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Abstract

In a simple one-sector economy operating at full capacity, workers and firms bargain *à la* [Nash (1950)] over wages and productivity gains taking into account the trade-offs faced by firms in choosing factor-augmenting technologies. The aggregate environment that arises from self-interested behavior by economic agents, thus producing decision rules on wages, productivity gains, savings and investment, is described by a two-dimensional dynamical system in the employment rate and output/capital ratio. The economy converges cyclically to a long-run equilibrium involving a Harrod-neutral profile of technical change, a constant rate of employment of labor, and constant input shares. The type of oscillations predicted by the model is qualitatively consistent with the available data on the United States (1963-2003), replicates the dynamics found in earlier models of growth cycles such as [Goodwin (1967)] [Shah and Desai (1981)], [van der Ploeg (1987)], and is verified numerically in simulations. Institutional change, as captured by variations in workers' bargaining power, has a positive effect on the rate of growth of output per worker but a negative effect on employment. Economic policy can also affect the growth and distribution pattern through changes in unemployment compensations, which also have a positive impact on labor productivity growth but a negative impact on employment.

Keywords: Goodwin Growth Cycle, Bargaining, Induced Technical Change, Factor Shares, Employment.
JEL Classification System: E24, E25, J52, O31.

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1 Introduction

Post-war macroeconomic data for the United States show that the labor employment rate cycles counterclockwise when plotted against the labor share, and that these types of cycles are found to persist even when one accounts for recent increases in wage inequality by subtracting the top 10%, 5%, and 1% of the wage distribution from labor share calculations, as shown in Figure 1. The same counterclockwise orientation is found if we plot the employment rate against the output/capital ratio, as Figure 2 shows.

[FIGURES 1 AND 2 ABOUT HERE]

The presence of such cyclical behavior in the relevant series has been the object of several studies, beginning with the seminal contribution by Richard [Goodwin (1967)]. In his paper, Goodwin developed a model of the labor share and employment rate based on Lotka-Volterra predator-prey dynamics which emphasizes the distributional conflict between capital and labor, and the role of unemployment as a discipline mechanism on wage demands by workers. High employment generates wage inflation which, as long as real wages increase more than labor productivity, increases the wage share in output. The resulting decrease in the profit share, it is argued, will act in reducing future investment and output. Lower labor demand will then correspond to lower output, leading the way to lower wage inflation or even wage deflation thus lowering the labor share. But a higher profit share will produce a surge in investment, which will generate higher employment, thus improving workers' bargaining power and consequently wages. At this point, the wage share has increased, and the cycle can repeat itself. It is well known that Goodwin's dynamical system generates *closed orbits*, that is perpetual cycles starting from any initial condition, and a feature that is crucial for generating the type of observed dynamics is the hypothesis that real wages grow with the employment rate. Such an assumption was justified appealing to Phillips curve considerations, but a more modern empirical counterpart to it can be found in the *wage curve* observed first by [Blanchflower and Oswald (1990)].

An important shortcoming of the Goodwin model of the growth cycle was the assumption of exogenous, Harrod-neutral (that is, purely labor-augmenting) technical change. [Shah and Desai (1981)], and later [van der Ploeg (1987)] built on the early literature on *induced bias* in innovation[1] to link technological progress to distributive shares in the model: technical change will be directed toward augmenting the productivity of the factor becoming more expensive. More recently, [Foley (2003)] took a similar approach, and [Julius (2005)] studied the Shah and Desai-like dynamics arising from a special case of Foley's model. A common finding is that induced innovation bias produces a pattern of technical change that is Harrod-neutral and features constant factor shares in equilibrium, thus matching the basic [Kaldor (1961)] facts of economic growth. At the same time, induced bias acts toward 'stabilizing' Goodwin's closed orbits, eventually determining convergence of the dynamics to the long-run equilibrium of the economy.

As relevant as they are toward an understanding of growth and distribution patterns in advanced capitalist economies, virtually all of the contributions in the above tradition don't address the question concerning what are the behavioral premises that are capable of generating the aggregate dynamics under consideration. Such

[1] [Kennedy (1964)], [Drandakis and Phelps (1965)] are probably the most representative articles of that wave of literature

a quest for microeconomic foundations is not only relevant *per se* from a methodological standpoint, but also and more importantly has the scope to highlight the role that institutional and/or policy factors play in shaping growth and distribution patterns that arise in general equilibrium. This paper deals with the role of negotiations between firms and workers on wages and productivity gains in determining the evolution of real GDP per worker, factor shares, capital accumulation, and employment. As such, it provides behavioral grounds that bridge the literature stemming from Goodwin's contribution with the growth and distribution approach of [Marglin (1984)], [Foley and Michl (1999)], with the ultimate scope of extending and refining a growth model rooted in the Classical tradition into a framework that is consistent with the observed dynamics of advanced capitalist economies such as the United States, and can be used for policy purposes.

With this aim in mind, I build an accumulation and innovation model centered around the assumption that workers and firms bargain in the axiomatic way introduced by [Nash (1950)] over wages and productivity, taking into account the fact that firms face trade-offs in augmenting the productivity of different factors. As far as production is concerned, I assume full capacity utilization and mainly focus on the no-substitution (Leontief) case, in which labor and capital enter production in fixed proportions, in order to depart as little as possible from the previous studies on the subject.[2] A deeper reason to focus on Leontief production functions builds on a view about production which starts from the consideration that at each moment in time production takes place with fixed input/output coefficients. Over time, however, profit-maximizing decisions about the direction of technical change are responsible for capital deepening, and will make the aggregate production function look Neoclassical, with diminishing returns to capital per worker in standard fashion.[3]

The main implications of the present analysis are that economic decisions on wage determination, innovation and capital accumulation eventually boil down to a two-dimensional dynamical system in output/capital ratio and the employment rate. The dynamics of the economy evolve so as to ensure a Harrod-neutral path of technical progress, and a constant long-run employment rate which adjusts so as to ensure the constancy of factor shares at the long-run equilibrium. Convergence to the equilibrium path of growth and distribution occurs cyclically. These oscillations are shown to be qualitatively consistent with the available empirical evidence for post-war United States, and can be verified numerically through simulations. Therefore, this contribution provides microeconomic foundations compatible with the aggregate behavior observed in the data and analyzed in the literature. More importantly, the behavioral foundations I propose allow to isolate the growth and distribution effects of one institutional and one policy variable, respectively the bargaining power of workers and the unemployment compensation. At an equilibrium of the model, an increase in the workers' contractual weight induces a higher long-run rate of labor-augmenting innovations, at the price of higher long-run unemployment. An increase in the unemployment compensation also pushes the economy toward higher labor productivity growth, although it determines higher structural unemployment.

[2] I also show in Section 3.3 that the main conclusions reached survive the introduction of instantaneous capital/labor substitution through a Neoclassical production function.

[3] The role of capital deepening over time resulting from biased technical change as opposed to instantaneous factor substitution has been emphasized by [Michl (1999)] as well as [Foley and Michl (1999)].

The paper is organized as follows. In Section 2, I first describe the economic environment, given by the technology for output production and the wage-productivity bargaining structure. Crucial to the model is to determine endogenously an outside option available for the workers during negotiations. I do so by comparing, as done for instance in [Shapiro and Stiglitz (1984)], or [Pissarides (2001)], the present value of an ('asset' representing an) employed worker with the present value of a worker currently outside the employment pool. In the baseline version of the model, there is no unemployment compensation. I then turn to savings decisions, made only by firm-owners in the baseline version of the model, in order to characterize investment in new capital stock. This is the final piece of the picture that allows to define a market equilibrium, derive the dynamical system describing the economy, characterize its long-run equilibrium, study its stability properties, analyze qualitatively the behavior of the system in the phase space, and carry comparative dynamics exercises for varying exogenous parameters. I then carry a calibration and simulation exercise to check for the ability of the model of replicating the observed dynamics of employment rate and output/capital ratio, and work out several extensions in Section 3 to include the effects of unemployment compensation, workers' savings, as well as to introduce capital/labor substitution. These extensions show that the main message of the framework proposed is robust with respect to such different assumptions. Section 4 concludes. Most of the results in this paper are stated as propositions: proofs are provided in the Appendices.

2 The Model

2.1 Technologies for Output and Innovation. Profit Maximization

Consider a representative firm in a simple one-sector economy populated by competitive firms, as well as workers with homogeneous skills. Workers supply labor, while firms own the means of production, hire labor and tie up stocks of a single capital good to produce a single final good Y homogeneous with capital. The capital stock is fixed in the short run, so that, assuming full capacity utilization, production of output takes place according to the instantaneous Leontief technique:

$$Y = \min\{AL, BK\} \tag{1}$$

where L, K denote labor and capital respectively, A and B are positive numbers summarizing the current stocks of knowledge in factor-augmenting technologies.

For a firm, it is profit-maximizing to choose factor demands such that $AL = BK$, so that no factor has idle (effective) units. If r denotes the rate of profit on capital invested and w is the wage to be paid to workers, the value of maximized profits will be given by $rK = B\left(1 - \frac{w}{A}\right)K$. Also, let the total labor force be denoted by $N \geq L$. Since firms will demand BK/A units of labor, the employment rate v in the economy will satisfy $v = \frac{BK}{AN} \in [0, 1]$.

