

*Roberto Lampa*

**A 'Walrasian Post-Keynesian' Model?  
Resolving the Paradox of Oskar Lange's 1938  
Theory of Interest**

November 2011  
Working Paper 15/2011  
Department of Economics  
The New School for Social Research

The views expressed herein are those of the author(s) and do not necessarily reflect the views of the New School for Social Research. © 2011 by Roberto Lampa. All rights reserved. Short sections of text may be quoted without explicit permission provided that full credit is given to the source.

# A ‘Walrasian Post-Keynesian’ model?

## Resolving the paradox of Oskar Lange's 1938 theory of interest

Roberto Lampa\*

### Abstract

This paper investigates Oskar Lange’s 1938 article, “The Rate of Interest and the Optimum Propensity to Consume”, with the intention of providing a thorough interpretation of this rather obscure work. We explore in depth Lange’s theory of interest and its relationship with both Keynes’ *General Theory* (1936) and Hicks’ synthesis (1937), developing two graphical models that show the non-linearity of Lange’s investment function as well as the consequentiality of his equilibrium solution. Through an unedited manuscript, we also reconstruct Lange’s beliefs about the chronic sub-optimality of the capitalist economy and his *scientific* endorsement of the socialist economy. We conclude that the purposes of Lange’s article predate and are independent of the *General Theory*. They consisted of a theoretical generalization and analysis of institutional data, intended to separate economic theory from the tacit assumption of a capitalist economy.

**Key words** Theory of Interest; Keynesian economics; Capitalism’s Instability; Neo-Classical Synthesis; Socialist Theory

**JEL codes** B22, B24, D50, E12, E21, P11

### 1. Introduction

Oskar Lange’s 1938 article “The Rate of Interest and the Optimum Propensity to Consume” is of great interest, from several perspectives. First of all, it is one of the earliest works dealing with Keynesian theory emanating from both American academia<sup>1</sup> and the Socialist field. Secondly, it represents the first attempt to generalize J.M. Keynes’ book, not only in a contemporary sense, but also independently of the well-known 1937 article by John R. Hicks, “Mr. Keynes and the

---

\* Post-doc Researcher in History of the Economic Thought, Dipartimento di Scienze Sociali e della Comunicazione (SS. n.7, Km 7+300 per Mesagne, 72100, Brindisi, ITALY), Università del Salento. [roberto.lampa@unisalento.it](mailto:roberto.lampa@unisalento.it)  
Invited Scholar (academic year 2011-2012), Department of Economics, New School for the Social Research (6 East 16th Street, New York, NY 10003) [lampar@newschool.edu](mailto:lampar@newschool.edu).

The author wishes to thank Professors Stefano Perri, Gary Mongiovi, William Milberg and Dr. Florencia Sember for their valuable advice. I’m also indebted with Jonathan Cogliano and Mary Borrowman. Of course, any error or typographical error should be attributed solely to the author. Working paper, do not quote or circulate without the author’s permission.

<sup>1</sup> Lange’s article actually predates Hansen’s presidential address to the American Economic Association (dated December 1938), which has been widely recognized as the key event in the reception of the *GT* in the U.S.A. Cf. Hansen, 1939.

‘Classics’: a suggested interpretation”. Finally and most importantly, it is the culmination of Lange’s previous contributions on both capital and the theory of interest, thus representing a key passage of his early (as well as ‘western’) economic literature.

These considerations notwithstanding, thus far it has generated only modest attention (or none at all) among the scholars who dealt with Keynes and the ‘Classics’<sup>2</sup>, and at best has been only vaguely associated with the Neo-Classical synthesis<sup>3</sup> through some formal analogies with Hicks’ pioneer work, merely reflecting elements of the *General Theory*.

However, we are convinced that a deeper investigation of Lange’s article, as well as a philological reconstruction of his earlier beliefs related to the same topic, may lead us to an alternative and consistent interpretation – as well as to new insights about the debate between Keynes and the Neo-classical synthesis – focusing on the ‘apparent paradox’ of this work, represented by three crucial features: first, its ambivalent relationship with the *GT*, given Lange’s sharing of the book’s implications (as recognized by Keynes himself), despite a clear analytical distance; second, both its formal differences and its radical departure from the policy proposals, when compared to Hicks’ 1937 article; and lastly, the appraisal that Lange’s article was not a *reaction* to Keynes’s work, since its purposes pre-dated the *GT* and were part of an ambitious scientific project which included a relevant and critical dimension, and used Keynes’ book to this extent.

Given the scarce existing literature about Lange’s model, this paper may therefore represent the first attempt to propose an interpretation and a detailed analysis of such a work.

Consequently, this article aims first to review the main contents of Lange’s generalization of the *General Theory* (Section 2), and to investigate its relationship with Keynes’s book (Section 3). Section 4 performs a comparative analysis with one of the most prominent works of the neo-classical synthesis (i.e. Hicks’ article). Finally, through a parallel of his 1936 work “The Place of Interest in the Theory of Production” and an unedited manuscript, Section 5 investigates the roots and scope of Lange’s reflection on the theory of interest.

## **2. The Walras-Keynes connection: Lange’s theory of interest**

Lange explicitly declares the three purposes of his article: first, it presents a generalization of the theory of interest; second, it demonstrates that both the Keynesian and ‘Classical’ theories are two limiting cases of such a generalised theory; and last, how this theory of interest applies to the study of under-consumption. With regard to the first goal, Lange proposes a four-equation model, in which  $Y$  and  $M$  are the known quantities, and  $i$ ,  $C$  and  $I$  are the unknowns:

---

2 See, for instance: Johnson, 1961; Leijonhufvud, 1968; Fisher, 1976; Coddington, 1976 and 1983; Chick, 1983.

3 See, in particular: Schumpeter, 1981, p.989, p. 1047 and p.1140; Ackley, 1961, p.376 and p.383n; Robbins, 1968 pp.30-32; Mac Dowell, 1957, p.409.

$$M = L(i, Y) \quad \text{assuming: } L_i \leq 0 \quad L_Y \geq 0 \quad (1)$$

$$C = \phi(Y, i) \quad \text{assuming: } 0 < \phi_Y < 1 \quad \phi_i \geq 0 \quad (2)$$

$$I = F(i, C) \quad \text{assuming: } F_i < 0 \quad F_C > 0 \quad (3)$$

$$Y \equiv C + I \quad (4)$$

Equation (1) represents the liquidity preference, whereas (2) is the consumption function. Lange specifies that  $M$ ,  $I$ ,  $C$  and  $Y$  are measured in (real) terms of a *numéraire* (wage units). Moreover, he underlines once and for all that no general relationship can be drawn between a change in the rate of interest and the level of consumption. Equation (3) is the marginal efficiency of investment and (4) is the equilibrium condition. Lange clarifies that he's making reference exclusively to the marginal efficiency of investment as defined in a "forthcoming paper" by A.P. Lerner. Finally, he emphasizes that the equality (4) is an identity and not an equation. At first sight, Lange's system may appear to be over-determined. However, he specifies that the solutions result from a *process* (and therefore not from a simultaneous resolution), as illustrated in three correlated diagrams:

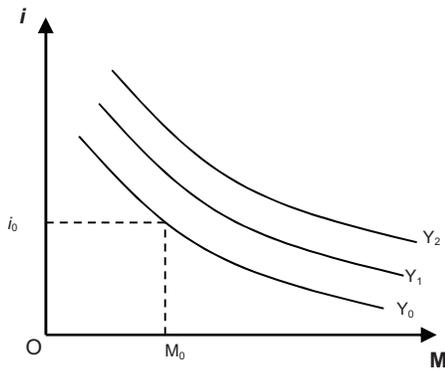


Fig. 1

The first of these represents a *family* of liquidity preference curves: given both a level of income  $Y_0$  and a quantity of money  $M_0$ , we get the rate of interest  $i_0$ .

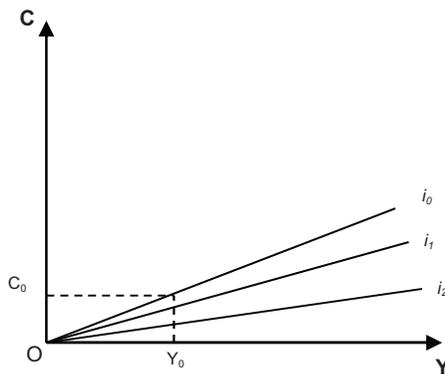


Fig. 2

In the second diagram, Lange plots the propensity to consume curves: given  $Y_0$  and  $i_0$ , we can then measure  $C_0$ .

