to consume out of their current income, lower than the workers' marginal propensity as usually assumed in models Kaleckian models thematizing the income distribution conflict, see e.g. Chiarella, Flaschel and Franke (2005). Also, we assume that workers are confronted with a higher relative tax burden than capitalists ( $t_w > t_c$ ), as it is the case in the majority of industrialized economies. Further, we assume that the responsiveness of aggregate investment with respect to expected profits is larger than to the capacity utilization, to the real interest rate and to deviations from a given debt target ( $i_\rho > i_u$ ,  $i_\rho > i_r$  and  $i_\rho > i_\lambda$ ). In contrast, the parameters concerning the wage and price Phillips curves are based on the econometric results for the U.S. economy by

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<sup>12</sup>This is an important feature of the present model as in purely demand-driven models the spending multiplier is typically an increasing function of the marginal propensities to consume. However, the present framework does not only consider aggregate demand, but also income distribution.

<sup>13</sup>All simulations were computed using the SND software, see Chiarella, Flaschel, Khomin, and Zhu (2002). The programming code is available upon request from the authors.

Table 1: Calibration Parameters

Asset holders' marginal propensity of consumption	$c_c = 0.1$
Workers' marginal propensity of consumption	$c_w = 1$
Home bias	$\sigma = 0.75$
Trend gross investment	$\gamma = 0.125$
Depreciation rate of capital	$\delta = 0.075$
Workers' paid taxes per domestic capital unit	$t_w = 0.2$
Asset holders' paid taxes per domestic capital unit	$t_c = 0.1$
Output-capital ratio	$y_p = 1$
Labor productivity	$x_a = 1.1$
Steady State capacity utilization rate	$u_o = 1$
Parameter in the new credit equation	$\alpha_f^o = 0.3$
Parameter in the new credit equation	$\alpha_f^1 = 0.3$
Investment responsiveness to expected profit rate	$i_\rho = 1.1$
Investment responsiveness to interest rate	$i_r = 0.1$
Investment responsiveness to capacity utilization	$i_u = 0.1$
Investment responsiveness to debt-capital ratio	$i_\lambda = 0.05$
Goods markets disequilibrium adjustment parameter	$\beta_y = 6$
Parameter in the wage Phillips curve	$\beta_{we} = 0.6$
Parameter in the wage Phillips curve	$\kappa_w = 0.5$
Parameter in the price Phillips curve	$\kappa_p = 0.5$
Parameter in the price Phillips curve	$\beta_{pu} = 0.2$
Parameter in the nominal exchange rate equation	$\beta_s = 1.5$
Parameter in the nominal exchange rate equation	$\beta_x = 0.5$
Steady state nominal interest rate	$\bar{r} = 0.03$
Steady state inflation rate	$\bar{\pi} = 0.0$
Countercyclical fiscal policy parameter	$g_c = 1.1$
Interest responsiveness to capacity utilization	$r_y = 0.1$
Interest responsiveness to price inflation	$r_p = 1.5$
State of confidence responsiveness to capacity utilization	$\psi_a = \psi_b = 0.1$
Adjustment parameter in the opinion dynamics	$\beta_{\pi_s^e} = 0.1$
Adjustment parameter in the opinion dynamics	$\beta_z = 1.3$

Franke, Flaschel and Proaño (2006) and Flaschel and Krolzig (2006), and reflect in particular the empirical finding that wage inflation is more reactive to labor market disequilibria than price inflation

with respect to goods market disequilibria. Further, concerning the nominal interest rate set by the monetary authorities, we assume a standard value for  $r_p = 1.5$ , as well as  $r_y = 0.1$ . Finally, we assume both in the nominal exchange rate and the aggregate opinion dynamics equations parameters which do not generate an explosive behavior of the whole system, and leave a proper estimation of such an issue for further research.

In the following we discuss by means of alternative numerical simulations some of the properties of this two-country framework.

#### 4.1 Baseline simulation

Figure 1 gives an overview of the transmission channels of the model. The calibration corresponds to the list of parameters presented in the previous section and the shock is the a decline in the interest rate from 3 percent to 2.7 percent in the home country. As expected, the decline in the interest rate in