As for the evolution over time of factor-augmenting technologies, let $\dot{B} = \beta B, \dot{A} = \alpha A$. Assume that at each moment in time the available profiles of technological improvements (α, β) belong to the *innovation set*:

$$I = \{(\alpha, \beta) \in \mathbb{R}^2 : \alpha \leq g(\beta)\} \tag{2}$$

where $g \in \mathcal{C}^2, \partial g/\partial\beta \equiv g_\beta < 0, g_{\beta\beta} < 0, g^{-1}(0) > 0$. Following [Kennedy (1964)], call Innovation Possibility Frontier (IPF henceforth) the boundary of the innovation set. Such an IPF represents the costs of inventive activity for a given R&D

budget, and it is supposed to be strictly concave, similarly to a familiar production possibility frontier, to capture a notion of increasing opportunity cost in relative factor augmentation.[4]

2.2 Bargaining over Productivity Gains and Wages

Workers and firms behave in the axiomatic way first studied by [Nash (1950)]. They face off in a market for costlessly enforceable labor contracts of length one period, and bargain at infinitely small intervals over productivity gains and wages over an infinite horizon.[5] In the present setup, it is assumed that both firms and workers know the existence of an innovation set, and they will take such set as a dynamic constraint in their negotiations. At each moment in time, the labor market closes after a single round of negotiations so that, if a deal is not struck, the production process will be interrupted for the period. If the negotiations succeed, firms will earn profits per unit of capital equal to the profit rate $r = B(1 - w/A)$, where the wage w has to be determined within the bargaining problem. We assume that production can be shut down at no cost, hence the fallback position for the firm is zero. On the other hand, the gain for each worker in each period in case of agreement is given by the difference between the wage and a non-negative outside option, which is endogenous to the model and represents the per-period flow of the present discounted value of unemployment. We let $\eta \in [0, 1)$ be a parameter denoting workers' bargaining power,[6] the discount rate be denoted by $\rho > 0$, and $V_U > 0$ be the present discounted value of (an asset representing) a worker out of the employment pool. Assuming that both bargainers are risk-neutral relative to their respective bargaining gains, we have the following problem to be solved:[7]

$$\begin{array}{lll} \text{Choose } w(t), \beta & \text{to max} & \displaystyle\int_0^\infty e^{-\rho t} \left\{ \eta \log[w(t) - \rho V_U] + (1-\eta) \log \left[B(t) \left(1 - \frac{w(t)}{A(t)} \right) \right] \right\} dt \\ & \text{s. t.} & \dot{B}(t) = \beta B(t) \\ & & \dot{A}(t) = g(\beta) A(t) \\ & & A(0), B(0) \text{ given} \\ & & \lim\limits_{t\to\infty} e^{-\rho t} B(t) \geq 0, \lim\limits_{t\to\infty} e^{-\rho t} A(t) \geq 0 \end{array} \tag{3}$$

The following proposition, proved in Appendix A.1, establishes the properties of the solution of the above problem.

Proposition 1. An optimal control solution for (3) involves time-paths for the real wage and the direction of technical change such that:

$$w(t) = \eta A(t) + (1 - \eta)\rho V_U \tag{4}$$

[4] A microfoundation for the IPF resulting from a general CES production technology is provided in [Funk (2002)].

[5] Using Nash bargaining as a wage-setting mechanism is pretty standard in the labor market literature. Examples include [Pissarides (2001)] in the matching literature, and [Oswald (1985)], [Blanchflower and Oswald (1990)] regarding labor unions. The advantage of Nash bargaining is that it focuses on the outcome of the bargaining problem. Also, the bargaining mechanism can be justified strategically as in [Binmore *et al.* (1986)].

[6] A traditional proxy for η is the rate of unionization, but one can also think about different aspects of labor legislation that increase the relative weight of workers in wage negotiations.

[7] Since the choice variables are w, β, it makes no difference if we consider profits, rK, or the profit rate, to appear in the firms' gain. Also, substitution of the constraint given by the IPF in the objective function is allowed because innovation set is strictly concave, so that the solution will be unique and on the boundary of I.

$$-g_\beta = \frac{1-\omega(t)}{\omega(t)} \tag{5}$$

where $\omega \equiv w/A$ is the wage share in firm's output.

The bargaining wage (4) is a linear combination of labor productivity and the outside option, thus providing a simple wage-setting rule that splits between bargainers the difference between a ceiling given by revenues per unit of labor and a floor to be determined endogenously in the model. On the other hand, equation (5) identifies a pair of continuous (implicit) functions, $\beta(\omega)$ and $\alpha(\omega)$, giving the bias of technical change at the firm level as a function of the wage share and the discount rate, such that $\alpha_\omega > 0, \beta_\omega < 0$.[8] Hence, the solution of our problem yields the standard 'induced bias proposition' according to which firms will direct technical change to augment the productivity of the factor whose share in costs increases ([Kennedy (1964)], [Drandakis and Phelps (1965)]).

2.3 Workers' Outside Option

Let $q : [0,1] \to [0,1]$, the probability of a worker out of employment (unemployed, or working in a traditional, as opposed to industrial, sector) to enter the employment pool, be a function of the employment rate: $q = q(v)$ such that $q(0) = 0, q(1) = 1, q_v > 0$. Assume for the moment that there is no unemployment compensation[9] and, as it is traditional in the labor market literature, let V_E, V_U be assets representing respectively a worker currently employed and a worker out of the employment pool. If the rate of time preference in the market for those assets is $\rho > 0$, equal to the discount rate in the bargaining problem, the present value V_U of a worker out of the employment pool must satisfy:

$$\rho V_U = q(v)(V_E - V_U) \tag{6}$$

with the standard asset-pricing interpretation.

Consider instead a worker that is currently employed, and denote the probability that (or the flow rate at which) her contract is terminated by $\lambda \in (0,1)$. The asset equation for a worker that is currently employed satisfies:

$$\rho V_E = w + \lambda(V_U - V_E) \tag{7}$$

Equations (6) and (7) can be solved for (V_U, V_E) as functions of the probabilities $q(v), \lambda$, the discount rate and the wage. We have:

$$V_U = \frac{q(v)}{\rho + q(v)} V_E; \quad \rho V_E = \frac{\rho + q(v)}{\rho + q(v) + \lambda} w$$

from which it is apparent that employed workers enjoy a rent over workers out of the employment pool. Plugging the value V_E into V_U, we find:

$$\rho V_U = \frac{q(v)}{\rho + q(v) + \lambda} w \tag{8}$$

[8] These claims are substantiated in Appendix A.2.
[9] We will relax this assumption in Section 3.1.

2.4 Capital Accumulation

In considering decisions about consumption and savings in this model, we make the simplifying hypotheses that the representative household has logarithmic preferences over consumption streams $\{C(t)\}_{t\in[0,\infty)}$, that capital is always utilized at full capacity, that there is no capital depreciation, and that the discount rate is constant and equal to ρ, as before. Also, given that this paper relates to [Goodwin (1967)] and subsequent research, in the baseline version of the model we assume that workers don't have any wealth to start with, and therefore consume all of their income. Therefore, savings decisions will be made only by profit-earning households. We will relax this 'Cambridge-style' postulate about savings in Section 3.2 to show that the model is robust to different savings scenarios.[10]

Under our hypotheses, the entrepreneurial households' income at each point in time will be given by $r(t)K(t)$, to be allocated to consumption and investment, denoted by $\dot{K} \equiv \partial K(t)/\partial t$. Being determined within the bargaining problem, the evolution of $B(t)$ is taken as given in choosing how much to consume and how much to save over time. This makes good sense intuitively if one thinks that, given the outcome of negotiations occurring in the workplace, savings decisions will be made within each household. Because of strict monotonicity of preferences, the household's budget constraint will be satisfied with equality at all t. Thus, the representative household faces the following problem:

$$\begin{array}{lll} \text{Choose } C(t) & \text{to maximize} & \displaystyle\int_0^\infty e^{-\rho t}\log C(t)dt \\ & \text{subject to:} & C(t) = r(t)K(t) - \dot{K}(t) \\ & & K(0) = K_0 > 0 \text{ given} \\ & & \lim\limits_{t\to\infty} e^{-\rho t}K(t) = 0 \end{array} \tag{9}$$

To find a solution for this problem, write down the current-value Hamiltonian:

$$H_C = \log C + \mu(rK - C)$$

in which the time-dependence is omitted since the Hamiltonian holds at any instant in time. The first-order condition on the control variable is $C = \mu^{-1}$, whereas the necessary condition for optimality on the costate variable μ is:

$$(\rho - r(t))\mu(t) = \dot{\mu}(t) \tag{10}$$

Also, the transversality condition $\lim_{t\to\infty}\exp(-\rho t)\{\mu(t)K(t)\} = 0$ must be fulfilled. Given strict concavity of the objective function and convexity of the constraint set, the sufficient conditions for optimality will also be satisfied. Hence, the solution of the optimization problem (9) is a system of differential equations in C, K formed by the (time-varying) Euler equation $\frac{\dot{C}}{C} = r(t) - \rho$, together with the transition equation $\frac{\dot{K}}{K} = r(t) - \frac{C(t)}{K(t)}$. To solve this system, let us use a guess-and-verify strategy. Our candidate solution is $C = a(t)K$, where $a(t)$ is a function to be determined. The