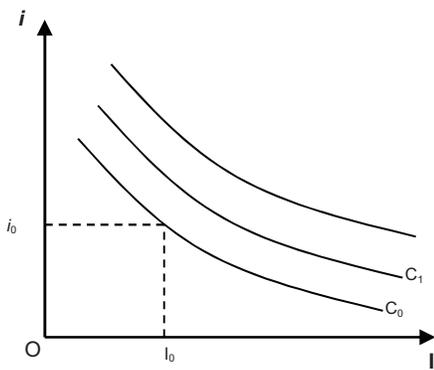


Fig. 3

The third diagram shows the marginal efficiency of investment curves: given  $C_0$  and  $i_0$  we obtain  $I_0$ . At this stage, Lange specifies that *if* it happens that  $C_0 + I_0$  equals the given level of income  $Y_0$ , then the system is in equilibrium; otherwise the liquidity preference curve shifts (since the quantity of money  $M_0$  is given), implying a new rate of interest, thus starting a process of *mutual adjustment*, clearly calling to mind Léon Walras's *tâtonnement*. Lange emphasizes that only if this process doesn't involve any time lag, the final result will be equilibrium; otherwise, the system will tend to cyclical fluctuations, similar to those described by M. Kalecki (1937)<sup>4</sup>:

"If [the] process of adjustment involves a time lag of a certain kind, a cyclical fluctuation instead of equilibrium is the result. Cf Kalecki, 'A theory of the business cycle'". (Lange O., 1938, note 1, p.17)

### 2.1 Two limiting cases

After the exposition of his 'generalised theory', Lange highlights that two strongholds of traditional theory still hold. First, an increase in the marginal productivity of capital is always accompanied by a rise of the rate of interest. Conversely, a decrease in the propensity to consume, i.e., an increase in the propensity to save, is accompanied by a fall of the rate of interest. This notwithstanding, Lange states that his theory of interest is not congruent with its traditional equivalent, since the former is "*quite general and formal*" when compared to the latter, allowing for two limiting cases. If the income elasticity of the demand for liquidity is zero (or the interest elasticity is infinite), the demand for liquidity becomes a function of the rate of interest alone. According to Lange, this case is compatible with Keynes' theory: since the quantity of money is given, the rate of interest does not react to changes in equations (2) and (3), but only to changes in total income. Conversely, when interest elasticity is zero, the demand for liquidity is a function of income alone. Thus, if this ratio is constant, it can be considered equivalent to the Cambridge equation.

---

<sup>4</sup> Unfortunately, Lange doesn't explore further such a crucial implication of his model. However, the connection between Lange's (static) theory of interest and Kalecki's (dynamic) analysis of the business cycle is the object of a parallel working paper, co-authored with Prof. Michael Assous.

## 2.2 Lange's general theory and the problem of under-consumption

Subsequently, Lange introduces the problem of under-consumption as the most important application of his generalised theory. Drawing on T.R. Malthus, he states that the real substance of under-consumption theories had been that investment *depends* on consumption. Consequently, the real issue was to discover the optimum propensity to consume (i.e. to save) which maximises investment. Therefore, starting from equation (2) and letting  $\partial C, \partial i$  be the variations of consumption and the rate of interest, Lange introduces the condition that maximizes investment:

$$\partial I = F_i \partial i + F_c \partial C = 0$$

That is, by substitution:

$$\partial Y = \partial C$$

And, finally:

$$-\frac{L_Y}{L_i} = -\frac{F_c}{F_i} \quad (5)$$

Therefore, the optimum propensity to consume is determined by the condition that:

“the marginal rate of substitution between the rate of interest and total income as affecting the demand for liquidity is equal to the marginal rate of substitution between the rate of interest and expenditure on consumption as inducements to invest” (Lange O., 1938, p.26)

Graphically:

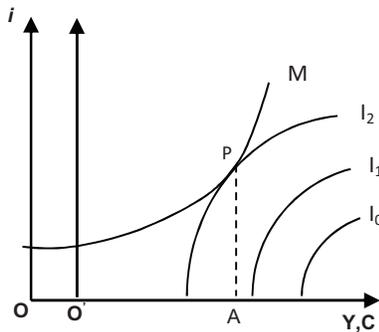


Fig. 4

Lange calls the curves  $I_0, I_1, I_2$  *isoinvestment curves*, since they indicate variations in the rate of interest (as well as consumption), which don't change the level of investment. The curve  $M$ , instead, is called the *isoliquidity curve*, since it represents the changes of both the rate of interest and the level of income, which don't affect the demand for liquidity (since the quantity of money is given, there's only one such curve). The position of the origins  $O$  and  $O'$  is the difference between total income ( $OA$ ) and the expenditure on consumption ( $O'A$ ), i.e., the investment:

“The optimum propensity to consume is, therefore, obtained by superimposing [the isoliquidity curve] upon [the isoinvestment curve] and moving it horizontally until the isoliquidity curve becomes tangent to the isoinvestment curve whose index (i.e. the level of investment) is equal to the length of  $OO'$ .” (Lange O., 1938, p.29)

Therefore, if the isoinvestment curves are concave and the isoliquidity curve is convex, there will be only one isoinvestment curve which satisfies this condition. The optimum propensity to consume is thus obtained from the tangency of the two curves. In other words, plotting the expenditure on consumption  $O'A$  and total income  $OA$  on a separate diagram, Lange obtains a point  $R$  through which the curve representing the optimum propensity to consume (i.e. that which maximises investment) has to pass. Any curve passing above  $R$  corresponds to a propensity to save below optimum (i.e. a propensity to consume above optimum) and vice versa:

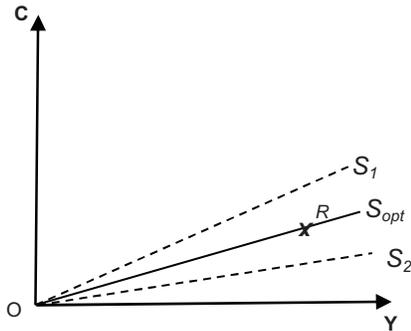


Fig. 5

### 2.3 The optimum propensity to save and a society of individuals...

Finally, Lange applies his solution to the previous limiting cases. When the income-elasticity of the demand for liquidity is zero, or the interest-elasticity is infinite (i.e. the case corresponding to Keynes's theory as Lange interprets it), a change in the propensity to consume does not affect the rate of interest at all, and the optimum propensity to consume is attained "*when the expenditure on consumption is such that a further increase does not any more increase the marginal efficiency of investment*" (Lange, 1938, p.31). When the interest-elasticity of the demand for liquidity is zero (i.e. when the "traditional" theory holds true), "*any decrease in the propensity to consume stimulates investments by causing an appropriate fall of the rate of interest*" (Ibidem), that is the propensity to save can never be excessive. However, Lange stresses that this is a very limiting and uncommon case, since in any other situation there must exist a (unique) optimum propensity to save. Thus, the very existence of such a point imposes a limit on investment, and any attempt to go beyond it – by increasing the propensity to save – will lead to a reduction of the investment itself. Consequently, Lange emphasizes that particularly in a society where the propensity to save is *freely* determined by *individuals*, there will be no force capable of keeping the propensity to save at its optimum.

### 3. Lange and the *General Theory*: a tortuous relationship

As shown in the previous section, the mere exposition of Lange's article already denotes an ambivalent attitude towards the *General Theory*. On the one hand, it reveals a remarkable interest in

Keynes's book. However, such an interest does not imply that Lange entirely accepts Keynes's *nouvelle vague*. In order to illustrate this, it is sufficient to set apart the purely theoretical dimension of his theory of interest (as discussed in sections 2-5 of his original article) from the analysis of under-consumption in a capitalist economy (sections 6-7).

### 3.1 The theoretical dimension

At first sight, Lange's model seems to draw on some Keynesian categories such as 'propensity to consume', 'marginal efficiency', and 'liquidity preference'. Nevertheless, a detailed investigation of each function reveals that such an analogy is purely formal.

First of all, Lange conceives the propensity to consume function as  $C = \phi(Y, i)$ . Therefore, the level of consumption also depends on the rate of interest, whereas Keynes's distinction between *loanable funds* and *capital assets* implied a certain scepticism of a direct relationship between saving (and/or consumption) and the rate of interest (i.e.  $C = a + cY$ ). From this perspective, Lange's qualifications about the propensity to consume reinforces the idea of a strict interdependence between the variables operating in the economic system, thus explicitly revealing the influence of the Walrasian theory:

"...[the] excess of income over consumption is conceived by Walras to be a function of both the rate of interest and income. (...) His equation thus corresponds to our equation (2)." (Lange O., 1938, pp.21-22)

Similarly, the marginal efficiency of investment function is also rather different from Keynes's *marginal efficiency of capital*, as Lange explicitly suggests in a footnote:

"They are frequently confused. However, the marginal efficiency of capital relates the rate of net return to a *stock* of capital, while the marginal efficiency of investment relates it to a *stream* of investment per unit of time. (...) cf. a forthcoming paper of Mr. Lerner" (Ivi, 1938, p.31)

The reference to Lerner, in particular, clearly indicates that Lange shares his idea of a direct relationship between the marginal productivity of capital and the rate of interest (Lerner, 1937, pp.348-50), as in 'Classical' theory:

"...the marginal efficiency of investment (which is correlated to the marginal net productivity of capital)..." (Ivi, p.18)

In contrast, Chapter 11 of Keynes's *General Theory* clearly states that the marginal efficiency of capital depends on capital's scarcity rather than on its marginal productivity, thus doubting that the level of investment could determine the rate of interest.