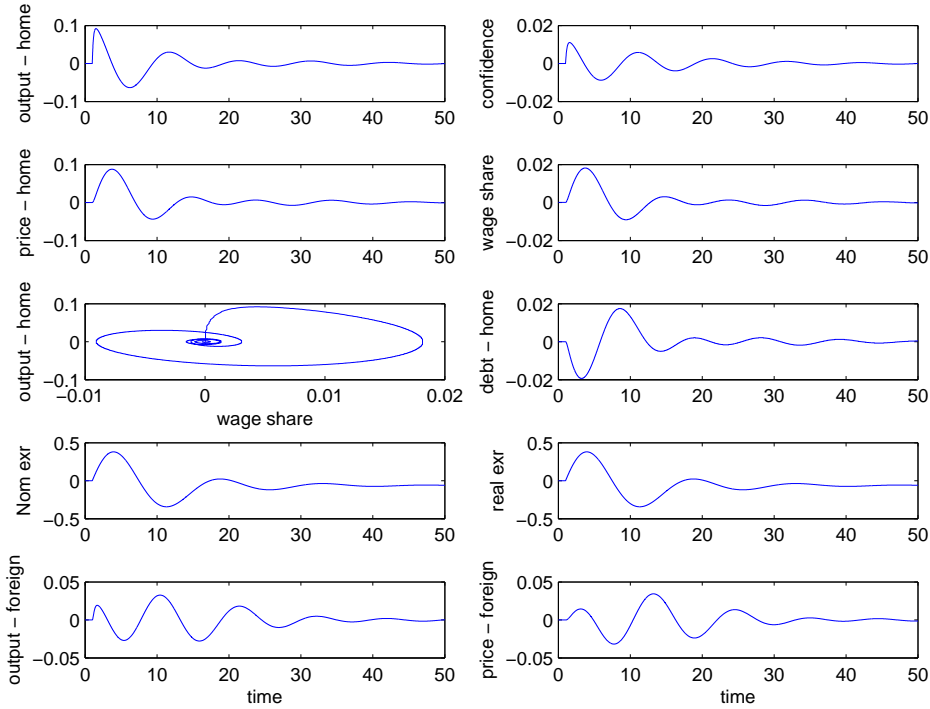


Figure 1: Dynamic adjustments to an exogenous decline in the nominal interest rate in the home country.

the home country affects economic activity in an expansionary manner via its effect on the domestic aggregate investment. Further, the interest rate decline also affects the profitability of firms in an indirect manner through its effect on the expected profit rate via the interest payment channel. In our calibration, this second effect is non-trivial as the parameter  $i_p$  is set equal to 1.1.

The resulting expansion in economic activity in the domestic economy is self-reinforcing through the reaction of the state of confidence – which depends positively on the domestic and foreign business cycles –, increasing domestic investment further. The improvement in the state of confidence leads to an increase the population of chartist relative to fundamentalist agents. This effect is however weak in the baseline calibration since the nominal exchange rate is assumed to depend weakly on the expected depreciation rate in the baseline calibration ( $\beta_{\pi^e} = 0.1$ ).

Due to the increased level of economic activity nominal wage and price inflation rise, as well as the labour share as nominal wage inflation is assumed to be more reactive than price inflation to its corresponding demand pressure (the flexibility of nominal wage inflation is governed by the parameter  $\beta_w = 0.6$ , while price inflation reacts to economic activity at a speed  $\beta_p = 0.2$ ). The interaction between labour costs and aggregate demand generates a cycle along the lines of Goodwin (1967). In the upper part of the cycle, the fast increase in the labour costs tend to crowd profitability and investment putting an end to the expansionary phase of the cycle. The Goodwin cycle is illustrated by the clockwise dependance between the labour share (on the horizontal axis) and economic activity (on the vertical axis) in the graph of the third row, first column.

Following the negative shock on the domestic interest rate the nominal exchange rate depreciates as it is mainly driven by the interest rate differential with  $\beta_i = 1.5$  in the baseline calibration. The depreciation of the nominal exchange rate tends to further increase aggregate demand via an improvement in the trade balance. Further, as a result of the fast rise in prices in the expansionary phase the competitiveness of the home country (measured by the real exchange rate) declines. This competitiveness effect together with the locomotive effect related to the fast growth in the home country generates positive spill-overs on the foreign country where output also expands. This spill-over effects are lagged in time, with the biggest increase taking place over the second business cycle.

Finally, the inflationary pressure in the expansionary phase of the cycle leads to a quick increase in the interest rate since  $r_p = 1.5$ , a dampening effect which is supported by the counter-cyclical fiscal policy behavior according to which fiscal authorities reduces public spending in the boom stabilizing the level of aggregate demand.

## 4.2 The role of the state of confidence

Figure 2 illustrates the transmission channels associated with the state of confidence in the economy  $\psi$ . Figure 2 shows the reaction of home production, foreign production, confidence and the chartist population following a decline in the interest rate from 3% to 2.85%. The difference between the first and the second column in Figure 2 is an increase in the parameter for the state of confidence from  $\psi_a = \psi_b = 0.1$  to  $\psi_a = \psi_b = 0.3$ .