[10] The key assumption here is that workers are assumed not to have any wealth to start with, and therefore to accumulate. Then, they solve the following problem:

$$\max_{c^w(t)} \int_0^\infty e^{-\rho t}u(c^w(t))dt \quad s.t.\ c^w(t) \leq w(t)$$

where c^w denotes workers' consumption. No matter their time-preference rate (reason for which there is no need to differentiate it across classes), if their utility function $u(\cdot)$ satisfies local non-satiation, workers will choose $c^w(t) = w(t)\ \forall t$.

only function satisfying the two differential equations is $a(t) = \rho \, \forall t \in [0, \infty)$. Such finding of a constant consumption/capital ratio can be justified observing that an isoelastic felicity function such as the Cobb-Douglas assumed here has the property that substitution and income effects exactly offset each other, so that the path of future prices such as interest rates makes no difference to current consumption decisions. This property is therefore responsible for the dramatic simplification of the intertemporal allocation problem.[11] Now, consider firms' propensity to save out of their profits, $s(t) \equiv \frac{r(t)K(t)-C(t)}{r(t)K(t)} = 1 - \frac{\rho}{r(t)}$. Using the firms' budget constraint, we have the following time-varying version of the Cambridge equation, giving the balanced growth path for our economy:[12]

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho = s(t)r(t) = \frac{\dot{K}(t)}{K(t)} \tag{11}$$

2.5 Market Equilibrium

A *market equilibrium* for our economy is: i) a sequence of wages $w(t)$ satisfying (4); ii) a value of the outside option V_U satisfying (8); iii) consumption and capital accumulation paths such that (11) is satisfied; iv) an allocation of labor such that profits are maximized given the wage and the capital accumulation path; v) a direction of technical progress satisfying (5); vi) a sequence of prices for the consumption good solving the accumulation problem (9), and vii) sequences of prices for capital- and labor-augmenting technologies which solve the maximization program (3).[13] Given that the solutions for all the problems above are well-defined and interior, a market equilibrium for our economy exists.

The wage at a market equilibrium is:

$$w = \eta \left[\frac{\rho + \lambda + q(v)}{\rho + \lambda + \eta q(v)} \right] A \tag{12}$$

from which it is easy to see that $w \in [0, A)$ as $\eta \in [0, 1)$, so that the rate of return on capital stock is positive at a market equilibrium. The following proposition summarizes the comparative statics results about the wage.

[11] For those who remain skeptic, the solution above can be alternatively found using $\exp\left\{-\int r(t)dt\right\}$ as an integrating factor for the transition equation on capital stock, and exploiting the transversality condition. In fact, the solution of the Euler equation for consumption will be $C(t) = C(0)\exp\left\{-\int (r(t) - \rho)dt\right\}$. Using $\exp\left\{-\int r(t)dt\right\}$ as integrating factor, we can solve the transition equation for capital up to a constant of integration. Because of the transversality condition, such constant turns out to be zero. Therefore, it is easy to determine $C(0) = \rho K(0)$, from which the solution for $K(t)$ follows immediately. A similar derivation, which produces the same result as here once the consumption good is assumed to exchange at the same price as the capital good, can be found in [Acemoglu (2009)], p. 667. In that context, too, what drives the result is the iso-elastic form of the utility function.

[12] It is worthwhile to stress once again the assumptions about saving decisions we made in this paper. It is assumed that saving and investment choices take place given the outcome of negotiation about wages and productivity gains. The reason why this assumption is made is not only its intuitive plausibility, but the fact that the Nash bargaining structure 3, although allowing bargainers to choose control variables taking advantage on information on both production and innovation environments, already gives the workplace a strongly cooperative character, because it involves the *joint* maximization of utility gains. The inclusion of investment decisions into a grand, single optimization problem would configure the firm as a cooperative, and describing such an environment is outside the focus of this paper.

[13] The equilibrium paths for $p_1(t), p_2(t)$, which denote prices for capital- and labor-augmenting technologies respectively, are represented by equations (24) and (25) in Appendix A.1).

Proposition 2. The wage at a market equilibrium is increasing in the employment rate and in worker's bargaining power, and is decreasing in the discount rate and in the probability of termination of the labor contract.

Proof. See Appendix B.1. □

The positive dependence of the equilibrium real wage on the employment rate is intuitive, and finds an empirical counterpart in the 'wage curve' estimated by [Blanchflower and Oswald (1990)] for the UK and the US. A positive correlation between equilibrium real wages and workers' bargaining power is also expected, given the bargaining structure of the model, just like only introductory knowledge of macroeconomics is required to make sense of the negative correlation between the equilibrium wage and the rate at which employment contracts are terminated. Finally, a higher discount rate increases the per period return of being out of the employment pool, thus reducing the equilibrium wage.

The profit rate at a market equilibrium is readily calculated:

$$r = (1-\eta)B\left[\frac{\rho+\lambda}{\rho+\lambda+\eta q(v)}\right] \tag{13}$$

and we have the following results as a direct implication of Proposition 2.

Corollary 1. The market equilibrium-profit rate decreases with workers' bargaining power, with the employment rate, and increases in the discount rate and the probability of termination of the labor contract.

Finally, the direction of technical change at a market equilibrium is:

$$-g_\beta = \left(\frac{1-\eta}{\eta}\right)\left[\frac{\rho+\lambda}{\rho+\lambda+q(v)}\right] \tag{14}$$

to obtain which we used of course (12). The RHS of (14) is obviously the ratio between equilibrium factor shares in our economy. Given that this ratio is a function of the parameters of the model, we can now state a number of results about the equilibrium growth rates of factor-augmenting technologies for varying exogenous variables.

Proposition 3. Under the assumptions made throughout this paper, the direction of technical change at a market equilibrium determines growth rates of factor-augmenting technologies $\beta(v;\eta,\lambda,\rho), \alpha(v;\eta,\lambda,\rho)$ such that:

$$\begin{array}{llll} \beta_v < 0, & \beta_\eta < 0, & \beta_\lambda > 0, & \beta_\rho > 0; \\ \alpha_v > 0, & \alpha_\eta > 0, & \alpha_\lambda < 0, & \alpha_\rho < 0. \end{array}$$

Proof. See Appendix B.2. □

The above results on the dependence of factor-augmenting technologies on employment rate, workers' bargaining power, and the rate of job destruction are entirely consistent with the idea of induced bias of technology, other than intuitive. First, the economic intuition behind the positive (negative) dependence on labor- (capital-) augmenting innovations on the employment rate is straightforward. The wage curve established in Proposition 2 ensures that the market equilibrium wage is increasing in the employment rate. It follows that the labor share w/A responds positively to increases in the employment rate, too. Therefore, our findings are in line with the standard argument according to which the higher the labor share, the more technical progress will be directed toward labor-augmenting blueprints for a given R&D intensity ([Kennedy (1964)]). Second, an increase in workers' bargaining power also results in a higher labor share and therefore leads to more labor-augmenting and less capital-augmenting innovations. Since 'labor-augmenting' is synonymous with 'labor-saving' when labor and capital are (gross) complements in production ([Drandakis and Phelps (1965)], [Kamien and Schwartz (1969)]), our findings about an institutional variable such as η add flavor to the distributional features of induced innovation,[14] because technical change is used by firms to mitigate the impact of rising workers' power in negotiations through reductions in labor demand.[15] Third, a higher rate of job termination reduces the equilibrium wage, and therefore leads to more capital augmentation and less labor augmentation. Similarly, a higher discount rate reduces the opportunity cost of being out of the employment pool, thus lowering the market equilibrium wage and therefore the rate of labor-augmenting technical progress via induced bias in innovation.

2.6 The Dynamical System

Consider the employment rate $v = \frac{BK}{AN}$. Logarithmic differentiation of v yields the dynamic equation:

$$\dot{v} = (\hat{B} + \hat{K} - \hat{A} - \hat{N})v$$

with the standard notational convention of 'hat' variables denoting growth rates. Assume that the model is labor-constrained: $\hat{N} \equiv n$, constant and exogenous. Plugging the growth rate of capital stock derived in (11), together with the equilibrium profit rate 13 and the results on the equilibrium direction of technical progress obtained in Proposition 3, we have the following nonlinear dynamical system in the state space (B, v):

$$\dot{B} = \beta(v; \eta, \lambda, \rho)B \tag{15}$$

$$\dot{v} = \left\{ \beta(v; \eta, \lambda, \rho) + (1-\eta)B\frac{\rho+\lambda}{\rho+\lambda+\eta q(v)} - [\rho + \alpha(v; \eta, \lambda, \rho) + n] \right\} v \tag{16}$$

2.7 Long-run Equilibrium

A *long-run equilibrium* for this economy is a pair (B, v) such that $\dot{B} = \dot{v} = 0$. At a long-run equilibrium,

$$\beta(v_{ss}; \eta, \lambda, \rho) = 0 \tag{17}$$

$$B_{ss} = \left[\frac{\alpha(v_{ss}; \eta, \lambda, \rho) + n + \rho}{(1-\eta)}\right]\left[\frac{\rho+\lambda+\eta q(v_{ss})}{\rho+\lambda}\right] \tag{18}$$

so that the first equation determines long-run employment rate, using which, given the exogenous parameters η, λ, ρ, we are able to pin down the long-run constant output/capital ratio. To check for existence, let us determine the nullclines on the phase space (B, v). The equilibrium locus for output/capital ratio is a horizontal line at $v_{ss} = \beta^{-1}(0; \cdot)$. On the other hand, it is not hard to show that the function $B(v; \cdot)$ as defined in (18) is strictly increasing in v, as both $\alpha_v, q_v > 0$. Hence, a long-run equilibrium's existence is ensured. The basic properties of such equilibrium are summarized in the following Proposition.