Also Lange's liquidity preference is, to a great extent, a different function. First, it is an *aggregated* function obtained by summing the individuals' function, just as "*a market demand function is obtained from the demand functions of the individuals*" (Lange, 1938, p.12n). Conversely, (in Chapter 13 of the *GT*) Keynes stressed that it was an *individual* function, in which the subjective

and psychological dimension (i.e. the expectations) played a pivotal role. Second, Lange specifies that his function “*holds only for a given distribution of income*”, whereas Keynes clearly indicated not only that such a ‘Classical’ assumption was “*a nonsense*”, but also that it was the main fallacy of the traditional theory of interest, since no variation in saving (and, therefore, in the rate of interest) can be possible without a corresponding variation in income (Keynes, 1973A, pp. 178-79). Lastly and most notably, both  $M$  and  $Y$  are expressed in real terms, or, to put it differently, they are “*measured in terms of wage-units or of any other numéraire*” (Lange, 1938, p.12). Therefore, Lange’s equation is lacking any reference to money as a store of value, which in turn implies that neither speculation on the asset market nor uncertainty is taken into account. Such a choice undoubtedly indicates a considerable distance from Keynesian theory, as the latter was essentially a *monetary* theory of interest. Yet again, Lange’s analytical tool box seems to lean towards Walras rather than Keynes:

“...the ratio of the price of each commodity (...) to the price of the (...) numéraire (...) may be thought of as determined by the Walrasian or Paretian system of equations of general economic equilibrium.” (Lange O., 1938, p.13)<sup>5</sup>

We can thus sum up that the qualifications made by Lange to each of the aforementioned functions constitute a sort of interdependent model, which somehow reproduces – albeit in a very simplified way – the Walrasian economies of exchange, production and capitalization rather than the Keynesian theory. Not coincidentally, Lange dedicates the whole of Section 5 in his article to show that, from a historical perspective, “*...the essentials of this general theory are contained already in the work of Walras*”. Moreover, further evidence of Lange’s attitude is also revealed by the reference to Brian Reddaway’s 1936 pioneer article, which contained the very first attempt to translate the *General Theory* into the terms of an interdependent model (Reddaway, 1936, pp. 34-5 and Millmow, 2003, pp. 136-138).

Nevertheless, two crucial assumptions of Lange’s model steer his work away from Walras, once again bringing it closer to the spirit of the *General Theory*. First and foremost, he chooses a different equilibrium condition:

“...Walras expresses in a separate equation the equality of the value of the *capitaux neufs* and the excess of income over consumption. This, however, is not equivalent to our equation (4) (...) For there is an important difference. In our system, as in the theory of Mr. Keynes, equation (4) **is an identity**” (*Ivi*, p.22, emphasis added)

Even though Lange doesn’t further discuss the theoretical implications of such an *important difference*, we can infer what they consist of by simply recalling Keynes’ theory. In the latter’s view, the substitution of the equilibrium identity for the equation was the equivalent of a shift in

---

<sup>5</sup> One might say that this distinction also reflects its belonging to two different traditions, i.e. Cantabrian-Marshallian and Continental-Walrasian.

analysis from the *ex-ante*, i.e. the entrepreneurs' decisions, to the *ex-post*, i.e. the *realised* results of the economic process (Keynes, 1973A, pp. 61-5 and pp.81-5). Given the definitions of income ( $Y=C+I$ ) and saving ( $S=Y-C$ ), together with the assumption that investment uses pre-existing resources, it followed that investment and saving are necessarily equal ( $I=S$ ). Therefore, the 'Classical' belief that 'saving creates its own investment opportunities' was substantially correct when conceived as a mere accounting relationship, which always held true as such. In the event of a discrepancy, a process of income adjustment would have reinstated the equality between saving and investment. From this perspective, Lange evidently shares Keynes's basic ideas:

"The identical equality of investment and saving holds for investment and saving actually performed. Investment and saving *decisions* can be different." (Lange O., 1938, p.14n, emphasis in original)

However:

"Whatever the investment and saving *decisions* are, the volume of total income always adjusts itself so as to equalise saving and investment *actually performed*." (Ivi, p.22, emphasis in original)<sup>6</sup>

As we have seen in the previous Section, he goes even further in remarking that, in the event of time lags, the final result of this adjustment process may not even be equilibrium. Second, Lange's model does not assume the flexibility of all prices: similar to Keynes, the level of money wages is both fixed and exogenously determined, as Lange refers to a given distribution of income, and opts for his *numéraire* to be measured in terms of wage units (thus  $p_w=1$ ). Therefore, his system of equations cannot be considered a traditional "*full-employment model*" (Keynes, 1973A, Chap.19), and as such is also relevant in the study of unemployment.

The combined effect is that, just as with Keynes' work, in Lange's model the equilibrium condition also ceases to be a determining matter for full-employment equilibrium (Gnos, 1998, p.43), as it simply refers to the actual level of production and not to the potential productive capacity of the economy. Thus, his theory of interest becomes consistent (in theory) with an analysis of both disequilibrium and underemployment equilibrium, as the reference to Kalecki may implicitly suggest (Kalecki, 1937, p.77).

In other words, by means of all these qualifications, Lange's interdependent model doesn't embody any *normative* idea of equilibrium, in contrast to Walras' (Jaffé, 1977, p.340), according to which economic equilibrium becomes a normative ideal to which the society has to aspire rather than a description of the actual functioning of the markets. Instead, it consists of a theoretical generalization, which is also compatible with the most relevant (and controversial) implications of the Keynesian approach.

---

<sup>6</sup> This implies that the  $I$  in equation (4) is a different entity from the  $I$  in equation (3): the former includes unplanned changes in the amount of the investment, whereas the latter refers only to planned investment expenditures.

### 3.2 *The analysis of under-consumption in a capitalist economy*

In support of our interpretation, after the exposition of his *generalised* theory of interest, Lange introduces what he considers its most relevant field of application: the problem of under-consumption in a capitalist economy. In the first place, he remarks on the importance of dealing with this topic, especially in light of Keynes's work. This notwithstanding (as anticipated in equations 2 and 3), he chooses not to share the Keynesian critique of the 'old' under-consumption theorists (Keynes, 1973A, pp. 324-26), since:

"Mr. Keynes treats investment and expenditure on consumption as two independent quantities (...) The real argument of the underconsumption theories is that investment depends on the expenditure on consumption" (Lange O., 1938, p.23)

However, when he comments on Fig.5, Lange simply outlines the mutual dependence of investment and consumption as a sort of 'indirect' relationship:

"...the function expressing the optimum propensity to consume is determined by only one point through which it has to pass. Any function which passes through the point R maximises investment. Any function, however, which does not pass through R makes total investment *smaller*." (Ivi, p.30, emphasis added)

Therefore, in order to fully understand his point of view, it becomes necessary to recover some crucial aspects of his analysis related to this topic. First, Lange states that, as in traditional theory, in his model an increase in the propensity to save induces a decrease in the rate of interest. However, his reasoning runs along more unconventional lines than the 'Classical' interaction of both supply (of) and demand (for) capital curves:

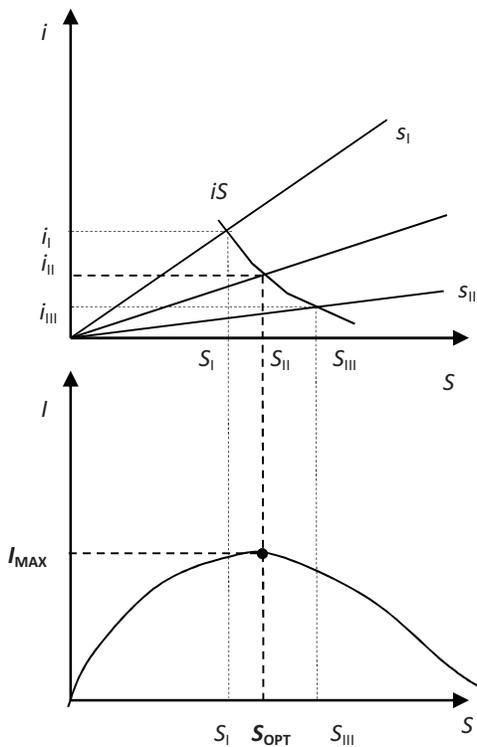
"...an increase in the propensity to save [implies that] expenditure on consumption is now lower. This causes (...) a lower quantity of investment (...). Total income decreases (...). Thus a downward shift of the liquidity preference curve in Fig.1 takes place. The consequence is a fall in the rate of interest." (Ivi, pp. 17-18)

In other words, in Lange's view the immediate effects of an increase in the propensity to save are a decrease in consumption, investment and total income. Therefore, as recognized by Keynes himself:

"The analysis which I gave in my *General Theory of Employment* is the same as the 'general theory' explained by Dr. Lange on p.18 of his article, except that my analysis is not based (as I think his is in this passage) on the assumption that the quantity of money is constant." (Keynes J.M., 1973B, p.232n)

Following this train of thinking, we deduce that it's only afterwards that the decreased level of the rate of interest will stimulate investment, consumption and total income. The final result of an increase in the propensity to save will then depend on the 'specific weight' of each of these two effects. Not coincidentally, Lange is persuaded – by drawing on Malthus (and also Karl Marx's realization crisis) – that excessive growth in saving (i.e. an excessive contraction of consumption, investment and total income) cannot be counter-balanced by the subsequent decrease in the rate of interest, as it destroys any incentive to invest, "*at least in a capitalist economy where investment is*