As Figure 2 clearly illustrates, when the state of confidence reacts more sensitively to the level of economic activity in both economies the dynamic adjustment of the two economies (represented

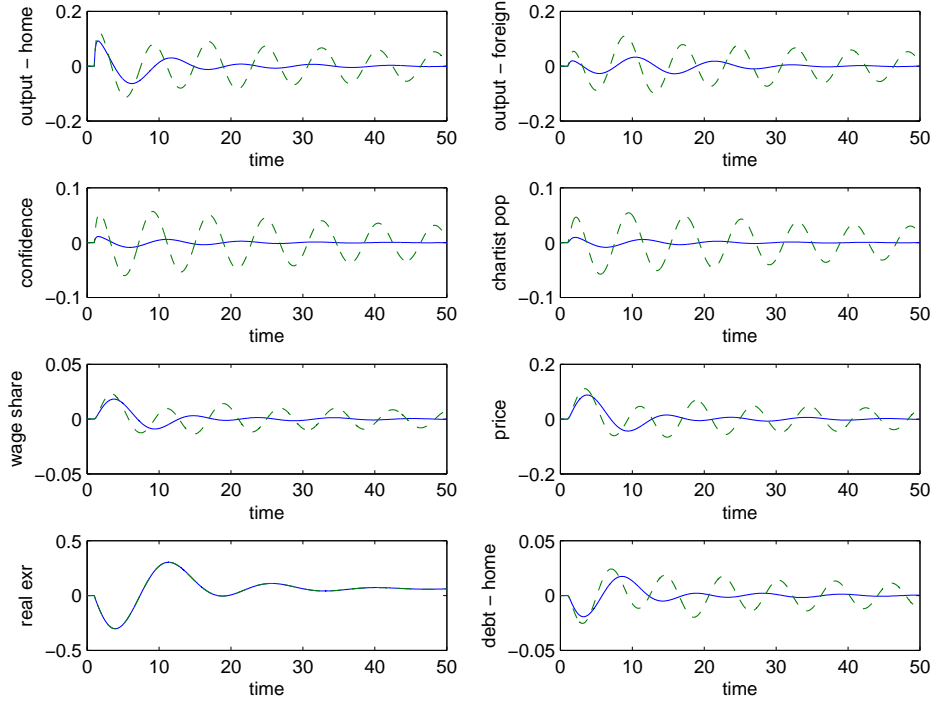


Figure 2: Dynamic adjustments to an exogenous decline in the nominal interest rate in the home country for  $\psi_a = \psi_b = 0.1$  (solid line) and for  $\psi_a = \psi_b = 0.3$  (dashed line).

by the subset of macroeconomic variables shown in Figure 2) is more volatile than in the baseline simulation. This is due to the reinforcing effect of higher economic activity on the state of confidence, and subsequently, on the expected profit rate and on aggregate investment. Further, as in the previous case, the foreign economy profits also from the reduction in the nominal interest rate in the domestic economy, as the foreign country benefits from both higher demand in the home country which translates into higher imports. This positive effect takes place despite the fact that the reduction of the interest rate at home leads to a depreciation of the nominal exchange rate, which is detrimental to the foreign country. There are signs of non-linearities in the model. In the home country, oscillations are dampening over time. Contrastingly, the amplitude of oscillations increase between the first and the second cycle in the foreign country before to lose magnitude.

It should be noted that since expectations adjusts slowly  $\beta_{\pi_s^e} = 0.1$  the relational size of the population does not affect significantly the dynamics of the nominal exchange rate in this simulation. The importance of exchange rate expectations is illustrated in the next exercise.



### 4.3 Exchange rate expectations and FX market population dynamics

Figure 3 illustrates the importance of expectations for the dynamics of the exchange rate. In the present figure, the speed of expectations adjustment  $\beta_{\pi_s^e}$  is increased from 0.1 to 2.0, keeping the parameters for the state of confidence at  $\psi_a = \psi_b = 0.3$ . As it can be clearly observed Figure 3, while