[14] The role of the direction of technical progress in distributional conflict is discussed, among others, in [Bowles and Kendrick (1970)].

[15] This is a key result in this paper, established formally in Proposition 8.

Proposition 4. Under the assumptions made throughout this paper, a long-run equilibrium in which $B \neq 0, v \neq 0$ exists and is unique. It features a Harrod-neutral profile of technical change, constant unemployment rate and constant input shares.

Proof. See Appendix B.3. □

Proposition 4 confirms the typical findings of models with an endogenous direction of technical change, such as [Drandakis and Phelps (1965)], in what basically says that our framework matches the [Kaldor (1961)] facts. An important characteristic of the model under investigation, however, is the role of the employment rate in adjusting so as to ensure the constancy of factor shares in the long-run and to annihilate the capital-augmenting component of technical progress.[16] Also, the employment rate, and therefore the labor share, are invariant with respect to the savings decisions made by capitalist households, as in [Julius (2005)]. As for the local stability properties of the long-run equilibrium, we have the following proposition.

Proposition 5. The long-run equilibrium of this economy is locally asymptotically stable.

Proof. See Appendix B.4. □

Also, it is easy to see that long-run output growth is equal to $\hat{Y} = \alpha(v_{ss}; \eta, \lambda) + n$, in turn equal to the growth rate of capital stock obtained in Section 2.4 along a balanced growth path.

2.8 Transition Dynamics

Let us now focus on the behavior of the dynamical system (15), (16) out of the long-run equilibrium. The main focus of this section is to show that convergence to the steady state in response to (not explicitly modeled here) shocks occurs cyclically. Given the low dimensionality of the phase space, we can consider qualitatively the dynamics of the two variables (B, v) in the phase plane. A graphical representation is provided in Figure 3. A numerical exercise is also worked out below.

[FIGURE 3 ABOUT HERE]

Clearly, the output/capital ratio is decreasing in v when the employment rate is not at its equilibrium level, as the function $\beta(v; \cdot)$ is decreasing in its argument. Therefore, the arrows in the phase diagram point west for $v < v_{ss}$ and east for $v > v_{ss}$. As for the behavior of the employment rate outside the isocline, it is sufficient to differentiate $\dot{v}$ with respect to B in (16) to see that the employment rate increases in output/capital ratio. Hence, the arrows in the phase space point north above the $\dot{v} = 0$ isocline and south below. Putting everything together, we see that the phase space is characterized by counterclockwise oscillations. Since we showed before that the long-run equilibrium is locally asymptotically stable, we conclude that the dynamics of the model describe a spiral converging to the equilibrium point. These results can be summarized in the following proposition.

Proposition 6. The long-run equilibrium of the system formed by (15), (16) is a stable spiral displaying counterclockwise oscillations in the phase space (B, v).

[16] Such adjustments take place also in [van der Ploeg (1987)] and [Julius (2005)], but with an exogenous wage and a reduced-form Phillips-style hypothesis of wage growth being an increasing function of the employment rate.

The counterclockwise transition dynamics in Figure 3 provides a qualitative match to the observed patterns in employment rate and output/capital ratio displayed in Figure 2. A better assessment of the extent to which the two variables under consideration actually cycle for some time while approaching the long-run equilibrium requires numerical calibrations of the parameters and functions appearing in (15) and (16) in order to see whether the eigenvalues of the Jacobian matrix of the system, evaluated at a long-run equilibrium, have imaginary roots. A simulation exercise using a standard parameter calibrations is carried in Appendix C. Key to the cycling behavior of the system are the magnitudes of (α_v, β_v). Each panel appearing in Figures 4 and 5 below, corresponding to a different value of workers' bargaining power parameter, shows several of the infinite possible trajectories for employment rate and output/capital ratio, each corresponding to a different pair (α_v, β_v).[17] Table 1 in the Appendix summarizes the calibration used.

[FIGURES 4, 5 ABOUT HERE]

2.9 Comparative Dynamics

We can now study how does the long-run equilibrium of our economy vary with the exogenous parameters of the model. The following results stand out.

Proposition 7. At a long-run equilibrium, the output/capital ratio varies directly with the employment rate, workers' bargaining power and population growth, and inversely with the rate of job termination.

Proof. See Appendix B.5. □

The fact that long-run capital/output ratio varies directly with population growth is obvious after glancing at (18). To make sense of the positive dependence of long-run output/capital ratio on workers' bargaining power, consider first that B_{ss} is directly related to the growth rate of labor productivity, in turn an increasing function of η. Second, and standard in this type of models ([Shah and Desai (1981)], [van der Ploeg (1987)],[Julius (2005)]), B_{ss} is decreasing in the savings rate. As firm-owners are the only saving agents in this baseline framework, and their income is inversely related to workers' bargaining power, the higher η the lower the savings rate, and therefore the higher B_{ss}.

The final question we ask is how does institutional change affects the long-run growth and distribution path of the model, that is how does the long-run equilibrium employment rate vary with workers' bargaining power.

Proposition 8. Assume that η increases of an amount $d\eta > 0$, while the probability of termination and the discount rate remain constant. Then, the corresponding variation in the long-run equilibrium employment rate dv_{ss} must be negative.

Proof. See Appendix B.6. □

The interpretation of the result is again straightforward. Induced bias of technology is the most powerful force at work in this model: even if workers, aware that firms innovate to save on factor costs, are able to bargain (cooperatively) over productivity gains, they will still have to trade a gain in wages due to an increase

[17] The software used for the simulations is *Mathematica* 7, and the code is available from the author upon request.

in bargaining power with a loss in the level of employment.[18] To make sense of the finding from a distributional standpoint, remember that factor shares have to remain constant in the long run. Hence, the upward pressure in the labor share arising from an increase in workers' bargaining power has to be compensated by a reduction in the employment rate in order to bring the dynamics back to its long run growth path.

3 Extensions

3.1 Unemployment Compensation

Let us introduce a positive term b denoting the unemployment compensation in real terms. First, equation (6) becomes:

$$\rho V_U = b + q(v)(V_E - V_U) \tag{19}$$

so that, proceeding as in Section 2.3 we find:

$$\rho V_E = \frac{[\rho + q(v)]w + \lambda b}{\rho + q(v) + \lambda}; \quad \rho V_U = \frac{(\rho + \lambda)b + q(v)w}{\rho + q(v) + \lambda}$$

At a market equilibrium, the wage is:

$$w = \eta \left[\frac{\rho + q(v) + \lambda}{\rho + \lambda + \eta q(v)}\right] A + (1 - \eta)\left[\frac{\rho + \lambda}{\rho + \lambda + \eta q(v)}\right] b$$

which of course reduces to (4) when $b = 0$. Note that, as long as the unemployment compensation is smaller than output per worker, the market equilibrium wage will lay between b and A, corresponding respectively to the case in which $\eta = 0$ and $\eta = 1$. As far as the comparative statics of the wage at a market equilibrium is considered, we have the following results, which generalize Proposition 2.

Proposition 9. Suppose that equation (19) holds with $b > 0$. Then, the wage at a market equilibrium is increasing in the unemployment compensation, increasing in the employment rate, decreasing in the rate of job termination and decreasing in the discount rate if and only if $A > b$.

Proof. See Appendix B.7 □

The requirement needed for this extended model to make sense is simply the unemployment compensation not to exceed output per worker, which is pretty sensible otherwise the whole economy would not be viable at all. The following proposition establishes that the previous results on the direction of technological change carry over in a model which considers unemployment benefits. As for the bias of technological change, the following generalizes Proposition 3).

[18]Note that, because of the fixed-coefficients assumption on output technology, effective inputs are complements in production, and therefore any increase in the labor-augmenting parameter A shifts the demand for labor inwards. Even if we assumed a smooth production function, however, the result of Proposition 8 would not substantially change with an elasticity of substitution smaller than one, which is also supported by most of the empirical evidence on economic growth[19]. An increase in labor-augmenting technologies would determine an increase in the demand for labor only with an elasticity of substitution greater than one.

Proposition 10. Suppose that equation (19) holds, and that $A > b$. Then, the direction of technical change at a market equilibrium determines growth rates of factor-augmenting technologies $\beta(v;\eta,\lambda,\rho,b), \alpha(v;\eta,\lambda,\rho,b)$ such that:[20]

$$\begin{aligned} \partial\beta/\partial v \equiv \beta_v < 0, \quad \beta_\eta < 0, \quad \beta_\lambda > 0, \quad \beta_\rho > 0, \quad \beta_b < 0; \\ \alpha_v > 0, \quad \alpha_\eta > 0, \quad \alpha_\lambda < 0, \quad \alpha_\rho < 0, \quad \alpha_b > 0. \end{aligned}$$

The only result that requires comments is the role of unemployment benefits in the growth and employment path of our economy. Higher values of b increase the value of workers' outside option, therefore pushing the bargaining wage up. As a result, the labor share will increase, and therefore labor productivity will grow via induced innovation bias. Finally, the following proposition holds.

Proposition 11. Suppose that equation (19) holds, and that the unemployment compensation $b > 0$ increases by an amount $db > 0$ while the other parameters in the model stay constant. Then, the corresponding variations in the equilibrium employment rate must be negative.