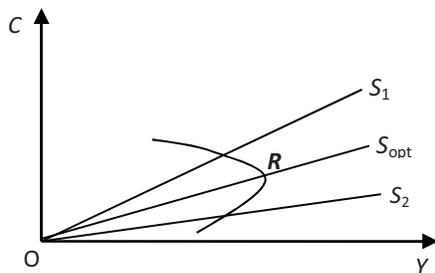
done for profit” (Lange, 1938, p.23). He thus firmly rejects the ‘Classical’ assumption that *any* abstinence from consumption implies automatically an increase in investment: according to him, such a direct relationship holds *only* until a certain limit (i.e. the *optimum* propensity to consume), beyond which the collapse of the demand for investment goods will drastically diminish investment itself (*Ivi*, p.14). On the other hand, he doesn’t believe either, as Keynes did, that total income can be increased indifferently by expanding either consumption or investment. Therefore, the real issue of his under-consumption theory becomes the determination of the *optimum propensity to consume*, so as to balance the two aforementioned effects and to maximise investment (and total income). At this stage, we are convinced that it becomes necessary to provide a graphical interpretation of our own, in order to sum up and highlight what Lange had in mind. Starting from Lange’s claim that “*a decrease in the propensity to consume (or, in other words, an increase in the propensity to save) is accompanied by a fall of the rate of interest*” (Lange, 1938, p.17), in the top section of our Diagram I we have plotted the propensity to save curves  $s_I$ ,  $s_{II}$  and  $s_{III}$  as a function of the rates of interest  $i_I$ ,  $i_{II}$  and  $i_{III}$ . Any increase in the propensity to save (represented by a shift to the curves below  $s_I$ ), is accompanied by a decrease in the rate of interest. Therefore, we are able to plot our  $iS$  curve, showing the level of both saving and the rate of interest corresponding to the different propensities to save:



**Diagram I:** the non-linearity of Lange’s investment function. The decreasing section of the investment curve coincides with the Keynesian case, since any increase in saving discourages investment.

Along this line, it becomes possible (in the bottom section of our Diagram I) to plot the non-linear relationship between saving (or consumption) and investment by means of a ‘humpbacked’ curve. As we can see in the first section of the curve, a decrease in the propensity to consume (i.e. an increase in the propensity to save) stimulates investment through a decrease in the rate of interest<sup>7</sup> which is stronger than the contraction of consumption itself (i.e.  $F_i\delta i > F_c\delta c$ ). This holds true until the curve reaches a peak, corresponding to the optimum propensity to save and to maximum investment (in which  $F_i\delta i = F_c\delta c$ ). Beyond this level, any further decrease in consumption (and, therefore, in the demand for investment) is stronger than the corresponding decrease in the rate of interest, thus inducing a contraction in investment itself (i.e.  $F_i\delta i < F_c\delta c$ ).

In order to make the comparison with the Keynesian theory clearer, it is of course possible to resolve in terms of total income rather than of investment<sup>8</sup>. In this case, it is sufficient to provide a re-elaboration of Lange’s fig.5<sup>9</sup>: our ‘arrow shaped’ curve shows how in correspondence with point R (Lange’s “*optimum propensity to consume*”), total income will reach a maximum. However, both an increase and a contraction in the propensity to consume will make total income smaller:



**Diagram II:** the relationship between total income and the propensity to consume

In other words, even if Keynes’ analysis of under-consumption is formally considered a theoretically limiting case (in which the propensity to consume is independent of the rate of interest), Lange’s under-consumption theory essentially shares Keynes’s rejection of the “*corollary of Say’s Law*” (Keynes, 1973A, pp.18-21). On a macroeconomic level, any mechanical coincidence between investment and saving cannot be assumed, not even ‘in theory’. On the contrary, Lange emphasizes the importance of studying the factual conditions that allow us to determine the level of

7 Strangely enough, the ‘Socialist’ Lange draws a less radical conclusion than the ‘Bourgeois’ Keynes, who considered any increase in saving an obstacle for productive investment, thus proposing a gradual “*euthanasia of the rentier*” (Keynes, 1973A, 374-77). According to Lange, such an opinion is a minority position, belonging to “*few underconsumption theorists*” such as Rosa Luxemburg. Cfr. 1938 p.23 (continues in note 2). A possible explanation lies in the different analytical premises: as evidenced by the top part of our Diagram I, Keynes’s “*paradox of thrift*” does not operate in Lange’s system, as an increase in the propensity to save is accompanied by an increase in the volume of savings.

8 As Lange himself states, an increase in the propensity to save induces a decrease both in investment and in total income, see above.

9 As further proof of his approach, it is already noticeable in Fig. 2 that Lange provides an equivalent solution (to the one included in Fig. 5) in terms of the level of the rate of interest. Interestingly, in Fig. 5 Lange calls the curve representing the optimum propensity to consume S (and not C, as per normal naming conventions).

consumption (and saving) capable of maximizing investment, thus implicitly admitting the theoretical possibility of under-consumption crises. Consequently, despite the use of different analytical tools, he finally arrives at conclusions which are clearly congruent with those of Keynes:

“...we arrive at the result that, with the exception of the special case covered by the traditional theory of interest, there exists (...) a maximum limit on investment per unit of time and any attempt to exceed it by raising the propensity to save frustrates itself...” (Lange O., 1938, p.32)

It is probably on the basis of these points of contact that Keynes himself was prompted to reflect that Lange’s article “*follows very closely and accurately my line of thought*” (Keynes, 1973B, p.232n. and Kowalik, 2008, p.866), notwithstanding the analytical differences. Lange was, after all, standing on the same “*side of the gulf*”, as he clearly rejected the notion that capitalism could be a “*self-adjusting system*” (Keynes, 1973C, pp. 486-92). In fact, we might add that Lange draws even more radical implications than Keynes from such a premise, as he remarks that in a capitalist economy the prevailing tendency can easily be chronic and long-lasting stagnation:

“In a society where the propensity to save is determined by the individuals, there are no forces at work that keep it automatically at its optimum, and it is well possible, as the underconsumption theorists maintain, that there is a tendency to exceed it.” (Lange O., 1938 p.32)<sup>10</sup>

In short, Lange’s analysis of under-consumption becomes a crucial integrative element of his interdependent model, in order to reveal his genuine view of the *General Theory*. On the one hand (as shown in the previous paragraph), the ‘purely theoretical’ dimension of his article clearly indicates a certain distance from Keynes’s analytical devices, as well as a clear proximity to the Walrasian interdependent model. However, by means of the equilibrium identity, as well as the assumption of fixed wages, his model endeavours to include the quintessence of Keynes’s book, i.e., the theoretical possibility of sub-optimal functioning of the economy, thus reaffirming his efforts in favour of a theoretical “*major synthesis*” (Kowalik, 2008, p.872). On the other hand, as far as Lange’s analysis shifts to a ‘less abstract’ level of investigation, the most relevant Keynesian conclusions seem to be implicitly shared and radicalized, even though Lange’s solution reveals a certain distance from the ‘standard’ Keynesian analytical tools (e.g. under-employment equilibrium), and a clear proximity both to the traditional theorists of under-consumption (Malthus or Sismondi) and to Marx.

In other words, even if general economic equilibrium theory remains a ‘necessary’ tool in order to study the pure *abstract* dimension of the theory of interest, on a macroeconomic level the *actual*

---

<sup>10</sup> In order to understand Lange’s circumlocution, it’s useful to be reminded that in his 1937 article “On the Economic Theory of Socialism”, he explicitly remarked that in a Socialist system the rate of accumulation had to be performed *corporately*, that is independently from the consumer’s preferences (i.e. propensity to save). See: Lange, 1937, p.127. Therefore, we can interpret that a “*society where the propensity to save is determined by the individuals*” is simply “*a capitalist economy*”.

possibility of such an equilibrium position is very limited and, to a great extent, unattainable<sup>11</sup>. Therefore, the traditional theory alone is no longer ‘sufficient’ to investigate the implications of the theory of interest in a capitalist economy. The Keynesian theory then becomes a powerful ally.

#### 4. Lange’s generalised theory vs. Hicks’s IS-LL model

“While writing this, there has come to my notice a forthcoming paper of Dr. Hicks on “Mr. Keynes and the Classics” (...). The form chosen in my paper seems, however, *more adapted for the study of the problems it is concerned with.*”  
(LANGE, 1938, p.1n. Emphasis added)

##### 4.1 Simultaneity vs. consequentiality

As we have shown in the previous sections, Lange’s theory of interest can (also) be interpreted as a generalization of the Keynesian theory. Once we take this into account, it naturally becomes necessary to consider the debate that started soon after the publication of the *General Theory*. Although most articles published from 1936-1938 refuted Keynes’s consumption function (more specifically, its independence from the rate of interest), it was only in Hicks’s contribution that the relationship between the traditional paradigm and the Keynesian approach played a pivotal role<sup>12</sup>. Therefore, since Lange explicitly deals with the same subject, a comparative analysis of the two articles becomes a useful exercise, in order to highlight the original features of his work.

Hicks notoriously characterized Keynes as a mere prosecutor of the Marshallian tradition, who laid an “*enormous emphasis on the qualifications, so that they almost blot out the original theory*” (Hicks, 1937, p.150), and proposed a three-equation model whose simultaneous resolution was represented by the intersection of the *LL* and the *IS* curves, in which both income and the rate of interest were determined together.

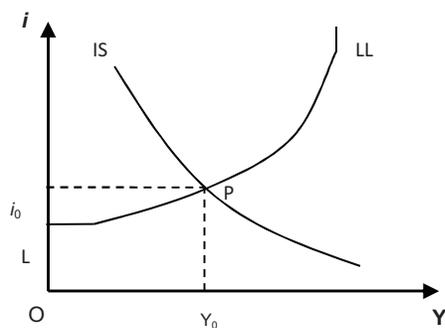
---

11 From this perspective, Lange’s idea clearly calls on that of Marx, expressed through the well-known Reproduction Schemes in *Second Book*.