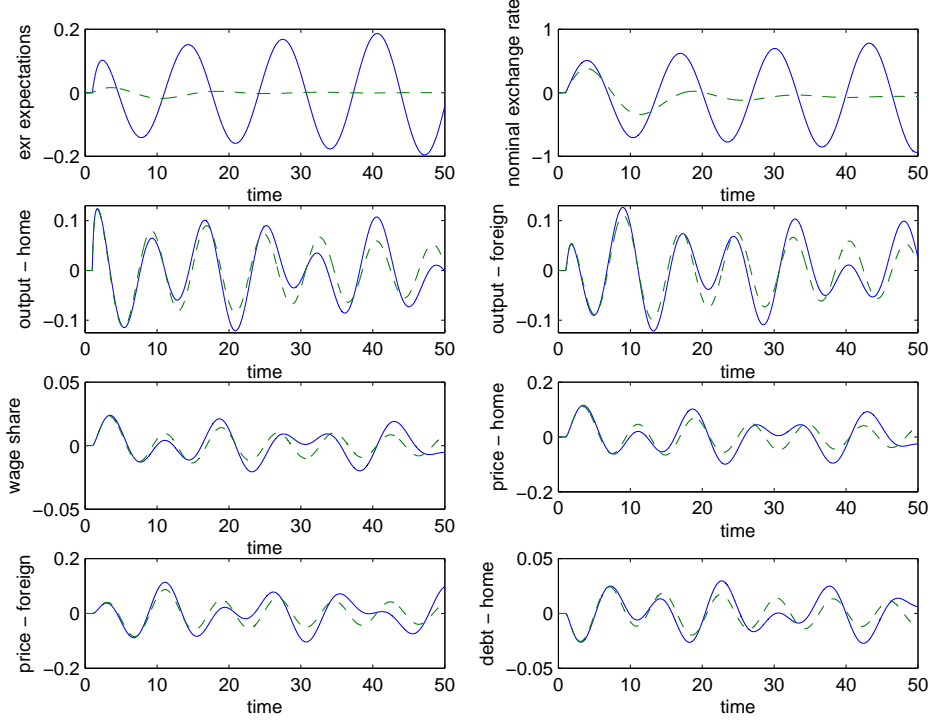


Figure 3: Dynamic adjustments to an exogenous decline in the nominal interest rate in the home country for  $\beta_{\pi_s^e} = 0.1$  (solid line) and for  $\beta_{\pi_s^e} = 2.0$  (dashed line) (both with  $\psi_a = \psi_b = 0.3$ ).

exchange rate depreciation expectations converge back to their steady state after about three cycles in the first case, when  $\beta_{\pi_s^e} = 2.0$  these depreciation expectations feature persistent oscillations with an increasing magnitude. Following the specification of the nominal exchange rate dynamics (see eq. (13)), these fluctuations are translated to fluctuations in the actual nominal exchange rate.<sup>14</sup>

Obviously, larger nominal exchange rate fluctuations affect the volatility of output in the domestic and the foreign economy. However, this effect is nonlinear as the graphs in Figure 3 clearly illustrate. Indeed, as it can be clearly observed, when  $\beta_{\pi_s^e} = 2.0$  the fluctuations of domestic and foreign output are larger than in the alternative case in some intervals and smaller in other intervals. Further, the dynamics of output as well as of the wage share (and therefore, of wage and price inflation) feature a highly erratic behavior, highlighting the nonlinear structure of the present framework.

<sup>14</sup>See Proaño (2009, 2011) for related studies of two-country dynamics using different behaviorally-founded exchange rate specifications.

Finally, it should be pointed out that despite of the sizeable value of  $\beta_{\pi_s^e}$  in the second case ( $\beta_{\pi_s^e} = 2.0$ ), the two-country framework remains remarkably stable. This is due to the specification of the aggregate opinion dynamics, which by construction induce a certain boundedness in the dynamic behavior of the system, at least for a certain range of parameters.

#### 4.4 Are Aggregate Opinion Dynamics Stabilizing?

Finally, we investigate to what extent the endogenous opinion dynamics mechanism contributes to the dynamic stability of the two-country model when the latter is subject to multiple shocks. This exercise is particularly interesting due to the nonlinear structure of the model, which becomes particularly relevant when subject to multiple shocks due to the synergy which arises from their simultaneous interaction. Specifically, Figure 4 illustrates the dynamics resulting from a negative shock to the domestic interest rate, a positive shock to the foreign output and a positive shock to the exchange rate depreciation expectations under the additional assumption that  $\beta_w = \beta_p = 0.6$ , what makes the wage-price spiral in the foreign country unstable.