Proof. See Appendix B.8. □

3.2 Allowing for Workers' Savings

We now relax the assumption that entrepreneurs are the only savers in this economy to show that the main conclusions reached above are robust to this different behavioral scenario.[21] Such an extension is entirely straightforward: it is enough to observe that all the income produced can be consumed or saved (invested). The accumulation constraint appearing in (9) becomes simply $BK = C + \dot{K}$. Hence, the Euler equation for consumption, giving also the growth rate of capital stock, is:

$$\frac{\dot{C}}{C} = B(t) - \rho$$

Equation (16) modifies as follows:

$$\frac{\dot{v}}{v} = \beta(v;\eta,\lambda,\rho) + (B - \rho) - \alpha(v;\eta,\lambda,\rho) - n$$

so that at a steady state technical change is still Harrod-neutral, and $B_{ss} = \alpha(v_{ss};\eta,\lambda,\rho) + n + \rho$. The steady-state Jacobian matrix under the new savings assumption is:

$$J_{ss} = \begin{pmatrix} 0 & \beta_v(\alpha_{ss} + n + \rho) \\ v_{ss} & (\beta_v - \alpha_v)v_{ss} \end{pmatrix}$$

Again, this matrix has negative trace and positive determinant, so that the long-run equilibrium of the system is locally asymptotically stable. As for the qualitative analysis of out-of-equilibrium behavior, the counterclockwise oscillations found above carry over in this different institutional scenario. It is easy to see that the long-run output/capital ratio varies directly with the employment rate, with population growth, and with the discount rate. Finally, Proposition 8 continues to hold.

[20]The proof of this proposition goes along the lines of Appendix B.2, and it is left as an exercise.

[21]For those familiar with history of thought, this extension amounts to chip into the 'Pasinetti *vs* anti-Pasinetti' debate of the 1960s.

3.3 Capital-Labor Substitution

The final exercise is to consider, instead of a kinked technology, a production function characterized by smooth isoquants. Let $Y = F(AL, BK)$, where F is linearly homogeneous in inputs measured in their respective efficiency units, and has constant elasticity of substitution. Defining $x \equiv \frac{AL}{BK}$, constant returns to scale imply $Y = BKF\left(\frac{AL}{BK}, 1\right) \equiv BKf(x)$. National income accounting requires $Y = wL + rK$. Thus, the profit rate, measured as before as profits per unit of capital, can be written as:[22]

$$r = B\left[f(x) - \frac{w}{A}x\right]$$

In Appendix B.9, I show that solving the bargaining problem under the new specification yields a wage satisfying:

$$w = \eta Af(x)/x + (1-\eta)\rho z$$

and a firm-level direction of technical change that fulfills:

$$-g_\beta = \frac{f(x) - xf'(x)}{xf'(x)}$$

Observe that, since $BKf(x) = Y = ALf(x)/x$, the bargaining wage gives again a rule that linearly combines output per worker and the per-period wage-floor, given by the discount rate times the value of the present discounted value of being out of the employment pool.

Once the bargaining wage is determined, profit-maximizing firms will demand labor so as to equalize its marginal product with the wage. Thus, $xf'(x)/f(x)$ will be the labor share in firm's output so that, again, firm-level direction of technical change and corresponding growth rates of factor-augmenting technologies will respond to the ratio of factor shares.[23] On this regard, it is worth to emphasize that, as it is standard in models of induced innovation ([Drandakis and Phelps (1965)], [Nordhaus (1967)]) even in the case of smooth capital-labor substitution it will be the shape of the IPF, and not the production function, which will determine the shares of productive factors in production.

Also, when the outside option V_U is determined as in Section 2.3, we can define a market equilibrium as in Section 2.5. Solutions for the equilibrium wage, profit rate, and direction of technical change are provided in Appendix B.9.

Differently from the fixed-coefficients case, in which firm's labor demand at each period of production was vertical at BK/A, the input-substitution model determines a downward-sloping labor demand curve. The profit-maximization condition $w = Af'(x)$ can be solved for x to determine how does the ratio of effective inputs behave in response to the real wage. Let $\chi(\omega) \equiv f'^{-1}(\omega)$ denote such demand for labor, decreasing in its argument. Then, the employment rate satisfies $v = \frac{BK}{AN}\chi(\omega)$. At a market equilibrium, equation (12) holds, and therefore the following equalities are satisfied: $\frac{w}{A} = \frac{xf'(x)}{f(x)} \equiv \omega(v)$, with $\omega_v > 0$ by what established in Proposition 2. The missing piece to characterize the equilibrium dynamics of the economy is the solution of the savings problem. Assume for simplicity that both entrepreneurs and workers save. Solving the optimal control problem of allocating consumption and savings given the accumulation constraint, we find $\frac{\dot{C}}{C} = B\left[f(x) - xf'(x)\right] - \rho = \frac{\dot{K}}{K}$,

[22]It is easy to verify that, when the elasticity of substitution equals zero, that is in the Leontief case, imposing $x = 1$ also implies $f(1) = 1$, so that $r = B[1 - w/A]$.

[23]It is somewhat tedious, but definitely not hard to show that, once marginal product of labor is equated to the bargaining wage, equation (14) holds.

using which we can finally write down the dynamic equation for the employment rate:

$$\dot{v} = \frac{1}{1-\theta_v}\left\{\beta(v;\eta,\lambda,\rho) + Bf(x)\left[1-\frac{xf'(x)}{f(x)}\right] - \rho - \alpha(v;\eta,\lambda,\rho) - n\right\}v \qquad (20)$$

where $\theta_v \equiv \frac{BK}{AN}\chi_\omega\omega_v$. This equation, combined with (15), gives the dynamical system that characterizes the economy. A long run equilibrium involves again a Harrod-neutral profile of technical change, and a steady level of output/capital ratio $B_{ss} = \frac{\alpha(\cdot)+\rho+n}{1-\omega(v_{ss})}$.[24] As $-\theta_v > 0$, there are no substantial changes in the transition dynamics relative to Section 2.8.

4 Conclusion

In this paper, I introduced axiomatic bargaining *à la* [Nash (1950)] as a mechanism of determination of productivity gains and wages into a one-sector model of growth and distribution where firms face trade-offs in introducing factor-augmenting technologies. Given such trade-offs, captured by the postulated existence of an IPF, firm-level bargaining results in induced bias in technology that carries over to the market equilibrium of the economy. Nash bargaining has the advantage of being simple, and therefore widely familiar as a modeling tool for wage-setting in models of the labor markets. Although the Nash bargaining solution arises from the maximization of bargainers' *joint* gains, and therefore appears to have intrinsic cooperative features, it has been shown that the outcome of more conflictual bargaining games such as the alternative offer model studied by [Rubinstein (1982)] closely reproduces the Nash solution.[25] Finally, the Nash solution is widely used in standard models of the labor market (see for instance, [Pissarides (2001)], Chapter 1), and in this particular case can be understood as any bargaining process that splits the difference between ceilings given by firms revenues net of material input costs and an economy-wide wage floor ([Julius (2009)]). Such floor in this paper is endogenously determined considering the present value of members of the labor force who are out of the employment pool.

In this model, the labor market equilibrium occurs at the intersection between a wage curve, which positively relates the level of real wages to the employment rate, and a labor demand curve, which is vertical in the non-substitution case, and downward sloping in the capital-labor substitution case discussed in Section 3.3. Also, the economy resulting in equilibrium from decision-making on wages and innovation, production, and savings, is completely described by a dynamical system in the employment rate and capital productivity. The system evolves so as to achieve a Harrod-neutral path of technical progress, and a constant rate of employment of labor. As this is a simple corn model, the role of equilibrium unemployment is not to prevent accelerating inflation, but to ensure the constancy of factor shares in output.[26] Other than constant factor shares, other features of the model are a positive

[24] As a consistency check, observe that in the Leontief case $\xi = 0 = f'(x), x = 1 = f(x)$, so that B_{ss} reduces to the sum $\alpha(\cdot) + n + \rho$ as in Section 3.2.

[25] A concise exposition appears in [Bowles (2004)]

[26] [Goodwin (1967)] would phrase this sentence by saying that the role of the equilibrium unemployment is to put the distributional conflict between capital and labor to rest. In this interpretation, equilibrium unemployment closely resembles a notion of 'reserve army' of labor, that shrinks or expands in response to the interaction between accumulation and technical change. See also [Bowles (1985)].

growth rate of labor productivity, and a positive rate of capital accumulation together with stationary output/capital ratio, so that the present framework matches the [Kaldor (1961)] facts. Finally, the dynamics of the model display cyclical behavior which replicates qualitatively the available evidence on capital productivity and employment rate for post-war United States, and can be simulated numerically. Even though in the baseline model I assumed profit-earners as the only saving class in the economy, the conclusions of the paper are robust when workers are allowed to save, as shown in Section 3.2, and when input substitution is present, as shown in Section 3.3.