12 Indeed, Harrod focused on the static nature of Keynes’ analysis: “*Saving essentially entails growth (...) No theory regarding the equilibrium amount of saving can be valid, which assumes that within the period in which equilibrium is established, other things, such as the level of income, do not grow but remain constant.*” Cfr. Harrod, 1937, p. 86.

Reddaway’s review was highly sympathetic to the *General Theory* and tried to translate it into a simple model: “*...we are (...) trying to describe a system where the variables mutually determine one another. This can best be seen by a sort of mathematical shorthand (...) Mr. Keynes quite rightly in my opinion, deprecates the spurious air of exactness introduced by too much mathematics. But in his endeavour to describe the system without this sort of shorthand he has tended to obscure the fact that the determination is mutual.*” Cfr. Reddaway, 1936, pp. 34-35

Finally, Meade’s article aimed to “*...construct a simple model of the economic system discussed in Mr. Keynes’ ‘The General Theory of Employment, Interest and Money’ in order to illustrate: (i) the conditions necessary for equilibrium; (ii) the conditions necessary for stability of equilibrium; and (iii) the effect on employment of changes in certain variables.*” Cfr. Meade, 1937, p.98



**Fig. 6**

These brief insights into Hicks's model are sufficient to capture the first relevant divergence from Lange's theory, represented by the simultaneity of the equilibrium solution.

As argued in Luigi Pasinetti's prominent work (Pasinetti, 1974, pp.45-53), such a choice raises the problem of compatibility between the simultaneous determination of the equilibrium position and the Keynesian theory, which instead considered a sequence of alternating money market and goods market decisions, coming one *after* the other, in order to get to the equilibrium values of  $i$ ,  $I$ ,  $Y$ . In contrast, Hicks determines these variables *contemporaneously* by means of the *IS* and *LL* curves, "*just as price and output are determined together in the modern theory of demand and supply*" (Hicks, 1937, p.153). However, a logical problem then arises (Leijonhufvud, 1968, pp.30-1), since the portfolio decisions (*LL*) are made in the context of a *stock* constraint, whereas the real market decisions (*IS*) are expressed through a *flow* relationship. Therefore, since they embody different time dimensions, the two curves can't be considered simultaneous.<sup>13</sup>

From this perspective, it's remarkable to see that Lange's solution doesn't incur this error due to two crucial features. In the first place, as a consequence of its Walrasian foundations, Lange's model leaves aside the study of money and bonds, thus referring exclusively to real variables. Therefore all market decisions are made within a *flow* constraint<sup>14</sup>. Secondly, the substitution of the equilibrium identity for the equation implies that, just as in Keynes's book, equilibrium becomes the result of a sequential process based on a three-stage sequence: given  $M_0$  and  $Y_0$ , we determine  $i_0$ ; we then obtain  $C_0$ ; and finally, we arrive at  $I_0$ . Hence, although Lange clearly refers exclusively to *logical* (and not to *historical*) time<sup>15</sup>, the process of determining equilibrium becomes quite different from that of Hicks, which is instead based on both the logical and historical simultaneous resolution of  $Y$  and  $i$ .

<sup>13</sup> Afterwards, Hicks recognized the validity of this criticism, see: Hicks, 1980, pp.150-1 and Hicks, 1988, p.5. However, an earlier defence of Hicks' solution is contained in: Patinkin D., 1958.

<sup>14</sup> As we have seen, Lange himself makes reference to the marginal efficiency of investment as it relates the rate of net return to a *stream* of investment per unit of time. *Vide supra*

<sup>15</sup> For equilibrium is reached when mutual adjustments occur *without* any time lag. *Vide supra*

Even though Lange's solution is probably inspired by the logic of the Walrasian *tâtonnement*, we can nevertheless conclude that it is more compatible with Keynes's theoretical framework than Hicks's, in light of its consequentiality. Furthermore, as we have shown in the previous section, Lange's process of adjustment is (in theory) consistent with both the case of disequilibrium and even under-employment equilibrium, whereas Hicks's simultaneous solution (as a consequence of his assumptions concerning the system of equations as well as the shape of the curves), scarcely allows for solutions other than full-employment equilibrium.

#### 4.2 Lange's model vs. IS curve: a graphical interpretation

Of course, to state that Lange rejects a simultaneous determination of the equilibrium magnitudes raises questions regarding his attitude towards both the *IS* and the *LL* (Hicksian) curves.

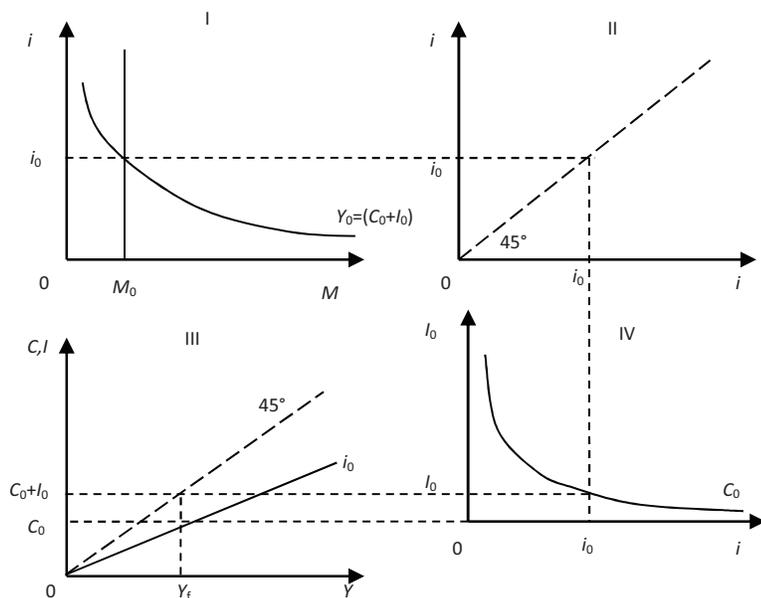
Starting from the latter, Lange refers to a family of indifference curves, which he calls *isoliquidity curves*. However, since the amount of money is given, there can be only one such curve (i.e. the *M* curve in Fig. 4). By means of this qualification, his solution seems similar to Hicks's *LL*. Nevertheless, as brilliantly shown by a recent article, it is more consistent with Keynes's original idea. In fact, the shape of Lange's curve is determined 'empirically', by the value of the interest elasticity of the demand for money, whereas Hicks's *LL* comes from the theoretical assumption of a 'floor' to the rate of interest (Boianovski, 2004, pp. 106-7). Accordingly, Lange's model is able to correctly describe the Keynesian theory as a limiting case in which the *whole M* curve becomes completely horizontal, while Hicks makes it coincide with an horizontal *section* of his *LL* curve (i.e. the leftmost one).<sup>16</sup>

On the other hand, Lange's model seems to be implicitly incompatible with the Hicksian idea of an *IS* curve for two other reasons. Firstly, on a logical level, his substitution of the equilibrium identity for the equation doesn't permit us to know *in advance* the value of *i* and *Y* such that *I* equals *S* on the real market. Therefore, Lange's general idea is considerably removed from that of Hicks, which instead tacitly assumed that, in the real market, the investment decisions were "*largely predetermined*" (Hicks, 1937, p.148 and 1980, p. 146). Moreover, when Lange's equilibrium identity is reformulated into an equation ( $Y=C+I$ ), a second logical issue arises. Recalling Hicks's definition of the *IS* curve as the diagram representing the relationship between income and interest given by "*the second two equations taken together*" [ $I_x=C(i)$  and  $I_x=S(Y)$ , where  $I_x$  is investment],

---

16 We could remark that a further 'apparent similarity' is that, just like Hicks (Cfr. 1946, *Value and Capital*, Chap.XII, pp.157-60), Lange also seems to accept the Classical dichotomy, as he suggests that the equilibrium 'monetary' magnitudes of his model might be expressed in terms of money by means of an equation of exchange ("*equivalent to the traditional equation of the quantity theory of money*"). However, this can be regarded as a purely formal exercise, as in 1942 Lange strongly rejects both the Classical dichotomy and such a 'Hicksian' solution, focused on the quantity theory of money. See: "Say's Law: a Restatement and Criticism", (1942) 1970, pp. 165-167

we could reason by analogy and try to sum Lange's functions (2) and (3). Therefore, in order to plot our 'Langean IS curve', we should now be able to study how a variation of  $i$  affects  $Y$ , which in turn presupposes that we should also know the relationships between  $i$  (on one hand) and  $I$  and  $C$  (on the other). Instead, we can easily see that in his equation (2) Lange assumes "no general rule" about the way a variation of  $i$  affects  $C$ , being:  $\phi_i \begin{matrix} \geq \\ < \end{matrix} 0$ . Consequently the derivative of  $C$  in terms of  $i$  would be indeterminate, thus preventing us from knowing, *a priori*, how a variation of the investment affects saving, i.e., to determine once and for all if the curve resulting from the summation of (2) and (3) would be positively or negatively sloped. Not coincidentally, Lange's reasoning runs along different lines. First of all, he deals with the relationship between the rate of interest, the expenditure on consumption, and investment simply by means of the marginal rate of substitution -  $F_c/F_i$ . Consequently, Lange is able to plot only a *family* of isoinvestment curves (representing the possible variations of both the rate of interest and consumption, which do not change the level of investment), whose slopes are positive "since  $F_c > 0$  and  $F_i < 0$ ". Then equilibrium can be attained exclusively through an adjustment process, as shown in Fig. 4, consisting of a horizontal shift of the  $M$  curve until it becomes tangential to the actual isoinvestment curve. In other words, Lange's solution seems to be focused on the equilibrium identity, whose role was just to show how "the volume of total income always adjusts itself so as to equalise saving and investment actually performed" (Lange, 1938, p.22). Since investment decisions are not given a priori, neither is it possible to plot only one IS curve, nor can equilibrium result from a simultaneous process of market clearing. In order to sum up and emphasize Lange's point of view, we think it's useful to give an alternative graphical interpretation of our own:



**Diagram III:** the consequentiality of Lange's equilibrium solution

In our Diagram III, Fig. I represents the liquidity preference curve: *given* a certain income  $Y_0$  and the quantity of money  $M_0$ , we get the rate of interest  $i_0$ ; Fig. II simply serves to shift  $i$  from the ordinate to the abscissa; Fig. III represents the propensity to consume curve: given the income  $Y_0$  and the rate of interest  $i_0$ , we determine  $C_0$ ; Fig. IV is plotted starting from Lange's marginal efficiency of investment curve and inverting the abscissa with the ordinate, so that the abscissa coincides with the level of consumption  $C_0$ . Given the rate of interest  $i_0$  and the consumption  $C_0$  we can thus determine  $I_0$  and finally, in Fig. III, the level of income  $Y_f$ , corresponding to  $I_0 + C_0$ . If this sum lies on the  $45^\circ$  line of Fig. III, then the system is in equilibrium; otherwise, if it is below or above it, the liquidity preference curve will shift to the new  $I_0 + C_0$  level, starting the *mutual adjustment* process (which will bring the system to equilibrium, in the absence of time lags). Once again, by means of our graphical model, we can easily see how Lange's process of determination of the equilibrium is intrinsically based on the consequentiality of the solution, rather than the simultaneous *IS-LL* model.

#### 4.3 *Is capitalism contracting? Implications of the models*

Another divergence between the models is the way they deal with the topic of a crisis in the capitalist economy, and therefore their implications in terms of policy.

Hicks's model can be regarded as a "*full employment model*", which assumed perfect price flexibility (Hicks, 1980, p.141). Thus Hicks firmly rejected Keynes's support of a public expenditure plan in order to counteract unemployment, since an expansive monetary policy would have been sufficient enough:

"If IS lies to the right, then we can indeed increase employment by increasing the quantity of money." (Hicks J.R., 1937, p.155)

However, Hicks recognized that in the very limiting case corresponding to the horizontal tract of the *IS* curve (i.e. the *minimum* rate of interest), Keynes's advice would become valid because of the inducement to hold cash for speculative motives (defined as the 'liquidity trap', discussed later on). Therefore, both the prescriptions of the *General Theory* and the relevance of the crisis were relegated to very special and unusual circumstances, ironically renamed the "*Economics of Depression*": underemployment became an exceptional event, solely induced by investment rigidity (Hicks, 1974, p.20; Pasinetti, 1974, p.48).

Lange instead dedicates a good ten pages of his article to what he considers its most important application: the problem of under-consumption in a capitalist economy. As we have seen in Section II, the core of his reasoning is the 'indirect' relationship between saving and investment:

"Since investment (...) is a function of both the rate of interest and expenditure on consumption, a decrease of the propensity to consume (...) has a two-fold effect." (Lange O., 1938, p.24)

Therefore, Lange's concise words state that an increase in saving implies both what we can call an 'interest effect' (which stimulates investment), and a 'consumption effect' (which discourages investment). As we have seen in the previous section, the net result on investment will then depend on the dimension of each effect: only in correspondence with the optimum propensity to save will the two effects be balanced, and investment (as well as total income) attain a maximum.

Given this 'Malthusian' difference in emphasis from Keynes, Lange's theory of under-consumption relegates the Keynesian theory to the limiting case in which the optimum propensity to consume hasn't been reached yet, so that "...any increase in the propensity to consume stimulates investment." (p.31). From a general point of view, one might thus infer that Lange's conclusion is not materially different from that previously expressed by Hicks.

However, instead of also affirming that crisis and underemployment can only be associated with such a (Keynesian) limiting case, Lange specifies that in a society where the propensity to save is freely determined by individuals, under-consumption can easily become a long-lasting and predominant scenario. In fact, in such an economy, even an expansive monetary policy could become but a poor and scarcely effective solution because of the monopolistic structure of the markets:

"How far such a policy is possible depends on the structure of the monetary and of the whole economic system." (Lange O., 1938, p.32)<sup>17</sup>

We therefore deduce that in a capitalist economy, an under-consumption crisis becomes 'the rule', whereas full employment is but 'an exception': not even an expansive monetary policy would be able to reverse this tendency. Such a conclusion is seemingly equivalent to a 180° reversal when compared to the implications of Hicks's article.

## **5. The apparent paradox of Lange's article**

To a certain extent, the analysis carried out in the previous sections has led us to an apparent paradox. On a formal analysis level, Lange's article can be legitimately interpreted as one of the early attempts at both generalizing the *General Theory* and integrating its main contents (as a limiting case) within the theoretical framework of general economic equilibrium. From this perspective, one may even argue that he was the forerunner of many GET economists, most notably Don Patinkin.

---

<sup>17</sup> A deeper investigation of the disturbing role of monopolies and oligopolies on the effect of an expansive monetary policy is included in the 1944 *Price Flexibility and Employment*. See, for instance, p.85: "*The growth of oligopolistic and oligopsonistic groups to a dominant position in present-day capitalism prevents a positive monetary effect of a change in prices from being translated into an increase in output or an increase in demand for factors of production.*"

This notwithstanding, once we shift to an applied level of investigation (i.e. when we introduce the institutional datum of a capitalist economy), Lange’s contribution reveals crucial differences from any other synthesis (both contemporary and successive) between Keynes and the ‘Classics’, as he puts a remarkable emphasis on the instability of capitalism, and seems to be skeptical about the possibility of relying exclusively on limited government intervention (e.g. an expansive monetary policy), in order to counteract both under-consumption and unemployment. In a nutshell, we might say that on these grounds Lange turns into a theoretician of ‘general economic *disequilibrium*’.

At first sight, these radical implications might sound completely astonishing (or even vaguely unjustified) in light of the article’s analytical premises. However, a consistent explanation becomes possible once we take into account a previous work on the theory of interest (published just a few months before) and some textual (as well as biographical) evidence related to the same topic.

### 5.1 *The place of interest in the theory of production*

From 1934-1936, Lange was a stipendiary at Harvard under the supervision of J.A. Schumpeter. The Austrian’s influence became evident in the article “The Place of Interest in the Theory of Production”. Although it was published in June 1936, an advance version of the work had already been presented and discussed on December 31, 1935 (Jones, 1936, p.186), hence its essentials were largely pre-existing to the *General Theory*.<sup>18</sup>

The starting point is a study of the conditions of maximum net output, in which production occurs independent of any institutional framework. To this extent, Lange assumes a *Robinsonnade* consisting of a “*communistic settlement of pioneer in a forest*” (p.160), where wood is the only commodity produced, and axes (produced by means of labour and axes) are the equipment. Given the production functions of such a ‘pure production’ case<sup>19</sup>:

$$x = F(m, l)$$

$$m + m' = \phi(m', l')$$

Lange shows that the maximum condition is:

$$\frac{\partial x}{\partial l'} = F_m \phi_{l'}$$

i.e., the marginal productivity of both direct and of indirect labour is equal. After a series of sections which draw from the results of its predecessor, Lange shows also that the rate of (real) interest is:

---

18 Strangely enough, none of the scholars who analysed the economic thought of Oskar Lange has ever noticed such a crucial detail. See, for instance: Kowalik, 1964; 1994; 2008. Keynes’ book had already been published in the U.S. (in February 1936) when the final version of Lange’s article appeared in the Review of Economic Studies.

19 Where  $m$  and  $l$  are the amounts of equipment and labour used per unit of time; and  $m'$  and  $l'$  are the amounts of equipment and labour used to produce equipment, per unit of time. Lange remarks on the analogy with the “*famous*” Simple Reproduction Schemes included in the Second Book of Capital.

$$i = \frac{F_m \phi_l - F_l}{F_l}$$

i.e., the ratio of the marginal *net* productivity of indirect labour to its marginal cost. Therefore, given the maximum condition, the rate of real interest will be zero when net output is at its maximum.