Figure 4 illustrates the importance of opinion dynamics for the stability of the two-country macroeconomic system. The first phase diagram in Figure 4 (first row, second column) illustrates the joint

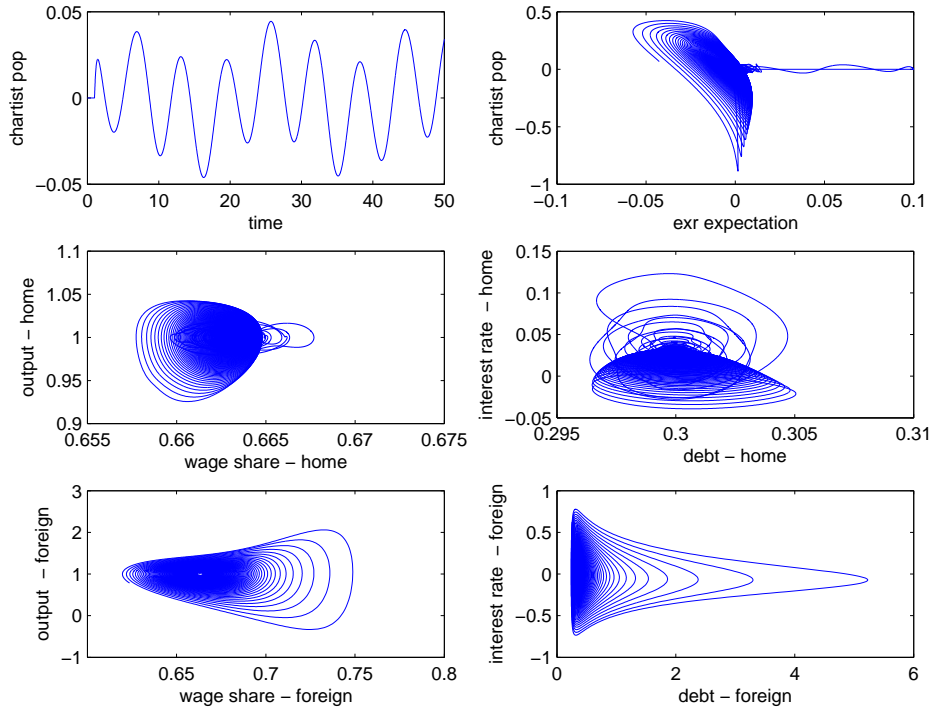


Figure 4: Dynamic adjustments to multiple shocks: The role of aggregate opinion dynamics for macroeconomic stability

dynamics of the exchange rate expectations (on the horizontal axis) and the population dynamics (on the vertical axis). As it can be observed, when the share of chartists increases depreciation expectations decrease, and viceversa. Interestingly, when the number of chartists becomes too large, the specification of the population dynamics given by eq.(19) leads to a nonlinear re-balancing in the agent population in the favour of fundamentalists agents. This is due to the fact that the impact of opinion dynamic on expectations formation is mainly related to the equal split between chartist and fundamentalist at the steady state. This stabilizes the dynamics of expectations in comparison to the case of purely adaptive expectations and contributes to the macroeconomic stability of the domestic economy, which indeed behaves in a stable manner, as the second row subgraphs in Figure 4 illustrates.

However, while the opinion dynamics is however unable to stabilize the dynamic of the economy in the foreign country. In the foreign country, the unstable wage-price spiral (since  $\beta_w^* = \beta_p^* = 0.6$ ) generates instability in the Goodwin cycle as illustrated by the phase diagram in the third row, first column of Figure 4. Another additional source of instability is the debt dynamics in the foreign country, which becomes quickly explosive as illustrated in the third row, second column subgraph in Figure 4. The opinion dynamics is simply not strong enough to stabilize these two effects and the model becomes explosive in the foreign country.

## 5 Concluding Remarks

In this paper we investigated the role of the state of confidence for the macroeconomic interaction of two large open economies interacting with each other through real and financial channels from a behavioral macrofinance perspective based on the work on aggregate opinion dynamics by Weidlich and Haag (1983) and Lux (1995).

Our analysis highlighted the international dimension of bounded rationality through the modeling of a state of confidence which depended on the business cycles of the domestic and foreign economy. Accordingly, our framework allowed for an international transmission channel seldom investigated in the literature, namely how foreign economy developments may affect not the *expectations*, but the *state of confidence* at home. As our numerical simulations showed, this channel does not only have sizable effects for the dynamics of the two economies, but may even bring about persistent cycles or unstable dynamics if the state of confidence becomes too sensitive to business cycle developments at home and abroad. Further, our framework was flexible enough to generate various types of persistent fluctuations which preserve the insight of basic seminal contributions of the literature such as the Goodwin distributive cycle, the related inflation-stagflation cycle, and debt cycles in particular.

As the importance of bounded rationality, social norms and other “non-rational” behavior by the economic agents for the dynamics of the economy at the macroeconomic level are becoming increasingly

accepted by the economics profession, this and other related approaches seem a promising field of research. In particular, the empirical quantification of these theoretic dynamics seems an interesting and worthwhile task to pursue.

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