Institutional change, in the form of variations in the parameter representing workers' bargaining power, has a considerable impact on economic growth, income distribution, and unemployment: I showed that a *coeteris paribus* increase in η produces a higher wave of labor-augmenting technical progress, while reducing the equilibrium employment rate. This framework is also able to emphasize one policy channel through which the general equilibrium pattern of growth and distribution can be influenced: an increase in the unemployment compensation results in higher labor productivity growth, at the price of higher equilibrium unemployment. To the best of my knowledge, such institutional and policy features have neither been analyzed in the early ([Kennedy (1964)], [Drandakis and Phelps (1965)], [Kamien and Schwartz (1969)], [Nordhaus (1967)]) nor in the more recent literature on the direction of technical change ([Acemoglu (2003)]). Further, the effect of labor market institutions and policies can be evaluated in general equilibrium,and such type of exercise is also novel in the literature stemming from [Goodwin (1967)].

While basically analyzing the same kind of framework, the previous contributions on growth cycles with endogenous technical change ([Shah and Desai (1981)], [van der Ploeg (1987)], [Julius (2005)]) appealed to institutional arguments in assuming an exogenously determined wage, which is fully within the realm of economic theories rooted in the Classical tradition. In this respect, the present analysis can be viewed as an attempt to 'dig deeper' into modeling explicitly the economic institutions that are responsible for wage setting in the economy. In endogenizing wages, thus linking them to labor productivity and the employment rate, the present model produces considerable simplification in the dynamical description of the economy relative to the previous literature on growth and distribution cycles, without either altering the typical findings of Harrod-neutrality and constant input shares that characterize long-run patterns of technical change and income distribution, nor obliterating the features of conflict over income distribution on which that literature focuses, and which are deemed to be at the source of the observed macroeconomic behavior under investigation.

Among the simplifying assumptions made throughout this paper, an important one is that of a full utilization of capacity. In relying on this assumption, the benchmark model presented here is not really a model of short-run but rather of medium to long-run fluctuations, and cannot deal neither with capital stock over- or underutilization, nor with the role of policy making in influencing the business cycle. Relaxing this hypothesis and allowing profit-maximizing firms to choose the rate of capacity utilization would enable to address in more depth the interaction between cycles and trends in this framework. In particular, a question that stands out is whether rising the unemployment compensation is harmful for employment when capacity is underutilized. On the other hand, although the framework proposed is able to replicate some features of the economy under consideration, more work needs to be done in testing empirically several of the propositions stated above. Finally, I aggregated across workers in modeling only one type of labor entering production of output in the economy. Considering heterogeneous workers, and consequently technical change directed toward different kinds of labor as well as capital, will extend the relevance of the model in addressing supply-side questions about wage inequality.

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A Solution of the Bargaining Problem

A.1 Proof of Proposition 1

Form the current-value Hamiltonian:

$$H_B = \eta \log(w - \rho z) + (1-\eta)\log\left(B\left(1 - \frac{w}{A}\right)\right) + \gamma_1 \beta B + \gamma_2 g(\beta) A \tag{21}$$

and let harmlessly $\gamma_1 \equiv p_1 e^{-\bar{\beta}t}, \gamma_2 \equiv p_1 e^{-g(\bar{\beta})t}$, $\bar{\beta}$ being the solution value for β to be determined. Necessary conditions for maximization are:

$$\frac{\partial H_B}{\partial w} = \frac{\eta}{w - \rho z} - \frac{1-\eta}{A}\left(\frac{1}{1 - \frac{w}{A}}\right) = 0 \tag{22}$$

$$\frac{\partial H_B}{\partial \beta} = p_1 e^{-\bar{\beta}t} B + p_2 e^{-g(\bar{\beta})t} g_\beta A = 0 \tag{23}$$

Solving (22) for w gives (4). Also, a necessary condition for optimality of the solution is the existence of continuous function $p_1(t), p_2(t)$ satisfying:

$$(\rho + \bar{\beta}) p_1 e^{-\bar{\beta}t} - e^{-\bar{\beta}t} \dot{p}_1 = \frac{1-\eta}{B} + \beta p_1 e^{-\bar{\beta}t} \tag{24}$$

$$[\rho + g(\bar{\beta})] p_2 e^{-g(\bar{\beta})t} - e^{-g(\bar{\beta})t} \dot{p}_2 = \frac{1-\eta}{A} \left(\frac{\omega}{1-\omega} \right) + g(\beta) p_2 e^{-g(\bar{\beta})t} \tag{25}$$

where the LHS of the two equations are equal to the familiar $\rho\gamma_i - \dot{\gamma}_i, i = 1, 2$, and $\omega \equiv w/A$ is the share of wages in firm's output. Also, the following transversality conditions must hold at an optimal path:

$$\lim_{t \to \infty} e^{-\bar{\beta}t} p_1(t) = \lim_{t \to \infty} e^{-g(\bar{\beta})t} p_2(t) = 0 \tag{26}$$

Because of concavity of the objective function and convexity of the constraint set, conditions (22)-(25) are also sufficient for a maximum of problem (3).
A stationary solution for (24) and (25) where $\dot{p}_1 = \dot{p}_2 = 0, \beta = \bar{\beta}$, satisfies:

$$e^{-\bar{\beta}t} p_1 B = \frac{1-\eta}{\rho}$$

$$e^{-g(\bar{\beta})t} p_2 A = \frac{1-\eta}{\rho} \left(\frac{\omega}{1-\omega} \right)$$

Divide the first equation by the second and use (23) to get (5).

A.2 Behavior of Rates of Factor-Augmentations at the Firm Level

In order to get started in studying the behavior of factor-augmentations, rewrite equation (5) as:

$$G(\beta, \omega; \rho) \equiv \frac{\omega}{1-\omega} + g_\beta = 0$$

Such an equation yields an implicit function $\beta(\omega, \rho)$ whose partial derivatives fulfill standard properties. We have that:

$$G_\beta = g_{\beta\beta} < 0$$

Clearly,

$$G_\omega = -\frac{1}{\omega^2}$$

Therefore,

$$\text{sign } (\beta_\omega) = \text{sign } \left(-\frac{G_\omega}{G_\beta} \right)$$

so that $\beta_\omega < 0$. Our assumption about the IPF imply $\alpha_\omega > 0$.

B Proofs of Various Propositions

B.1 Proof of Proposition 2

We have:

$$\frac{\partial w}{\partial v} = \eta(1-\eta)\frac{q_v(\rho+\lambda)}{[\rho+\lambda+\eta q(v)]^2}A > 0$$

$$\frac{\partial w}{\partial \eta} = \frac{\rho+\lambda+q(v)}{[\rho+\lambda+\eta q(v)]^2}(\rho+\lambda)A > 0$$

Furthermore,

$$\frac{\partial w}{\partial \lambda} = \frac{\eta(\eta-1)}{[\rho+\lambda+\eta q(v)]^2}A < 0$$

Finally,

$$\frac{\partial w}{\partial \rho} = \frac{\eta(\eta-1)q(v)}{[\rho+\lambda+\eta q(v)]^2}A < 0$$

B.2 Proof of Proposition 3

Using the same approach as in Appendix A.2, we can construct a function $\Gamma(\beta, v; \rho, \lambda, \eta) = 0$ such that:

$$\Gamma_\beta = g_{\beta\beta} < 0$$

Since

$$\Gamma_v = -\frac{1-\eta}{\eta}\left[\frac{q_v(\rho+\lambda)}{(\rho+q(v)+\lambda)^2}\right] < 0$$

we have that:

$$\text{sign } \beta_v = \text{sign }\left(-\frac{\Gamma_v}{\Gamma_\beta}\right) < 0$$

and our assumptions about the IPF ensure that $\alpha_v > 0$. Also,

$$\Gamma_\eta = -\frac{1}{\eta^2}\left[\frac{\rho+\lambda}{\rho+q(v)+\lambda}\right] < 0$$

Hence,

$$\text{sign } \beta_\eta = \text{sign }\left(-\frac{\Gamma_\eta}{\Gamma_\beta}\right) < 0$$

and $\alpha_\eta > 0$. Further,

$$\Gamma_\lambda = \frac{1-\eta}{\eta}\left[\frac{q(v)}{(\rho+q(v)+\lambda)^2}\right] > 0$$

so that

$$\text{sign } \beta_\lambda = \text{sign }\left(-\frac{\Gamma_\lambda}{\Gamma_\beta}\right) > 0$$

Therefore, $\alpha_\lambda < 0$. Finally,

$$\Gamma_\rho = \frac{1-\eta}{\eta}\frac{q(v)}{\rho+q(v)+\lambda} > 0$$

so that

$$\text{sign } \beta_\rho = \text{sign } \left(-\frac{\Gamma_\rho}{\Gamma_\beta} \right) > 0$$

so that $\alpha_\rho < 0$.

B.3 Proof of Proposition 4

Existence and uniqueness of a long-run equilibrium are ensured by the fact that the functions appearing in the dynamical system (15), (16) are both $\mathcal{C}^1$ in their arguments (Hirsch, Smale and Devaney [Hirsch, Smale and Devaney (2004)], p.144). A glance at equations (17) and (18) reveals that the long-run equilibrium is of the purely Harrod-neutral type, with zero capital-productivity growth and a growth rate of labor-augmenting technologies equal to $\alpha_{ss} = g(0; \cdot)$. The equilibrium employment rate is constant and equal to $\beta^{-1}(0; \cdot)$. Finally, once the long-run employment rate of the model is achieved, the wage share will be constant, and given by $\eta \frac{\rho+\lambda+q(v_{ss}}{\rho+\lambda+\eta q(v_{ss})}$.