At this point, Lange introduces a further datum: the existence of a “*capitalist enterprise economy*”. He stresses that the most outstanding feature of such a system is the role played by monetary capital. Following Schumpeter (and also Marx), Lange defines it as a prerequisite of production (a “*command*”), which allows the entrepreneurs to purchase other factors indispensable to production itself. However, in this case the very existence of a positive rate of interest will be an index of the scarcity of capital. Thus the capitalist economy becomes intrinsically characterised by “*a shortage of capital which affects the distribution of original resources*”<sup>20</sup>, whereas the purely abstract theory of production presupposed a perfect saturation with capital. Subsequently, in order to explain how the theory of interest can be deduced from the theory of ‘pure’ production, Lange treats them as long-term and short-term equilibria, so that the rate of interest:

“...being an index of the shortage of capital, is also an index of the distance of the actual state from a long-period equilibrium.” (Lange O., 1936A, p.190)

In other words, the accumulation of capital becomes the most relevant process within capitalist dynamics, since the passage from one period to the other implies an increase in capital in order to increase profits by means of investment:

“The accumulation of capital provides the bridge between short-period equilibrium and long-period equilibrium in the theory of interest.” (Ivi, p.191)

However, Lange emphasizes that the path to long-period equilibrium (i.e. a perfect saturation with capital) is an obstacle race, almost an *anarchic* law of capitalist expansion. Firstly, he notes that the annual accumulation of capital is but a small fraction of the existing stock of capital, so that the movement towards long-period equilibrium could potentially be of a secular type. Moreover (again revealing the Schumpeterian influence on his article), Lange remarks that even such a secular process could be slowed down by the plausible existence of Kondratieff cycles. Finally, an extension of the durability of equipment and the delay period can also further remove the determination of long-term equilibrium. Thus in 1936 Lange had already implicitly discussed both the validity of the ‘corollary of Say’s Law’ and the traditional belief that there are no limits to the expansion of investment:

---

20 One may notice an evident incompatibility between Lange’s 1936 point of view and the Keynesian theory, due to the pivotal role played by the *scarcity* of capital in the process of accumulation.

“These considerations expose the fallacy of the superstitious belief current among many economists, that there would be an infinite demand for money capital if the rate of interest were zero, or that a saturation with capital could be attained only when all commodities were to become free goods.” (Lange O., 1936A, p.190)

The brief and general comparison between the two articles is sufficient to show that, despite the analytical distance, the object and scope of 1938’s “The Rate of Interest and the Optimum Propensity to Consume” was the same as 1936’s “The Place of Interest in the Theory of Production”. Even the structure of the two articles is, to a great extent, coincident: in both cases, the works are divided into two parts, depending on the level of abstraction. Each of the first sections is based on a purely abstract level (in order to have a generalised theory), whereas the second deals with the case of a capitalist economy, emphasizing the restraining role of such an institutional datum in order to achieve the very abstract purposes of the economic theory itself. Besides, in both cases Lange focuses on the destabilizing role of capital accumulation and attacks the ‘corollary of Say’s Law’, thus stressing how, in a capitalist economy, long-lasting sub-optimal functioning can easily become both a plausible and an enduring scenario.

### 5.2. *Lange’s scientific project and an unedited manuscript*

The connection evidenced between the two articles allows us to further explore what we defined as “the apparent paradox” of Lange’s 1938 contribution.

In 1936 he unequivocally declared his scientific project:

“... only after the theory of interest has been established independently of the effects of money creation can a satisfactory elucidation of the influence of money creation on interest and production be achieved.” (Lange O., 1936A, p.160)<sup>21</sup>

Therefore, Lange’s early attempt was primarily aimed at providing economists with a *universal* theory of interest. Subsequently he split the abstract analysis of the problem (i.e. independent of any institutional data) from the particular case of a capitalist economy. From this perspective, the publication of Keynes’ *General Theory* only prompted a change in the form chosen by Lange (in 1938), essentially for reasons of simplicity:

“By introducing liquidity preference into the theory of interest, Mr. Keynes has provided us with an analytical apparatus of great power to attack problems which hitherto have successfully resisted the intrusion of the economic theorist” (Lange O., 1938, p.12)<sup>22</sup>

In other words, given Lange’s project to rewrite the theory of interest in a more general way, it simply happened that in the passage from 1936 to 1938, Schumpeter gave way to Keynes.<sup>23</sup>

---

21 The second step of the project (i.e. the “*elucidation of the influence of money creation on interest*”) was developed through 1942’s *Say’s Law – a Restatement and Criticism*

22 At the time, a similar attitude towards Keynes’ book had already been expressed in a well-known 1937 article. See: Lange, 1937, pp.127, 130 and 135

Therefore, in contrast to Hicks (and also H.R.F. Harrod, Reddaway, and J.E. Meade), whose article was a *reaction* to Keynes' book, Lange's purposes *predated* the *General Theory*, so that the latter became but a powerful tool in order to achieve them, in a simpler and more rigorous way.

Nevertheless, in Lange's view a "*satisfactory*" theory of interest should not only be 'universal', but also 'realistic'. Thus, the abstract analysis of the problem has to be followed by the investigation of a capitalist economy. However, as we have shown in both 1936 and 1938, the introduction of such institutional datum becomes a *condition of crisis*. In other words, Lange's scientific project (i.e. to isolate the abstract and universal purposes behind the theory of interest) tacitly embodies a relevant critical dimension: to show that a capitalist economy is the worst institutional instrument in order to achieve a rational allocation of (capital) resources. Just like Marx in the Second Book of *Capital*, Lange considers full employment equilibrium as an event which is *possible in theory*, but almost *unattainable in practice*, at least in a capitalist economy. Notwithstanding his sober and polite language (both in 1936 and in 1938), similar criticism of capitalism is a recurrent topic in his contemporary reflection. In particular, an unpublished manuscript<sup>24</sup> (dated approximately 1941-1942) contains the most revealing of his deepest convictions:

"In earlier societies [investment] opportunities were limited by the scarcity of economic resources (...). Capitalism, however, has revolutionised this situation. Technical progress has diminished substantially the scarcity of economic resources. But it has substituted, instead, another scarcity: an increasing scarcity of the opportunities to utilize them. *This is due to the institutional structure of the capitalist economy* in which the utilization of resources is decided not by their direct utility but by private profitability. If the latter fails resources remain idle: private profit enters, *like a brake*, between resources and the satisfaction of wants. Thus, under capitalism, the utilization of resources is limited not only by their physical scarcity but also, before this limit is reached, by the scarcity of profitable ways to utilize them." (Lange O., 1936-1944, p.12, emphasis added)

The manuscript takes Lange's critique of the sub-optimality of capitalism further, focusing on both the 'anarchic character' of the accumulation of capital and the monopolistic structure of the markets:

"In addition, the monopolistic structure of present capitalist enterprise diminishes the amount of employment provided by a given profit prospective. Thus chronic mass unemployment becomes the dominant feature of present day capitalism. As the accumulation of capital proceeds, opportunities for new investment become increasingly scarce and the employment situation deteriorates." (*Ivi*, p.18)<sup>25</sup>

---

23 Clear evidence of such a change is also provided by a subsequent review of Schumpeter's *Business Cycles*. See: Lange, 1941 pp. 190-3

24 Through private correspondence with the author, the official biographer and editor of the Collected Writings of Oskar Lange (Prof. Tadeusz Kowalik) affirmed that he didn't have access to the manuscript during his last visit in Chicago, dated 1996. The following quotations can therefore be considered the first ever published from the archive. The author wishes to thank Daniel Mayer (Associate Director) and the Special Collections Research Center, University of Chicago Library, for their valuable support to the present research.

25 Both of these topics were in turn widely discussed by Lange in many of his contemporary works. For instance, in 1942's "The Economic Operation of a Socialist Society" he wrote: "*The other element of truth [in the Marxian doctrine of anarchy of production] is exactly the question of capital accumulation.*" Cf. Lange, 1987, p.16

The natural progression of his analysis is that economic theory demonstrates the necessity of institutional change, or to put it differently, that the most evident reasons for socialism are provided by the abstract and universal purposes of economic theory itself:

“No political arrangement (...) can secure lasting peace as long as the scramble for contracting investment and employment opportunities continues. This scramble, however, can be stopped (...) if the deterioration of economic opportunities can be halted and turned into expansion. With plentiful natural resources and ever-expanding technology there is nothing to prevent such a turn, *except the institutional structure of the capitalist economy*. The utilization of economic resources has simply to be based on considerations of social utility instead of upon private (and largely monopolistic) profitability. This implies a reorganization of the economic system on Socialist lines.”<sup>26</sup> (*Ivi*, p.23, emphasis added)

From this perspective, Lange’s reflection on the theory of interest can therefore be interpreted as the necessary premise to a broader project: once he had clarified the abstract purposes of economic theory, he not only wished to highlight the sub-optimality of capitalism, but also to set the stage for an alternative policy proposal, contained in the well-known (and contemporaneous) article “On the Economic Theory of Socialism”, which was re-published (not coincidentally) in 1938. Lange’s challenge to capitalist economies in turn clearly calls to mind (and reverses) the logic of Mises’s attack on socialist systems<sup>27</sup>. In his view, it is evident that after the breakdown of 1929, it is *in the name of* economic theory that one should abandon capitalism and shift to Socialism. Thus the theory of interest becomes the most natural battlefield. Although it is commonly acknowledged that Lange was the first socialist economist to seriously take up the gauntlet of the Austrian School, it should be remarked that his challenge started *from within* the traditional economic theory and *before* his celebrated article on socialist economies dated 1936-37. It is our view that this element becomes a consistent key to interpretation in order to explain both the ‘apparent paradox’ discussed at the beginning of this section, and Lange’s tortuous relationship towards Keynes’s (and Hicks’s) works, as shown in the previous sections.