B.4 Proof of Proposition 5

Linearize the above system around its non-trivial rest point to obtain the Jacobian matrix:

$$J_{\{Bss,vss\}} = \begin{pmatrix} 0 & \beta_v \left(\frac{\rho+\alpha+n}{1-\eta}\right) \left[\frac{\rho+\lambda+\eta q(v_{ss})}{\rho+\lambda}\right] \\ (1-\eta)\frac{\rho+\lambda}{\rho+\lambda+\eta q(v_{ss})} & \left(\beta_v - \frac{\eta(1-\eta)}{\rho+\lambda+\eta q(v_{ss})}(\rho+\lambda)q_v - \alpha_v\right) v_{ss} \end{pmatrix}$$

This matrix has a negative trace, as $\beta_v < 0, -q_v < 0, -\alpha_v < 0$, and a positive determinant. Therefore, its two eigenvalues have real parts that are of the same sign and sum up to a negative number, and this proves the claim.

B.5 Proof of Proposition 7

We have that:

$$\frac{\partial B_{ss}}{\partial n} = \frac{1}{1-\eta} \left[\frac{\rho+\lambda+\eta q(v_{ss})}{\rho+\lambda}\right] > 0$$

Further,

$$\frac{\partial B_{ss}}{\partial v} = \frac{\alpha_v}{1-\eta} \left[\frac{\rho+\lambda+\eta q(v_{ss})}{\rho+\lambda}\right] + \frac{\eta q(v_{ss})}{\rho+\lambda} \left(\frac{\rho+\alpha(\cdot)+n}{1-\eta}\right) > 0$$

Also,

$$\frac{\partial B_{ss}}{\partial \eta} = \left[\frac{\alpha_\eta}{1-\eta} + \frac{\alpha(\cdot)+n+\rho}{(1-\eta)^2}\right] \left[\frac{\rho+\lambda+\eta q(v_{ss})}{\rho+\lambda}\right] + \frac{q(v_{ss})}{\rho+\lambda} \left(\frac{\rho+\alpha(\cdot)+n}{1-\eta}\right) < 0$$

Finally,

$$\frac{\partial B_{ss}}{\partial \lambda} = \frac{\alpha_\lambda}{1-\eta} \left[\frac{\rho+\lambda+\eta q(v_{ss})}{\rho+\lambda}\right] - \frac{\eta q(v_{ss})}{(\rho+\lambda)^2} \frac{\rho+\alpha(\cdot)+n}{1-\eta} < 0$$

B.6 Proof of Proposition 8

Since $\beta(v_{ss}; \eta, \lambda, \rho) = 0$, we must have that $\beta_{v_{ss}} dv_{ss} + \beta_\eta d\eta = 0$, from which:

$$\frac{dv_{ss}}{d\eta} = -\frac{\beta_\eta}{\beta_{v_{ss}}}$$

As both $\beta_\eta, \beta_{v_{ss}} < 0$, the claim is proved.

B.7 Proof of Proposition 9

We have:

$$\frac{\partial w}{\partial b} = (1-\eta)\frac{\rho+\lambda}{\rho+q(v)+\lambda} > 0$$

$$\frac{\partial w}{\partial \eta} = (A-b)\frac{(\rho+\lambda)[\rho+\lambda+q(v)]}{[\rho+\lambda+q(v)]^2} > 0 \iff A > b$$

$$\frac{\partial w}{\partial v} = \eta(1-\eta)\frac{(A-b)q_v(\rho+\lambda)}{[\rho+\lambda+q(v)]^2} > 0 \iff A > b$$

$$\frac{\partial w}{\partial \lambda} = \eta^{-1}\frac{\partial w}{\partial \rho} = \eta(\eta-1)\frac{(A-b)q(v)}{[\rho+\lambda+q(v)]^2} < 0 \iff A > b$$

B.8 Proof of Proposition 11

Since $\beta_{v_{ss}} dv_{ss} + \beta_b db = 0$,

$$\frac{dv_{ss}}{db} = -\frac{\beta_b}{\beta_{v_{ss}}}$$

As both $\beta_b, \beta_{v_{ss}} < 0$, the claim is proved.

B.9 Bargaining in the Model with Substitution

The statement of the bargaining problem with substitution is similar to the no-substitution case, and therefore omitted. The current-value Hamiltonian H_S associated with such problem is:

$$H_S = \eta \log(w - \rho V_U) + (1-\eta)\log\left[f(x) - \frac{w}{A}x\right] + \gamma_1 \beta B + \gamma_2 g(\beta) A$$

where γ_1, γ_2 are defined as in Appendix A. Setting $\partial H_S/\partial w = 0$ yields:

$$w = \eta A f(x)/x + (1-\eta)\rho V_U \tag{27}$$

and setting $\partial H_S/\partial \beta = 0$ gives (23). Proceeding as in the no-substitution case, a stationary solution in which the adjoint variables γ_1, γ_2 don't change over time satisfies:

$$\rho B p_1 e^{-\bar{\beta}t} = (1-\eta)\left[\frac{f(x) - xf'(x)}{f(x) - xw/A}\right] \tag{28}$$

$$\rho A p_2 e^{-g(\bar{\beta})t} = (1-\eta)\left[\frac{xf'(x)}{f(x) - xw/A}\right] \tag{29}$$

Dividing (28) by (29) and using (23), one obtains $-g_\beta = \frac{f(x)-xf'(x)}{xf'(x)}$.

Once a bargaining deal is struck, profit-maximizing firms will demand labor up to the point where its marginal product $Af'(x)$ is equal to the wage. The wage share in firm's output is then $wL/Y = xf'(x)/f(x)$, so that the firm-level optimal direction of technical change satisfies (5). Once the outside option V_U is determined through (8), we can define a market equilibrium as in section 2.5, at which real wage and profit rate are:

$$w = \eta \left[\frac{\rho+q(v)+\lambda}{\rho+\lambda+\eta q(v)}\right] A\frac{f(x)}{x}; \quad r = (1-\eta)Bf(x)\left[\frac{\rho+\lambda}{\rho+\lambda+\eta q(v)}\right] \tag{30}$$

C Numerical Simulations

For this exercise, we assume the following parameter calibration: $n = .02$, $\alpha(v_{ss}, \cdot) = .02$, which match first moments in population growth and labor productivity growth in the US. As far as rates of job creation and job destruction are concerned, Davis, Haltiwanger and Shuh [Davis *et al.* (1997)] suggest to calibrate $\lambda = .113, q(v_{ss}) = .092$. The plot in Figure 2 points toward specifying $v_{ss} = .94$. If we assume a linear function $q(v)q_v v = \gamma^{-1}v^{\gamma}$, then we can internally calibrate q_v. Finally, we want to use the national US average for the unionization rate as a proxy for η. The current value is 12.1%, although this figure has been steadily decreasing in recent decades. I run simulations corresponding to $\eta = .\{0.1, 0.2, 0.3\}$. The following table summarizes the parameter values used for the simulation runs displayed in Figures 4 and 5.

D Tables and Figures

Parameter	Moment to Match	Source
n	.02	standard
ρ	.016	standard
$\alpha(v_{ss}, \cdot)$	.02	standard
λ	.103	[Davis *et al.* (1997)]
$q(v_{ss})$	.091	[Davis *et al.* (1997)]
q_v	0.0968	internally
η	$\{.1, .2, .3\}$	BLS

Table 1: Parameter Calibration

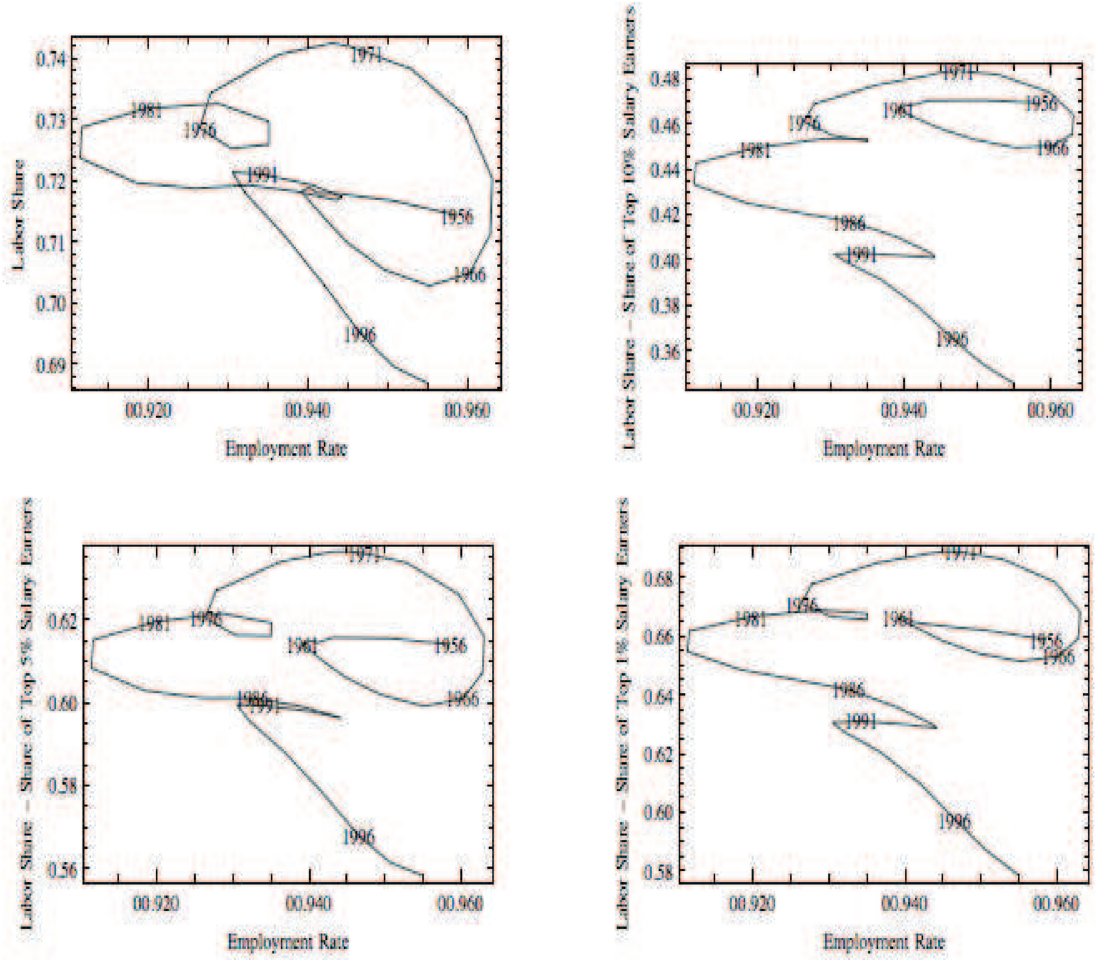

Figure 1: Cycles in employment rate and the wage share in the US (1956-1998). Sources: Piketty and Saez (2003) (Labor Share), annual average of BLS monthly data (Employment Rate).

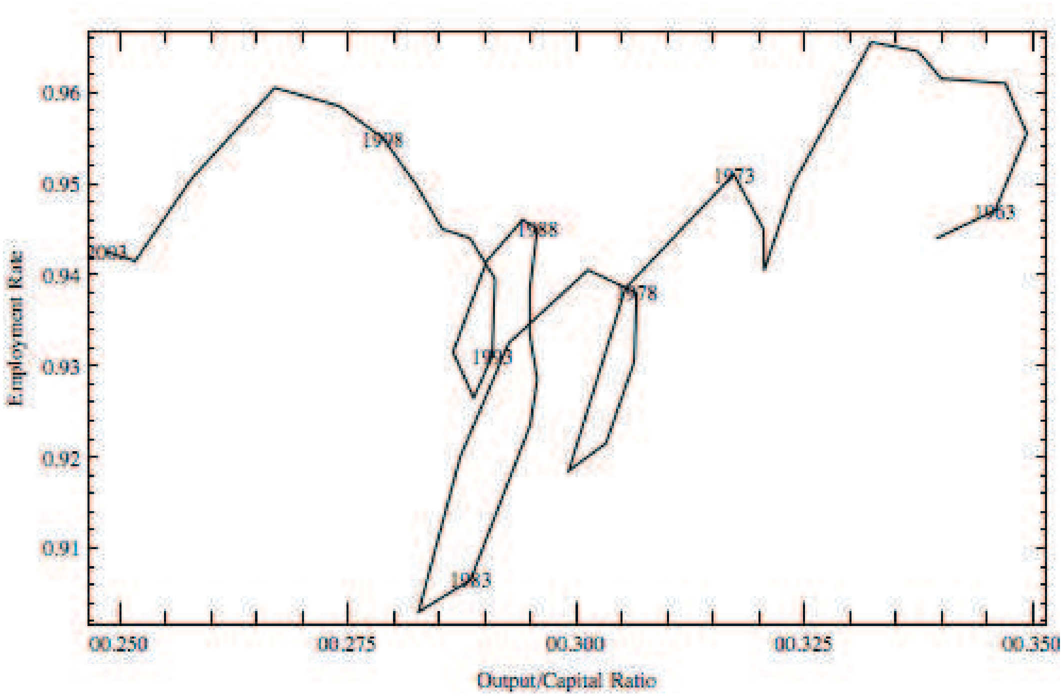

Figure 2: Counterclockwise growth cycles in the phase space (B, v) in the US (1963-2003). Source: Extended Penn World Table (Output/Capital Ratio), BLS (Employment Rate).

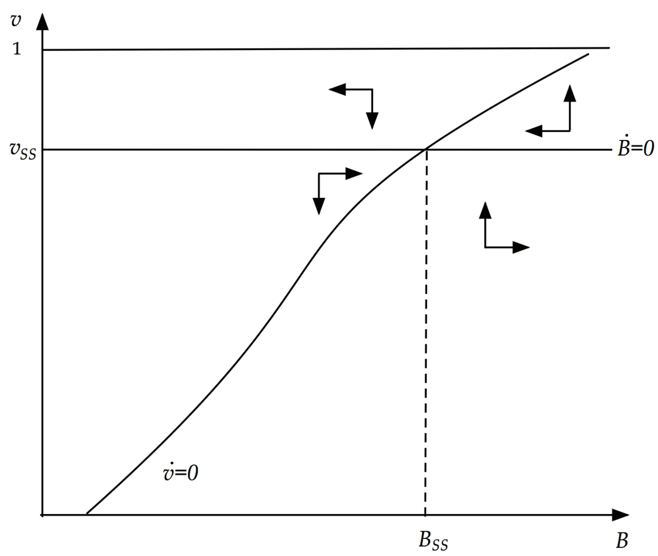

Figure 3: Phase Diagram for the Dynamical System (15), (16).

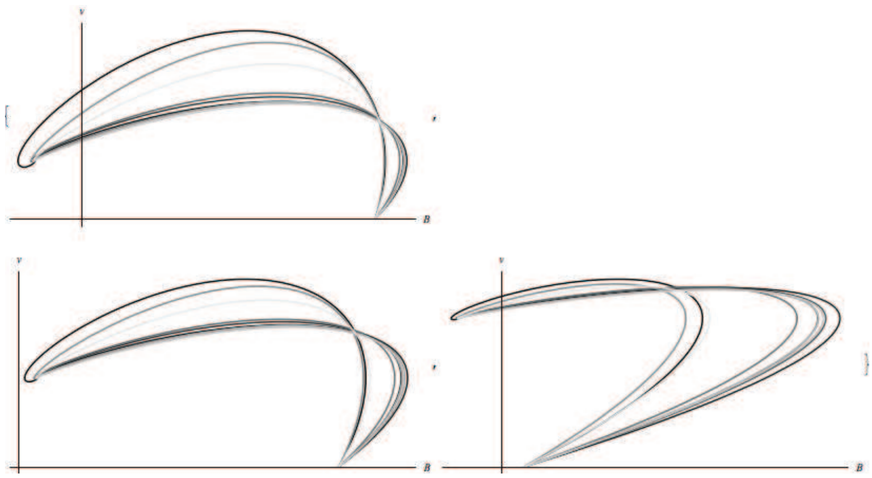

Figure 4: Numerical Simulations for the Dynamical System (15), (16), for varying (α_v, β_v) (10 different trajectories). Each panel corresponds to a different value of $\eta = \{0.1, 0.2, 0.3\}$ respectively.

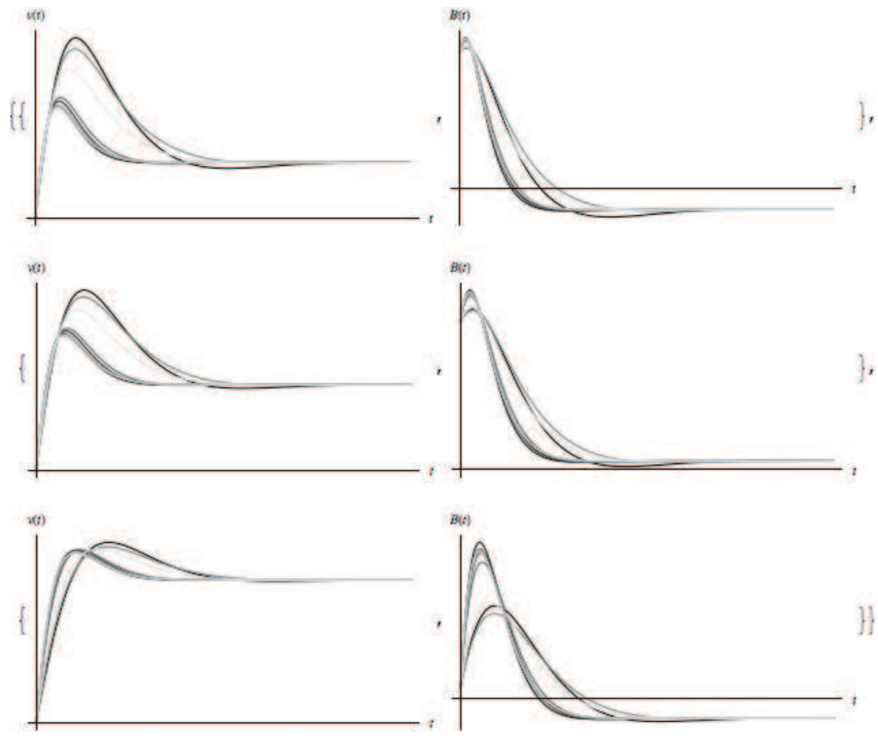

Figure 5: Numerical Simulations for employment rate and output/capital ratio over time, for varying (α_v, β_v) (10 different trajectories). Each pair of plots $\{v(t), B(t)\}$ corresponds to a different value of $\eta = \{0.1, 0.2, 0.3\}$ respectively.