## 6. Concluding remarks

The article examined in this discourse is the culmination of Lange’s reflection on the theory of interest, written after the publication of the *General Theory* and contemporaneous with the first attempts at the ‘generalization’ of Keynes’ book.

---

26 Yet in 1936 Lange had strongly recommended a socialist economy, since it “*corporately*” performs both saving and the accumulation of capital. Therefore, a Central Planning Board (and not consumers) ought to decide the rate of accumulation in order to prevent serious problems, such as those evidenced by capitalist dynamics. See Lange, 1936B, p.65

27 One should keep in mind that Mises’s article on the socialist calculation debate was re-published and translated into English just a year before Lange’s article, in 1935 (See: Hayek, 1975). Furthermore, Lange was a great connoisseur of both Austro Marxism and the debates related to the ‘Red Wien’ Commission for Socialization, to which the original article of Mises was implicitly addressed.

As in these latter works, Lange provides a theoretical synthesis in which Keynes's theory becomes but a limiting case of an interdependent Walrasian model. Nevertheless, through an original analysis of the institutional datum assumed by economic theory (i.e. a capitalist system), he is able to save the core of Keynes' analysis, that is the tendency towards a chronic under-consumption crisis.

This original outcome reverses Hicks' conclusion on the *General Theory*. The apparent paradox, consisting of an identical starting point and an opposite final result, can be explained by considering Lange's scientific project expressed in a previous work regarding the theory of interest (whose general pattern is largely coincident with the subsequent 1938 paper), published before Keynes' book, as well as in an unpublished manuscript, which revealed his deepest convictions about the destabilizing role of a capitalist economy.

In this sense, we can not only affirm that Lange's theory of interest is "*essentially pre-Keynesian*" (Toporowski, 2005, p.189) in terms of its formal and analytical aspects, but also that its very purposes to a great extent predate and are independent of the *General Theory*.

In short, they consisted of a theoretical generalization (capable of guaranteeing *universality*) and an analysis of institutional data (in order to have *realism*), intended to separate economic theory from the tacit assumptions of a capitalist economy. We may even conclude that Lange's attempt was aimed at generalizing the theory of interest for a 'world to come', whose features were portrayed in a series of contemporary works about socialist theory.

## **Bibliography**

- Ackley, G. 1961. *Macroeconomic Theory*, New York, Mac Millan
- Boianovsky, M. 2004. The IS-LM Model and the Liquidity Trap Concept: From Hicks to Krugman, *History of Political Economy*, vol. 36, Annual Supplement, 92-126
- Chick, V. 1983. *Macroeconomics after Keynes: A Reconsideration of the General Theory*, Oxford: Philip. Allan (Publisher) and Cambridge, MA: MIT Press
- Coddington, A. 1976. Keynesian Economics: the search for first principles, *Journal of Economic Literature*, 14 (4), 1258-73
- Coddington, A. 1983. *Keynesian Economics: The Search for First Principles*, London, George Allen and Unwin
- Fisher, M.R. 1976. Professor Hicks and the Keynesians, *Economica*, vol.43, no.171, 305-14
- Gnos, C. 1998. The Keynesian Identity of Income and Output, in *Historical Perspectives on Macroeconomics: 60 years after the General Theory*, Jolink A. and Fontaine P. (eds.), London Routledge, 40-48
- Hansen, A. H. 1939. Economic Progress and Declining Population Growth, *American Economic Review*, vol.29, no.1, 1-15
- Harrod, R.F. 1937. Mr. Keynes and Traditional Theory, *Econometrica*, vol. 5, no.1, 74-86

- Hayek, F.A. (ed.) 1975. *Collectivist Economic Planning: Critical Studies on the Possibilities of Socialism*, New York, A.M. Kelley pub.
- Hicks, J.R. 1937. Mr. Keynes and the 'Classics': a suggested interpretation, *Econometrica*, vol.5, no.2, 147-59
- Hicks, J.R. 1946. *Value and Capital*, 2<sup>nd</sup> ed., Oxford University Press
- Hicks, J.R. 1974. *The Crisis in Keynesian Economics*, Oxford, Basil Blackwell
- Hicks, J.R. 1980. "IS-LM: An explanation", *Journal of Post Keynesian Economics*, vol.3, no.2, 139-54
- Hicks, Sir J.R. 1988. A Conversation with Sir John Hicks about 'Value and Capital', *Eastern Economic Journal*, vol.14, no.1, 1-6
- Jaffé, W. 1977. The Normative Bias of the Walrasian Model: Walras versus Gossen, *Quarterly Journal of Economics*, 91, 371-87
- Johnson, G.J. 1961, The General Theory after twenty-five years, *American Economic Review*, vol.51, no.2, 1-17
- Jones, H.E. 1936. Meetings of the Econometric Society in New York City December 1935 and St. Louis January 1936, *Econometrica*, vol.4, no.2, 184-92
- Kalecki, M.1937. A Theory of the Business Cycle, *Review of Economic Studies*, 4, no.2, 77-97
- Keynes, J.M. 1973A. *The General Theory of Employment Interest and Money*, in Collected Writings (ed. Moggridge D.), vol. VII, MacMillan St. Martin's Press, London
- Keynes, J.M. 1973B. *The General Theory and After: Part II. Defence and Development*, in Collected Writings (ed. Moggridge D.), vol. XIV, MacMillan St. Martin's Press, London
- Keynes, J.M. 1973C. *The General Theory and After: Part I. Preparation*, in Collected Writings (ed. Moggridge D.), vol. XIII, MacMillan St. Martin's Press, London
- Kowalik, T. 2008. Lange Oskar Ryszard (1904–1965), *The New Palgrave Dictionary of Economics*, Second Edition, Eds. S.N. Durlauf and L.E. Blume, Palgrave Macmillan, vol. 4, 865-76
- Kowalik, T. (ed.) 1998. *Economic Theory and Market Socialism: Selected Essays of Oskar Lange*, Edward Elgar publishing
- Kowalik, T. (ed.) 1964. *On Political Economy and Econometrics. Essays in Honour of Oskar Lange*, Warsaw, Polish Scientific Publishers
- Lampa, R. 2010. 'Rigore Scientifico e Rilevanza Sociale: le due dimensioni della riflessione economica nel giovane Oskar R. Lange (1931-1945)', PhD dissertation, Università degli Studi di Macerata
- Lange, O. 1936A. The Place of Interest in the Theory of Production, *Review of Economic Studies*, vol.3, no.3, 159-92
- Lange, O. 1936B. On the Economic Theory of Socialism: Part One, in *The Review of Economic Studies*, vol.4, no.1, 53-71
- Lange, O. 1936-1944. After the War, *Papers* [Box 1; Folder 1], Special Collections Research Center, University of Chicago Library, 1-37
- Lange, O. 1937. On the Economic Theory of Socialism: Part Two, *Review of the Economic Studies*, vol.4, no.2, 123-42
- Lange, O. 1938. The rate of interest and the optimum propensity to consume, *Economica*, vol. 5, no. 17, 12-32

- Lange, O. 1941. Review to: *Business Cycles: a theoretical, historical and statistical analysis of the capitalist process*, in *Review of Economic and Statistics*, vol.23, no.4, 190-93
- Lange, O. 1944. *Price Flexibility and Employment*, The Principia Press, Bloomington, Indiana
- Lange, O. (1942) 1970. Say's Law: a Restatement and Criticism, in *Papers in Economics and Sociology*, Oxford, Pergamon Press, 149-70
- Lange, O. (1942) 1987. The Economic Operation of a Socialist Society, in *Contributions to Political Economy*, 6, 13-24
- Leijonhufvud, A. 1968. *On Keynesian Economics and the Economics of Keynes: A Study in Monetary Theory*, New York, Oxford U. Press
- Lerner, A.P. 1937. Capital, Investment and Interest, reprinted in (1953) *Essays in Economic Analysis*, London MacMillan, 347-53
- Lerner, A.P. 1953. On the Marginal Product of Capital and the Marginal Efficiency of Investment, *Journal of Political Economy*, vol. LXI, no.1, 1-14
- Mac Dowell, G. 1957. Consumption and the rate of interest, *Canadian Journal of Economics and Political Science*, vol.23, no.3, 409-12
- Marschak, J. 1942. Identity and Stability in Economics: a Survey, *Econometrica*, 10 (1), 61-74
- Meade, J.E. 1937. A simplified model of Mr. Keynes' System, *Review of Economic Studies*, vol.4, no. 2, 98-107
- Millmow, A. 2003. "W. Brian Reddaway-Keynes Emissary to Australia, 1913-2002", *Economic Record*, vol.79, no.244, 136-38
- Pasinetti, L.L. 1974. *Growth and Income Distribution: Essays in economic theory*, Cambridge University Press
- Patinkin, D. 1958. Liquidity Preference and Loanable Funds: Stock and Flow Analysis, *Economica*, 25(100), 300-18
- Reddaway, W.B. 1936. The General Theory of Employment, Interest and Money, *Economic Record*, vol.12, 28-36
- Robbins, L.C. 1968. *The Economic Causes of War*, New York, Howard Fertig
- Schumpeter, J.A. 1981. *History of Economic Analysis*, Routledge
- Toporowski, J. 2005. Between Breit and Keynes: Oskar Lange's Theory of Interest, in Sadowski Z. (ed.), *Oskar Lange a wspolczesnosc*, Warszawa: Wydawnictwo Polskiego Towarzystwa Ekonomicznego, 185-90